A micro-econometric analysis of care for the dependent elderly living at home in France: How do formal and informal care interact?

Working Paper*

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Abstract

This paper focuses on the trade-off between formal care and informal care for disabled elderly people living in the community. We propose a simple microeconomic model which describes the simultaneous decision process between potential informal caregivers and disabled elderly person living at home. The model predicts a strict substitution between the two kinds of care. To test the substitution effect and assess its intensity, we estimate a recursive simultaneous equation model linked by a copula function, using data from the French “Handicap Santé Ménages” survey (2008). In contrast with previous literature, the estimated model allows to simultaneously identify both potential causality directions. Thanks to the copula function, strong assumptions about a linear link between the two residuals of both equations do not have to be made. Our results confirm that informal and formal home care are globally substitutes. Nevertheless, the predicted increase of formal care following a reduction of one hour of informal care remains very small. Informal care supply is also weakly sensitive to variations of formal care hours: The substitution effect is high only for individuals receiving much more informal than formal care.

JEL classification: I11, I12, J22

Keywords: Long-term Care, Informal Care, Formal Care, Simultaneous Equations Model, Copula Function

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1 Introduction

The goal of this project is to study the relation between professional care and informal care for elderly people living at home. By informal care we mean here care brought by a non-professional caregiver. This may be a family member, some friend, or some other acquaintance of the care dependent person. In the following we will list several arguments which justify our interest for this problem.

1. Ask for care may increase in the next few years
In France, as in most developed countries, population aging is an ever already phenomenon which will continue for the next decades. According to the French National Institute of Statistics (INSEE), the proportion of persons aged 65 and older has already reached 13.9% in 1990 and 16.6% in 2008. The preliminary estimates for January 2011 are 16.9%. In 2025, the elderly population will represent 21.7% of the French population and this figure can even raise to 26.2% in 2050. This inescapable aging could have many consequences in terms of public health. Even if it is now possible to age healthy, population ageing could increase the number of elderly individuals in a situation of dependency. The dependency which is at issue here could be defined by the situation where an elderly person cannot carry out its duties or accomplish daily living tasks without the assistance of another person, because of an anatomic, physiologic or affective disorder. It concerns around 1.2 million people\(^1\) in 2010. This represents 7% of the population aged 60 and over.

2. Caring for elderly dependent people is costly
Two kinds of dependency can be distinguished in the case of elderly people: physical dependency and mental dependency. The physically disabled elderly persons who are not autonomous belong to the group of physically dependent people. Those affected by a cognitive disorder who cannot accomplish alone daily living tasks anymore belong to the group of mentally dependent people. Some people are at once physical and mental dependent.
Alzheimer’s disease is the main cause of dementia and mental dependency for elderly people. It provokes the progressive and irreversible loss of mental abilities, in particular of the memory. Alzheimer’s disease is very different from other types of dependency, in particular physical dependencies. It requires a stronger support from caregivers and an adapted and constant medical supervision. This illness can also affect younger people but most of the cases are 65 years old or more and even 75 or older. Diagnoses are often made very late. These numerous cases represent a real public health problem. Among those aged 75 and older in France, the estimated prevalence of Alzheimer’s disease is 13.2% for men and 20.5% for women in PAQUID (c.f. Ramaroson et al. (2003) [19]), a french study published in 2003.

What is the range of alternatives for elderly dependent individuals to be helped, in particular for those affected by Alzheimer’s disease? An elderly dependent person can be helped by its family or by professional caregivers, at home or in institutions. In every case being helped has a cost, which can be very high. This high cost is often borne by the elderly person itself or by its

\(^1\)1174 thousands individuals on the 30th. of June 2010, estimates based on the number of persons receiving the “ Allocation Personnalise d’Autonomie ” (APA).
family. However the social security and local collectivities can participate. The French benefit for autonomy (APA), an allowances proposed by the French national fund for retirement provision (CNAV), social help for housing and housing benefit paid by the French family allowance fund (CNAF) are the most important part of the available allowances dedicated to elderly persons. Individuals affected by Alzheimer’s disease do not benefit from specific public benefits, apart from those mentioned before dedicated to elderly dependent people. However, the disease is recognized as a “chronic illness” (ALD). That is why medical care provided at home for individuals affected by Alzheimer’s disease is completely reimbursed by the French state health insurance office.

An important part of public finance is devoted to dependency and long-term care. Through the APA, regional councils paid 5.240 billion Euro to the 1,174,000 people who receive it in 2010. Tax reliefs are sometimes offered to informal caregivers to reduce the cost of caring an elderly dependent person. The amount of these reliefs was evaluated at 250 million Euro in 2009.

More generally speaking, public expenditure devoted to dependency is estimated at 24.7 billion Euro in 2010, which represents 1.3% of the French Gross Domestic Product. In 2008, the amount of this public support reached 21.4 billion Euro. Between 2008 and 2010, it increased by 15.4%.

Figure 1: Public expenditure devoted to long-term care and dependency in 2010 in France

Reading: In 2010, the National Fund for Solidarity and Autonomy (CNSA) spent 2.7 billion Euro devoted to help dependent people.

3. Several sociodemographic evolutions question the future role of family in caring for elderly dependent persons
Home health care is an alternative to nursing home care. Home health care can be formal, which means that it is brought by professional caregivers, and/or informal, which means that it is brought by one or several members of the circle of acquaintance. Informal caregivers are often family members, partners or children. Home health care is often chosen by elderly dependent people and their family. Most of the European countries encourage elderly dependent people to
stay at home because it seems to reduce public expenditure. Given the actual economic situation in the world, governments will certainly count on families to maintain an adequate level of care for elderly dependent persons.

The biggest part of informal care brought by family members or close relatives consists in helping for daily living tasks or in a moral support. However they can sometimes help their elderly dependent relative financially. Informal care is more diversified than formal care brought by professionals, which concerns more specifically personal care or domestic chores. That are reasons why informal caregivers play a major role in the help of elderly dependent people.

But in the next few years, the number of informal caregivers is expected to go down. The increase of active seniors will make children less available to help their elderly dependent parent. The smaller number of siblings and the growing physical distance between parents and children are other explanations to this phenomenon. Presence and availability of informal caregivers will more and more impact the choice of elderly dependent persons to stay at home or not. They also have an influence on the type of care - formal or informal - received, and on the “cost” of the offered informal care. In terms of opportunity cost, the cost of informal care changes with the situation of the informal caregiver on the labour market for example.

2 Research questions and literature review

In the context of economic problems, where the number of elderly dependent people should increase and where informal caregivers should lack, it seems to be urgent to understand the organization of care for elderly dependent persons living at home. A better understanding could allow to anticipate and evaluate the effect of future public policies devoted to act on this care structure and to reduce its cost.

We put ourselves in a standard microeconomic analytical framework, where we consider formal care provided at home and informal care as two factors of production. We would like to study simultaneously the decisions taken by the elderly dependent persons about the quantity of formal care and informal care received at home. More than simply analyzing their determinants, we would like to understand how these two types of care are related. In particular, we would like to answer to the question of their possible complementarity or substitutability: How would the quantity of one kind of care evolve if the quantity of the other increases?

2.1 Literature review

Although studies have been published for about thirty years, there has not yet been a decisive answer to the following question: Are informal and formal care complements or substitutes? Early authors like McAuley and Arling (1984) [16] and Kemper (1992) [13] could only establish that the grade of the dependency influences the number of care hours provided, but they did not include formal care as an explanatory variable of informal care and vice versa. We can imagine three broad categories of possible answers to our main question.
1. Formal care and informal care are complements

2. Formal care and informal care are substitutes

3. Informal care is a substitute of certain types of formal care and a complement to others

There seems to be a widespread agreement that hours of informal and formal care are negatively correlated, at least to some extent and specific services. To the knowledge of the authors, no article concludes with a real complementary between the two types of care. On the contrary, many authors propose a real substitution effect and belong to the second group. The first article which goes into this direction is Greene (1983) [9]. Using a formal model which includes measures to account for endogeneity, he finds a “substantial tendency for substitution”. In the last ten years, several articles have been published and seem to support Greene’s finding. Van Houtven and Norton published two articles in 2004 [21] and 2008 [22]. In the first paper, they find that informal care is a substitute to formal long-term care (nursing home entry and home based care) as well as health care (hospital and doctor visits). Their second article goes into the same direction but looks more on policy implications. The findings are comparable and include also that “informal care by children reduces Medicare long-term care and inpatient expenditures of single elderly”. Lo Sasso and Johnson (2002) [14] and Charles and Sevak (2005) [5] conclude that informal care reduces the risk of entering into a nursing home and that it can thus be seen as a substitute to institution. Holly et al. (2010) [11] finally support the idea that the two types of care are substitutes in their recent working paper and emphasize on the importance for a good control of the endogeneity and the different institutional settings when comparing different countries.

The third group which confirms that informal care can be seen as a substitute for some but not all formal care services has also found some advocates lately. Bolin et al. (2008) [3] for example find that informal care acts as a substitute for formal home care but that it also increases doctor and hospital visits and is thus a complement to health care services. Jiménez et al. (2011) [12] find on the contrary that informal care is a substitute for formal nursing home care but also a complement to formal home care.

So while there is no final answer to this question we can see a strong tendency towards them being substitutes. Nevertheless we will add to the existing literature in the following way:

1. New evidence for France

2. Endogeneity and double recursivity taken into account

3. Distinction between physical and mental dependency

ad 1) Until now there is only little evidence available for France. The few articles who have considered France (for example Holly et al. (2010) [11] and Bolin et al. (2008) [3]) only as one country among others and did not model specifically the French case. It would be a mistake to consider the results obtained with American data as valid for the French situation. French care institutions are very different as American ones or as the ones of most of the European countries. In the United States, the public supply of formal home care is mainly
restricted to disabled persons through Medicare, or is means-tested via Medi-
caid. The private market of formal home care is well developed but also very
costly for elderly dependent people. In the United States, an elderly dependent
person may consider the probability of benefiting from refunded formal home
care before deciding the amount of informal care received. In France as in many
European countries, the co-payment for an elderly dependent person receiving
formal home care is lower thanks to the French benefit for autonomy (APA)
in particular. This benefit is not means-tested but the patient’s contribution
towards medical costs increases with the income : High incomes receive a lower
amount. However, the APA can help a large part of the elderly dependent peo-
tle to bear the costs associated with formal home care. Moreover medical care
associated with Alzheimer’s disease, recognized as a “chronic illness” (ALD)
and main factor of mental dependency, are entirely reimbursed by social secu-
rity. Only the non-medical part of home care has to be paid by these dependent
persons, such as the homemaker, and the APA can help to cover it. Finally,
benefiting from public allowances for formal home care seems to be less difficult
in France than in the USA for a large number of elderly dependent persons.
They are related to needs or degree of dependency but are exclusively devoted
to low incomes. Consequently, it appears difficult to know how informal and
formal home care are related in France: Does the amount of informal care re-
ceived impact the quantity of formal home care asked by the elderly dependent
individual? Or is the inverse causality dominant? The question of their com-
plementarity/substitutability remains also open. Contrary to what we could
think, formal home care and informal care are not necessarily substitutes. We
can imagine them as complements if formal home care consists more in personal
care and informal care more in domestic tasks for example. Several normative or
emotional considerations can also have an impact on the degree of participation
of family members. An informal caregiver can say for example: “I help 1 hour
each day, whatever the other quantities of care my parent receives.” These
considerations are not in favor of substitution. Finally, if the quantity of formal
home care increases, an increase of the quantity of informal care provided can
also be necessary to avoid the placement in an institution.

ad 2) Formal approaches until now have either lacked the very important
fact to control for endogeneity in the decision process (especially the very early
papers, see e.g. McAuley and Arling (1984) [16] and Kemper (1992) [13]) or
took this into consideration but could only model the impact of one kind of care
on the other and not the two directions at the same time using instrumental
variables and 2SLS techniques (see e.g. Bolin et al. (2008) [3] and van Houtven
simultaneous equation model using copula functions. While there seem to be
some problems in the formalization of their likelihood function, this seems to be
a good way to take account of the simultaneous-decision-problem and including
the recursivity in the two directions.

ad 3) We will study explicitly how different types of care dependency lead
to different care needs. A special focus will here be laid on people affected
by a dementia/Alzheimer’s disease. Our aim is to understand better the
differences between this kind of dependency and physical dependency, in terms
of supportive behaviors of caregivers. The question behind for the public funding
authority is the one about the necessity to build or not specific public policies
for each kind of dependency (mental or physical for example).
2.2 Why is it interesting for the public funding authority?

Studying the relation between formal home care and informal care could help the public administration to answer to several questions in terms of public health policies:

- Is there a crowding-out effect of familial solidarities by public solidarities? Will a public subsidy helping elderly dependent persons to pay formal home care decrease the quantity of care brought by their family? Such a crowding-out effect could be disadvantageous if the aim of the public policy is to increase the global quantity of care received by the elderly dependent people and to delay the placement in an institution. But the crowding-out effect can be looked for by the public funding authority if its goal is to relieve overworked informal caregivers. The help of professional caregivers could indeed allow them to relax a little bit.

- Will the expected decrease of the number of informal caregivers be balanced by a higher demand for formal home care?

Studying the inter-relationship of the two different types of care will give us a better understanding of how people who are in need of care for various reasons might behave.

3 Theoretical Model

3.1 Conceptual framework

The aim of this article is to answer the question of the possible complementarity or substitutability between formal home care and informal care for elderly dependent people in France. How does the quantity of one vary when the one of the other changes?

We base our conceptual framework on the model from Van Houtven and Norton (2004) [21], which is an extension of the classic Grossman (1972) [10] model of health demand, altered to include formal and informal caregiving. We focus on the case where there is one elderly dependent person (the parent) living alone, one informal caregiver (a child for example), and several formal caregivers. In our model, there is no financial transfer between the parent and the child. The child must decide which quantity of informal care he or she provides, while the parent decides on the quantity of formal care and pays for it. We suppose that the two decisions are taken simultaneously when an elderly parent needs assistance. We focus on how the parent’s choice of formal care may change if the supply of informal caregivers changes, and also on how the supply of informal caregivers would react if the parent decided to ask for more or less formal care. The superscript c references an individual child while p references an elderly dependent parent.

We assume that the child is characterized by the following utility function:

\[ U_C = u(X^c, L^c) + \beta h(I, F, E^p) \]
The child’s utility function $U_C$ depends on the private consumption of a composite commodity $X_c$ and leisure time $L_c$. The child is assumed to be altruistic. Like Pezzin and Schone (1999) [18], altruism is introduced in the model by allowing for a special type of interdependence between the parent’s and the child’s utilities that operates through the consumption of the public good $h$ (see also Becker (1992) [2]). In particular we assume that $h$ represents the parent’s well-being. The child’s utility depends positively on its parent’s well-being $h$. Informal caregiving $I$ and formal caregiving $F$ affect positively $h$. The parent’s well-being $h$ depends also on its stock of human capital $E_p$ (adapted from Grossman (1972) [10]). We assume that informal caregiving $I$ does not have a direct (negative) influence on the child’s utility, but only an indirect effect through $h$. Nevertheless since informal care is time-consuming, an increase in $I$ decreases $L_c$, which reduces the child’s utility.

The child maximises its utility function subject to the following full income budget constraint:

$$X_c \leq \omega(24 - I - L_c) + R$$

where $\omega$ is the child’s wage, and $R$ the child’s non-labour income. The full income budget constraint incorporates a time constraint which states that time is spent either consuming leisure ($L_c$), providing informal care ($I$) or working in the market.

The other decision, faced by the elderly dependent parent at the same time, is how much formal care to use. We assume that the parent is characterised by the following utility function:

$$U_P = v(X_p) + h(I, F, E_p)$$

The non-altruistic parent’s utility function $U_P$ depends on the private consumption of a composite commodity $X_p$ and on its well-being $h$. Leisure is not an additional choice variable for the parent, since it is assumed that the elderly parent does not face a time allocation decision. Informal care $I$, formal care $F$ and the stock of human capital $E_p$ have an influence on the parent’s well-being $h$.

The parent maximises its utility function subject to the following budget constraint:

$$X_p + p_F F \leq Y_p$$

where $p_F$ is the price of formal care and $Y_p$ the parent’s income. Most of the elderly dependent persons are retired, so we can consider $Y_p$ as constant.

We assume that the utility functions of both parent and child are increasing in each argument ($\frac{\partial u}{\partial X_c} = u_{X_c} > 0$, $u_{L_c} > 0$), $\frac{\partial U_C}{\partial h} = \beta > 0$, $\frac{\partial v}{\partial X_p} = v_{X_p} > 0$, $\frac{\partial h}{\partial I} = h_I > 0$, $h_F > 0$ and $h_{E_p} > 0$). We also assume that $u$, $v$ and $h$ are continuous, twice differentiable and quasi-concave. This implies that $u_{X_c}X_c < 0$, $u_{L_c}L_c < 0$, $u_{X_p}X_p < 0$, $h_{II} < 0$, $h_{FF} < 0$ and $h_{E_E}E_p < 0$. We finally assume that every cross-derivative equals 0 ($u_{X_c}I$, $u_{X_c}I$, $u_{L_c}I$, $u_{L_c}I$, $u_{E_p}F$, $u_{E_p}F$, $u_{L_c}E_p$) except $h_{II} = h_{IF} < 0$: The more the parent receives formal care (resp. informal care) the smaller is the effect of an additional hour of informal care (resp. formal care) on its well-being.

We then resolve the interdependence of utilities by assuming a Cournot-Nash
equilibrium for the determination of the level of formal care and informal care used. This equilibrium corresponds to a utility maximizing strategy by parent and child such that each takes the other’s strategy as given. The simultaneous decisions of the parent and the child lead to a set of conditional demands or reaction functions which represent the child’s best response based on beliefs about the parent’s supply of formal care:

\[ I = f^I(R, \omega, p_F, \bar{F}, E^p) \]

The parent’s reaction functions will depend on \( \bar{I} \):

\[ F = f^F(Y^p, p_F, \bar{I}, E^p) \]

At the equilibrium, parent’s and child’s expectations about each other’s choices are confirmed, implying that \( \bar{F} = F^* \) and \( \bar{I} = I^* \). Determining the partial derivative of \( F \) with respect to \( \bar{I} \) \( (\frac{\partial F}{\partial \bar{I}}) \) from the parent’s reaction function \( f^F \) indicates whether formal care is a substitute to informal care. For example, in a situation where the number of informal caregivers decreases. In the same way, determining the partial derivative of \( I \) with respect to \( \bar{F} \) \( (\frac{\partial I}{\partial \bar{F}}) \) from the child’s reaction function \( f^I \) indicates whether informal care is a substitute to formal care. It could help us to predict how informal caregivers would react if the parent asked for more formal care.

For individuals characterised by an interior solution, the first-order conditions that give the optimal amount of formal and informal care are:

\[ \beta_h I - u_L c = 0 \]  
\[ u_L c - \omega u_L c = 0 \]  
\[ h_F - p_F v_X p = 0 \]

By adopting a partial equilibrium perspective, we can specify from these conditions our two reaction functions \( f^I \) and \( f^F \). Through these functions, both impacts of an exogenous positive variation of \( \bar{I} \) on \( F \), and of \( \bar{F} \) on \( I \) are given by:

\[ \frac{\partial F}{\partial \bar{I}} = -\frac{h_{FI}}{h_{FF} + p_{F}^2 + v_{X p} X p} < 0 \]
\[ \frac{\partial I}{\partial \bar{F}} = -\frac{\beta_h I}{\beta_h I (u_{L} c + \omega^2 u_X c)} < 0 \]

Given the assumptions made, these expression are both strictly negative. The model predicts that every exogenous shock increasing the caregiving time of one type of care (formal or informal) leads to a reduction in the caregiving time of the other.

### 3.2 A recursive statistical model

Several characteristics impact the choice of the statistical model. The variables of interest are the weekly hours of informal care and formal care. In fact we decided to log-transform formal and informal care hours. Hypotheses of normality and homoscedasticity of the error terms are indeed better satisfied. Since \( \ln(0) \) is not defined, hours of care \( Y \) are transformed in the following way:

\[ Y^{new} = \ln(Y + 1) \]
Both of these transformed variables take only non-negative values and have a distribution with a large mass at zero. In order to take this into account, we model our variables of interest using latent variables, only observed if they are strictly positive.

In contrast to Van Houtven and Norton (2004) [21], who can only use binary variables, we have got continuous variables for care available: On the one hand the (log-transformed) weekly hours of informal care received by an elderly dependent people and on the other hand the (log-transformed) weekly hours of formal home care received. In this study, each variable can therefore be used as an explicative variable of the other in a simultaneous equations model with latent variables. It allows us to study the direct effects of both informal care on formal care, and vice versa.

However, in a model where the use of formal home care is explained by the quantity of informal care received, the variable of informal care is endogenous. This endogeneity occurs because the choice of the pair of hours of formal care and informal care is made at the same time. Van Houtven and Norton (2004) [21] treat this problem by using instrumental variables (IV) for informal care. They construct several over-identification tests to determine the best instruments. They choose to use the number of children and the binary variable indicating whether or not the oldest child is a daughter to instrument the quantity of informal care received. Despite these over-identification tests, IV models remain very sensitive to the choice of instruments. The simultaneous equations model with latent variables here allows to solve the problem of endogeneity in an alternative way from the use of instrumental variables.

The model takes the following form:

\[
\begin{align*}
    y_{i1}^* &= \beta_1^* X_{i1} + \alpha_1 y_{2i} + u_{1i} \\
    y_{i2}^* &= \beta_2^* X_{i2} + \alpha_2 y_{1i} + u_{2i}
\end{align*}
\]

\[
y_{1i} = \begin{cases} 
    y_{i1}^*, & \text{if } y_{i1}^* > 0 \\
    0, & \text{else.}
\end{cases}
\]

\[
y_{2i} = \begin{cases} 
    y_{i2}^*, & \text{if } y_{i2}^* > 0 \\
    0, & \text{else.}
\end{cases}
\]

with:
- \(y_{1i}\) and \(y_{2i}\): neperian logarithms of home formal care hours and informal care hours (plus one)\(^2\)
- \(y_{i1}^*\) and \(y_{i2}^*\): latent variables associated with \(y_{1i}\) et \(y_{2i}\) (only observed if they are positive)
- \(X_{i1}\) and \(X_{i2}\): vectors of explaining variables (constant included),
- \(\beta_1^*\) and \(\beta_2^*\): vectors of parameters,
- \(\alpha_1\) et \(\alpha_2\): parameters describing the relationship of complementarity/substitutability between home formal care and informal care. If the model is correctly specified, \(\alpha_3\) represent the causal effect of informal care\(^3\) on the one of home formal care\(^4\).

The coefficient of \(\alpha_2\) represents the causal effect of formal home care on informal care.

\(^2\)We consider \(\ln(1 + y_{ki})\), \(k \in (1, 2)\) to take into account individuals, who declare receiving care quantities between 0 and 1 hour per week.

\(^3\)More precisely, of the neperian logarithm of informal care plus one.

\(^4\)On the neperian logarithm of home formal care plus one.
(u_{1i}, u_{2i}) follows a joint law whose density function is \( f(u_{1i}, u_{2i}) \) and whose cumulative distribution function is \( F(u_{1i}, u_{2i}) \).

The marginal distributions of the error terms are assumed to be Normal. Coefficients are estimated with the maximum likelihood estimation procedure (Stata 11).

### 3.3 The copula function

One of the particularities of our model is the use of a copula function, a very popular tool in statistical modelling of financial processes for example. A bivariate copula is a function that join or “couple” bivariate distribution functions to their one-dimensional marginal distribution functions. In the model, the two marginal distributions associated with the error terms are “coupled” with their joint distribution by a copula function. According to Nelsen (2006) [17], one of the main interest of copula functions is to capture specific properties of the joint distribution. Clauss (2009) [6] explains that the dependency between random variables is perfectly described by their joint distribution. The copula allows to extract the dependency structure from the joint distribution and so to distinguish dependency from marginal behaviour.

We assume that our error terms \( u_{1i} \) et \( u_{2i} \) follow normal marginals (conditionnaly to exogenous variables):

\[
\begin{align*}
    u_{1i} &\rightarrow N(0, \sigma_1) ,
    u_{2i} &\rightarrow N(0, \sigma_2)
\end{align*}
\]

For \( k \in (1, 2) \), let’s denote \( f_k \) and \( F_k \) density and cumulative distribution functions associated with the error term \( u_{ki} \). Then \( \forall k \in (1, 2) \):

\[
\begin{align*}
    f_k(u_{ki}) &= \frac{1}{\sigma_k \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left( \frac{u_{ki}}{\sigma_k} \right)^2 \right) \\
    F_k(u_{ki}) &= \Phi\left( \frac{u_{ki}}{\sigma_k} \right)
\end{align*}
\]

, where \( \phi \) and \( \Phi \) denote the density and cumulative distribution functions of a Standard Normal.

Thus, following Sklar (1973) [20], the copula \( C_\theta \) “couples” the joint distribution to its one-dimensional marginal distributions:

\[
F(u_{1i}, u_{2i}) = C_\theta(F_1(u_{1i}) , F_2(u_{2i})) = C_\theta \left( \Phi\left( \frac{u_{1i}}{\sigma_1} \right) , \Phi\left( \frac{u_{2i}}{\sigma_2} \right) \right)
\]

This copula is unique if the marginal cumulative distribution functions are continuous, which is the case in our model. We limit our study to several simple copula functions: Archimedian copulas. These are parametric and depend on a unique \( \theta \) parameter. We test several copulas, each allowing a specific dependency structure between the error terms, and try to find the one which
corresponds the best to our data:

- the Product copula, valid if the error terms are independent.
- Gumbel and Clayton copulas allowing positive dependency between error terms.
- Frank and Farlie-Gumbel-Morgenstern (FGM) copulas allowing positive or negative dependency.

Precisions related to copulas are given in Appendix.

3.4 Coherence and identifibiality

Two important conditions underlined by Amemiya (1974) [1] and Fontaine (2011) [7] have to be fulfilled in order to have a valid model and consistent estimates. First, the model may present a risk of incompleteness in the sense, that, for a given vector of exogenous variables (both observed and unobserved); it does not always predict a unique time allocation. This incompleteness stems from the fact that the model defines the optimal allocation as the intersection of two non-linear functions, one giving the optimal formal caregiving time as a function of informal caregiving time and the other giving the optimal informal caregiving time as a function of formal caregiving time. This non-linearity may potentially lead to zero or several intersection points. In this case, the model predicts no or multiple equilibria. To overcome this difficulty, it is necessary to impose the following “coherence condition” (Amemiya (1974) [1], Maddala (1983) [15]) prior to estimating the model:

\[ 1 - \alpha_1 \alpha_2 > 0 \]

This condition ensures the completeness of the model regardless of the individual (observed and unobserved) characteristics.

Moreover, if we include the same explanatory variables in each equation (i.e. if \( X_{1i} = X_{2i} \)), then the identification of the parameters is only due to the censure characterising the formal and informal caregiving time. It is theoretically possible to proceed this way. But it may be not optimal. To ensure the identification of the parameters, and according to Amemiya (1974) [1] (Assumption 3.3. (p. 1007)), we decide to exclude from the formal caregiving time equation, variables that appear to be theoretically and empirically correlated with informal caregiving time but unrelated to formal caregiving. Following Van Houten and Norton (2004)[21] and Bonsang (2009) [4], we exclude the number of children and the proportion of daughters from the formal caregiving time equation. Correspondingly, we exclude from the caregiving time equation the income that appears to be correlated with formal caregiving time but not with informal caregiving.
4 Data

We use the 2008 Disability - Healthcare data on households (Handicap Santé Ménages - HSM 2008) of the French national institute for statistics and national studies (INSEE), a database which is based on questionnaires. The main aim of this database is to give as many informations as possible about care dependent people in France. It includes thus as well informations about the health status, the socio-economic status, the living situation as informations about the care which the individual receives. In total, 29,931 individuals answered the questionnaire. Since the focus of our work lies among others on the care which Alzheimer’s disease patients receive, we included a newly (c.f. Gramain (2011) [8]) constructed dummy variable. The HSM data already includes an indicator variable for Alzheimer’s disease, but the number of individuals who report this illness is very low. It can be assumed that the actual number of people suffering from this illness is much higher, but that it is very often not diagnosed. Gramain used other variables from the HSM database which are known to be highly correlated with this disease and constructs thus the new variable. For an exact definition of the variable please refer to her article.

4.1 Population of interest

The aim of our work is to describe how elderly people in France are cared for. For this work we defined elderly people as being at least 60 of age and excluded thus younger people from the analysis who might nevertheless be care dependent. Furthermore we excluded all completely independent people. In order to not exclude accidentally individuals who are only slightly care dependent we used a very broad definition. People were considered at least as being in danger of becoming care dependent when they reported at least one problem with an activity of daily living (in the following ADL) or an instrumental activity of daily living (in the following IADL), or if they had Alzheimer’s disease. The ADL include fundamental tasks which are necessary for an individual to live and survive on its own like taking care of personal hygiene, eating, walking, and the like. Instrumental activities of daily are not necessary to survive but enable the person to live on its own. These include for example cooking, shopping and using transportation. Furthermore there was an important number of people who did not report the actual number of received care hours. These people were also excluded from the data sample. The decision to only look at single living persons was made to take account of the potentially very different care arrangements according to the household composition: If the care-dependent person lives together with a spouse or a child this will almost automatically lead to a much higher number of informal care hours and a relationship between informal care and formal care which will be very different from the one which single living care dependent people experience. Secondly, a modeling of people who are living with their spouses was furthermore not possible in a satisfying way because there was no information about the health status of the spouse available. Since we can imagine that the age of the dependent person and of the spouse is highly correlated, we also assume that there might be a significant proportion of spouses who is in need of care themselves and thus not comparable to completely independent spouses who very often serve as the most important caregiver. Finally it is clearly visible in the data that informal caregivers who
are living together with the care recipient had very often problems to declare the hours which they spent on caring. This leads to a high proportion of missing values for this particular group of caregivers. The declaration of care hours is difficult in the case of co-living caregivers. The distinction between care and regular household duties can in some cases be very difficult. It also seems like many spouses see some caring tasks as marital duties and report thus only a very low amount of care hours even if they actually help a lot more. We have thus decided to only look at single-living care dependent people and stay on statistically safe grounds. This finally leaves us with a sample of 1705 individuals who meet all the criteria and who answered the questionnaire sufficiently.

4.2 Explanatory variables

We can group the explanatory variables in several classes. First, the group of variables related to the elderly person’s health and dependency state: the self declared health state, the number of ADL limitations, the number of IADL limitations, the fact of suffering (or not) from Alzheimer’s disease. Seven ADLs are considered (bathing, dressing, cutting the food, eating the meals, going to restroom, waking up or going to bed, sitting down) and twelve IADL (going grocery shopping, preparing the meals, doing common household chores, doing less common domestic chores, doing administrative tasks, taking medications, moving around in all of the rooms of the floor, leaving the home, using a method of transportation, finding the way, using the phone, using a computer). We suppose that the effect of an additional limitation in ADL or in IADL on the number of hours of care received is not linear: It depends on the limitations that the elderly dependent person already has. Therefore, we prefer to use categorical variables. Variables related to demographic and socio-economic characteristics of the elderly dependent person are also introduced: the level of education, income (categorical variable), living place, the fact of living (or not) overseas (DOM). A third group of variables concerns children, as potential informal caregivers. The number of children is indeed introduced as a continuous variable. We could focus only on children living close to their dependent parents, since their probability of care is greater. However the variable of proximity is potentially endogenous: the child can come to live closer to its dependent parent if the informal care he has to supply increases. To prevent the estimations from a bias, we did thus exclude this variable. The proportion of daughters is also introduced, since daughters are known to often give more help than sons to their care dependent parents. Eventually, control variables are added: the age of the elderly dependent person and its gender.

Table 3 gives descriptive statistics for the most important variables. At first sight it is surprising that 80% of the individuals in the data sample are women. An explanation can be found when we go back to the construction of our sample. We are only looking at elderly individuals not living with a spouse. The fact that women have a high life expectancy plus the fact that women tend to be younger then their partners leads naturally to a greater proportion of women among our sample. The individuals have at the mean slightly more than two children, with a few more sons than daughters. About 9% of our sample is not living in metropolitan France but in one of the overseas departments. The average age of all individuals in our sample is 79 years. Almost all individuals in our sample reported at least one IADL limitation (98%) and the mean of
IADL limitations is just over 4, 40% reported problems with an ADL and 6% are having Alzheimer’s disease. The high figure for IADL limitations had to be expected due to the construction of our data sample and the fact that some IADL are very far fetched. While more individuals receive some kind of formal care (about 66%) than informal care (about 42%) we can see that the number of actual care hours is in average higher for informal care. Formal home care and informal care quantities consist of the sum of care hours provided by all informal and formal caregivers per week.

Table 4 approaches for a first time the question of whether informal care and formal care hours act as substitutes or complements. It gives the Pearson correlation between the natural logarithm of formal and informal care hours by group of dependency. If we look at all individuals we find that both types of care seem to be not correlated at all. If we calculate the same correlation but control for the dependency of the person, by its number of limitations in its ADL, we find on the contrary a negative and very significant correlation. This suggests a substitution effect between the two types of care if controlling for the dependency of the individuals. Still, this simple correlation should not be confused with a causal effect. The next section will give the results from the complex recursive model.

5 Results and Interpretation

Table 5 shows the results from our estimations. The first estimation (double recursive) is the full recursive model described in 3.2, where informal care may explain formal care and formal care may explain informal care. The second model serves as a sensitivity analysis of our results. In this case we exclude the possibility that informal care is influenced by the number of formal care hours. When estimating the model, it is possible to use many different copula functions. We tested five different archimedean copulas which are frequently used in applied research: Frank, FGM (Farlie-Gumbel-Morgenstern), Product, Clayton, and Gumbel. The copula functions Clayton and Gumbel assume a positive correlation between the error terms, Frank gives the possibility of a negative or positive relationship, FGM makes the hypotheses of a relatively weak correlation and Product finally assumes that the two error terms are uncorrelated. Please refer to the appendix for more information on copula functions. We found that Clayton and Gumbel did not converge to an optimum. It is thus likely that the correlation between the error term is negative. The other three copulas result in very similar values of log likelihood (see table 6). We decided to report the results using the Frank Copula, which gives a slightly greater log likelihood than the other two copula functions. Nevertheless, the estimations using the other two copula functions give very comparable results and are available upon request from the authors.

In the following sections we first have a look at the results of the fully recursive model before considering the results from our second model. Since our models include latent variables and are recursive, we then discuss the problem of calculating marginal effects before interpreting them.
5.1 Model 1: Fully Recursive

The results are very coherent with our expectations. If we start with the question of a substitution effect, we can see that both $\alpha_1$ and $\alpha_2$ are significant (at least at a 10% level) and negative. While the coefficient of informal care in the formal care equation is significant at a lower level, we find that the coefficient of formal care in the informal care equation is almost double as great. This indicates that the influence of formal care on informal care is not as clear as it is the other way around. Individuals may react strongly on a change of formal care hours, but it is not always the case. This may underline the various attitudes towards care by informal caregivers: Some might see it as a duty to be there for the person in need no matter how much help s/he receives from professional caregivers. Others on the contrary will evaluate the need of the dependent person and thus adjust their behaviour more to changes of professional care.

If we have a look at other variables which explain formal care we can observe the expected relationships. Variables which describe the dependency status of the individual (self-reported health, Alzheimer, sum of I/ADL limitations) are all significant and show that an individual will receive more professional care the more dependent s/he is. Only the indicator variable corresponding to Alzheimer’s disease does not significantly explain formal care hours. Education increases formal care hours. Interestingly though we can only observe an effect between people without any degree and all others: Elderly dependent persons without diploma may have more difficulty to access to the market of formal care. There is no significant difference between individuals who own a post secondary school degree and those with only a primary or lower secondary school degree. As expected, the higher the income is, the more formal care hours an elderly dependent person receives. People living in a big city or overseas (DOM) seem to receive less formal care hours than others. Age explains positively formal care hours. Age can be assumed to be correlated with the health status of the individual and may thus capture some of the real dependency which is not already explained by the aforementioned variables.

Informal care is also largely explained by health and dependency status of the individual. Nevertheless we can see several interesting differences with the significant explanatory variables of formal care. Alzheimer has a significant impact on informal care hours. This has not been the case with formal care. Still, it seems reasonable. A large part of the care work caregiver of elderly dependent suffering from dementia consists of paying attention and monitoring. This burden, at least in the earlier stages of dementia, is primarily carried by informal caregivers. A second interesting difference is that an increase of IADL limitations gives much bigger coefficients than what we have seen for formal care. Education decreases informal care hours, which seems consistent with the existing literature. Individuals living in cities (except Paris) receive in general more informal care than people living in rural areas or small towns. In general, the main caregivers of elderly dependent people living alone are children. It is also known that care is more often provided by daughters than by sons. Both effects can be seen in our model: The number of children and the proportion of daughters explain significantly and positively the quantity of informal care received. Age does not seem to influence informal care hours a lot in contrast to formal care hours. Only the 90 years and older receive significantly more care than the younger dependent individuals.
The \( \theta \) can be interpreted as a dependency parameter between the two error terms of both equations. The negative value here suggests a negative association and thus a negative relationship for the unobserved explanatory variables. Nevertheless, the Kendall’s Tau (see Appendix B for more details) remains very small, suggesting a weak negative association between our error terms. One key missing variable here might be the personal preference for formal or informal care which would indeed lead to a positive coefficient in one equation and a negative coefficient in the other equation.

5.2 Model 2: Recursive \((\alpha_2 = 0)\)

As a sensitivity analysis we estimated a model which has already been proposed in the literature (see e.g. Van Houtven and Norton (2004) [21]). This model has the underlying assumption that the informal caregivers will decide unilaterally on the number of care hours which they want to provide without taking account of formal care hours. The results are very reassuring. Not only are the signs of the coefficients in the same directions, also the size are very comparable. This suggests that our model is in general very stable. Still, since the formal care explains significantly informal care hours in our main model we prefer model one over model two.

5.3 Interpretation and Marginal Effects

After these first results it is important to note that the interpretation of the coefficient in our models has to be made with caution. This has two reasons: 1.) We are looking at Tobit-like models and 2.) We are in a context of simultaneous equations where the outcome of one equation may influence the outcome of the second. The first problem means that the observed coefficients cannot be directly interpreted as marginal effects on the observed dependent variable but as the marginal effects of the latent and thus unobserved variable. An interpretation as a marginal effect of the observed variable is thus biased. The coefficients seem to be greater in absolute terms than the real effect actually is. The standard formula to transform these variables in marginal effects of the observed dependent variable is given by:

\[
\frac{\partial E(y|x)}{\partial x_k} = \beta_k \Phi \left( \frac{x\beta}{\sigma} \right),
\]

where \( \beta_k \) is the coefficient of variable \( x_k \). The marginal effect can thus be seen as the product of the coefficient and the probability that \( y > 0 \). Still, the results from this calculations do not describe the real effect since the two equations influence each other (problem 2). In our special case where we find that informal and formal care influence each other negatively and significantly we would thus underestimate the real effect if we don’t take this recursivity into account. We thus decided to employ two different methods to calculate marginal effects and use them for two different situations. The first one is close to equation 7 and will be used to calculate the marginal effect of an exogenous change of informal or formal care hours. The second will use an iterative calculation of care hours for a discrete change of one of the exogenous dependent variables.
Table 1: Marginal Effects for a variation of care hours

<table>
<thead>
<tr>
<th></th>
<th>Direct Effect</th>
<th>Indirect Effect</th>
<th>Total Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1$</td>
<td>$S$</td>
<td>$S \cdot \frac{\alpha_1 \alpha_2}{1-\alpha_1 \alpha_2} \cdot P(Y_2 &gt; 0)$</td>
<td>$S \cdot (1 + \frac{\alpha_1 \alpha_2}{1-\alpha_1 \alpha_2} \cdot P(Y_2 &gt; 0))$</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>$S \cdot \alpha_2 \cdot P(Y_2 &gt; 0)$</td>
<td>$S \cdot \frac{\alpha_1 \alpha_2}{1-\alpha_1 \alpha_2} \cdot P(Y_2 &gt; 0)$</td>
<td>$S \cdot \frac{\alpha_2}{1-\alpha_1 \alpha_2} \cdot P(Y_2 &gt; 0)$</td>
</tr>
</tbody>
</table>

$S$: shock (exogenous)

5.3.1 Marginal Effects of Care Hours

In the case of care hours we are mainly interested in two effects: How do informal care hours change when we increase formal care by one hour for every individual? How do formal care hours change when we decrease informal care hours by one hour. These questions are closely related to problems which a policy maker faces: How does the care composition change (and thus the costs) if s/he increases the social benefits in a way that every dependent person receives one more formal care hour? And secondly, if we expect a decrease in informal care due to societal changes, what are the changes in the demand for formal care? A change of only one hour is in our context a rather marginal variation of care hours and we can thus use equation 7 to calculate a direct effect. Since our two equations are influencing each other, we cannot stop here, but observe also endogenous / indirect variations. Table 1 gives the formulas for the exogenous / direct variations, the endogenous variations, and the total effect which is the sum of the former two.

5.3.2 Discrete Effects of Exogenous Variables

Since we only use categorical variables in our estimations we cannot calculate marginal effects for these variables. Here we are more interested in the discrete effect. How does having Alzheimer’s disease change the care composition? Do people who have one daughter receive more hours of informal care than people without any daughter? The usual formula to calculate discrete effects is given by:

$$DE(\hat{\beta}, X) = f(\hat{\beta}, X|x_i = k) - f(\hat{\beta}, X|x_i = base),$$

where $\hat{\beta}$ are the estimated coefficients, $X$ is the vector of the explanatory variables, and $f$ is the function which calculates the value for the dependent variable $Y$. It is thus simply the difference of the estimated values for $Y$ if all individuals switch the category for one variable. The problem in our recursive model is that while usually we can take the explanatory variables as given, this is not the case for the number of care hours. So for example, while estimating the number of formal care hours, we have to estimate at the same time the number of informal care hours since a change in one explanatory variable will change both values. We calculated the number of care hours in an iterative way, which is described in table 2. Convergence (to at least 7 positions after the decimal point) was achieved relatively quickly. It never took more than 40 iterations, most of the time about 10 iterations were needed for convergence. After the estimation of
Table 2: Iterative process to calculate optimal number of hours

<table>
<thead>
<tr>
<th>First Step</th>
<th>Following Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{Y}_1^1 = \max(0; X_1 \hat{\beta}_1)$</td>
<td>$\hat{Y}_1^1 = \max(0; \hat{Y}_1^1 + \hat{\alpha}_1 \hat{Y}_2^{t-1})$</td>
</tr>
<tr>
<td>$\hat{Y}_2^1 = \max(0; X_2 \hat{\beta}_2)$</td>
<td>$\hat{Y}_2^1 = \max(0; \hat{Y}_2^1 + \hat{\alpha}_2 \hat{Y}_2^{t-1})$</td>
</tr>
</tbody>
</table>

Superscript denotes the step, subscript the equation

the two dependent variables we can calculate the discrete effect following equation 8. We don’t have to apply this iterative process to calculate the marginal effects when changing the number of care hours marginally since in the other variable of care hours is given and does not need to be re-estimated.

5.3.3 Limitations of the calculations

While we find that the applied methods to calculate marginal and discrete effects give relatively reliable results, they are still not without flaws. In the case of the marginal effects of hours of care (cf. table 1) we assume that the Probability $P(Y_2 > 0)$ stays the same. This might not be reasonable for individuals with a relatively weak but positive value of $Y_2$. Still it is a simplification we have to make in order to avoid programming an iterative process as in the case of the discrete effect. We prefer this because the iterative process is also not perfect.

5.3.4 Results and Interpretation

Using the before mentioned formulas we calculated marginal and discrete effects for important variables. We calculated the effects for every individual in our database. This has the advantage that we can interpret the results in much more detail than if we would have calculated a mean effect from the beginning. We get the full distribution of marginal and discrete effects.

We start by having a look at the interdependence of formal care and informal care. We make two assumption: First, since the supply of informal caregivers is likely to decrease in the future due to demographical (less children) and societal (e.g. children living further away) changes we have a look at what happens if informal care is decreasing by one hour for every individual which receives a positive amount of informal care until now. Table 7 gives the results on formal care. We can see that the increase of formal care is relatively small. For those people who receive 0 or less than the median number of professional care hours the effect is almost negligible. Formal care hours will increase by only up to 3 minutes. Even for people who already received more than the median number of care hours we can see a rather small change of 4 to 14 minutes depending on their initial amount of informal care hours.

The second assumption which we can make is that every individual receives one more hour of formal care than they used to before. This could be for example due to an increase in public subsidy towards care dependent people. As table 8 shows, we find in this case a reduction of informal care hours for those who used to receive a positive number of informal care hours before. This reduction
is most important for those who used to receive a relatively large number of
informal and a relatively small number of formal care hours. Here the change
is at the median also an hour. For all the other groups we find median changes
of 5 to 19 minutes.

We also had a look at the marginal effect for three core variables: How does
the care composition change if an individual is going from not having Alzheimer
to having this disease? How does an increase in ADL limitations increase the
formal and informal care hours? What is the impact of having one more child?
The results can be found in table 9. Here we report the distribution of the effect
to show that individuals react very differently to these changes. Alzheimer for
example will increase formal and informal care hours for many people, but
as we can see, there are a lot of people where this does not change the care
composition. still for over 50% of the individuals in our sample we can see an
increase of at least 30 minutes. In the case of informal care we can also see that
there are some individuals who will receive a lot more. An increase of over 10
hours per week seems possible. ADL limitations influence formal care stronger
than informal care. 75% of the individuals under research receive at least one
hour more of formal care per week when they go from one limitation to three
or more. In the more extrem values we find increases of 8 hours and more. A
similar effect is also visible for informal care, although it is less important. Since
the activities of daily living include core tasks like personal hygiene which are
more often associated with professional than with informal care. The number
of children should increase informal caregiving since children are in general the
main caregivers of elderly single living persons. Via the increase of informal care
hours we expect formal care hours for some individuals to slightly decrease. This
second effect is as we can see relatively small. For 75% of our sample this effect
is smaller than 7 minutes and for more than 50% there is no effect at all. The
change on informal care hours is a little bit more important and can be as high
as 3 hours per week. Still, many people would not profit from more informal
care only because they are having one more child. A further investigation into
this shows that most people who did not receive informal care before won’t
receive informal care afterwards either. This could be partially explained by
personal preferences. Someone who does not won’t to receive care by a family
member before won’t change his opinion just because there is one more child.

5.4 Importance for Policymaking

The analysis of the marginal effects underlines at least three important findings:

1. A decrease of informal care hours is only partially compensated by an
increase of formal care.

2. An increase of formal care leads to a decrease of informal care. Especially
for those caregivers who used to aid more than the average.

3. Public policy needs to reflect the fact that different kinds of dependency
demand different care compositions.

The first finding is closely linked to the idea of a decrease of informal care in
the future due to demographic change and other societal factors. If this is the
case than public policy should be aware that this loss of informal care may
not be compensated by formal care for all individuals. The consequence could be an earlier institutionalization for many individuals, a solution which is often more expensive and not preferred by the care dependent individuals. The public financier could avoid this by increasing subsidies towards care dependent people. This could be by either increasing the help to buy formal care (and thus answer by substituting informal care by formal care) or by giving more help to informal caregivers (and thus to increase the number of people who are able to give care). The second finding underlines that an increase of public subsidy towards the use of formal care would not necessarily lead to a decrease of informal care. It is mainly those caregivers who used to aid a lot who will profit from the increase of formal care. Together with finding one we can thus assume that this increase of public aid could help in fighting the loss of informal care without making a lot of informal caregivers give up their help. The third finding goes back to the marginal effects of Alzheimer and ADL limitations. These stand here as two typical ways of how the health status of an individual could decline in two different ways. It has been shown that individuals react differently to these two kinds of dependency. In both cases we can see an increase in the demand for formal and informal care. But while people who develop an Alzheimer’s disease tend to increase informal care far more than formal care, we can see the inverse for ADL limitations. Public policy should always keep in mind that different types of dependency demand different types of care. Together with the fact that every individual has different attitudes and preferences towards care, it is advisable to give as much freedom of choice to the individuals in need as they can bear and to help them in the decision making when needed.

6 Conclusion

Using the HSM 2008 data we find that formal and informal care have to be seen as substitutes. Nevertheless, our calculations show that this effect is rather weak and far from being the same for all individuals. The effect from a variation in informal care on the demand for formal care is very small. This means that a possible reduction of informal care hours would not be equally replaced by formal care hours for almost all individuals. The same holds true the other way around: Although the effect of formal care on informal care is a little bit more pronounced, we can see that an increase of formal care reduces mainly the care hours for carers who are heavily burdened by care duties. In terms of public policy our findings imply that a possible decrease of informal care in the future could possibly not be automatically compensated by formal care if the state does not intervene. On the other hand it has been shown that an increase of formal care does not lead to a great crowding out effect of informal caregivers. Only caregivers who give a lot of informal care may decrease their help a little bit as an answer to this.

Furthermore we have also shown that Alzheimer’s disease is increasing formal and informal care hours. Still, and in contrast to an increase of physical dependency, we find that Alzheimer’s disease increases mainly informal care. Again, we find that many people would not necessarily change both amounts of care hours. Personal preferences can be seen as a main reason for this. An interesting question for future research would be to investigate if the effect of substitution varies between different kinds of dependency. Do people who suffer
from a dementia optimize their care composition differently than people who are physically care dependent?

We intend to refine the microeconomic model as well as our quantitative estimations. A generalization of the microeconomic model would be appreciated. Until now it describes only the special case of one informal caregiver. We would like to include the possibility of multiple caregivers in the future and get thus closer to our econometric model. The econometric modeling could also be enriched by specifically estimating a two stage model separating thus the question of why someone receives a specific kind of care from the question of how many hours of care s/he will receive. This could give a better insight in the decision process at work for a care dependent person. It would also allow for an inclusion of explanatory variables which are related to the (main) informal caregiver.
# Tables

## Table 3: Some descriptive statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Min</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>0.80</td>
<td>0.40</td>
<td>0</td>
<td>1</td>
<td>1705</td>
</tr>
<tr>
<td>Age</td>
<td>79.00</td>
<td>8.49</td>
<td>60</td>
<td>101</td>
<td>1705</td>
</tr>
<tr>
<td># Children</td>
<td>2.31</td>
<td>2.06</td>
<td>0</td>
<td>13</td>
<td>1705</td>
</tr>
<tr>
<td>Proportion of Daughters</td>
<td>0.40</td>
<td>0.37</td>
<td>0</td>
<td>1</td>
<td>1705</td>
</tr>
<tr>
<td>Alzheimer</td>
<td>0.06</td>
<td>0.24</td>
<td>0</td>
<td>1</td>
<td>1705</td>
</tr>
<tr>
<td>ADL limitations</td>
<td>1.00</td>
<td>1.61</td>
<td>0</td>
<td>7</td>
<td>1705</td>
</tr>
<tr>
<td>IADL limitations</td>
<td>4.09</td>
<td>2.85</td>
<td>0</td>
<td>12</td>
<td>1705</td>
</tr>
<tr>
<td>Living in a DOM</td>
<td>0.09</td>
<td>0.28</td>
<td>0</td>
<td>1</td>
<td>1705</td>
</tr>
<tr>
<td>Receive informal care</td>
<td>0.42</td>
<td>0.49</td>
<td>0</td>
<td>1</td>
<td>1705</td>
</tr>
<tr>
<td>Hours of informal care</td>
<td>8.46</td>
<td>18.43</td>
<td>0</td>
<td>142</td>
<td>1705</td>
</tr>
<tr>
<td>Receive formal care</td>
<td>0.66</td>
<td>0.47</td>
<td>0</td>
<td>1</td>
<td>1705</td>
</tr>
<tr>
<td>Hours of formal care</td>
<td>7.15</td>
<td>15.29</td>
<td>0</td>
<td>168</td>
<td>1705</td>
</tr>
</tbody>
</table>

Sample: Single living individuals aged 60 or older

## Table 4: Pearson correlation between the natural log of formal and informal care by group of dependency

<table>
<thead>
<tr>
<th>ADL limitations</th>
<th>All</th>
<th>0</th>
<th>1 or 2</th>
<th>3 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.0013</td>
<td>$-0.1418^{***}$</td>
<td>$-0.1355^{***}$</td>
<td>$-0.1047^*$</td>
</tr>
</tbody>
</table>

Significance Levels: 10%, **: 5%, ***: 1%
Sample: Single living individuals aged 60 or older

## Table 5: Copula

<table>
<thead>
<tr>
<th>FORMAL CARE</th>
<th>Double Recursive</th>
<th>Recursive (alpha2=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self reported health status (REF: Good / Very Good)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fairly Good</td>
<td>0.180 (1.45)</td>
<td>0.175 (1.41)</td>
</tr>
<tr>
<td>Bad</td>
<td>0.322** (2.57)</td>
<td>0.316** (2.53)</td>
</tr>
<tr>
<td>Very Bad</td>
<td>0.246 (1.56)</td>
<td>0.239 (1.52)</td>
</tr>
<tr>
<td>Person has Alzheimer(REF: No)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>0.224 (1.53)</td>
<td>0.225 (1.55)</td>
</tr>
<tr>
<td>Sum of ADL limitations (REF: 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$-0.425^{***}$ (-4.06)</td>
<td>$-0.422^{***}$ (-4.03)</td>
</tr>
<tr>
<td>2</td>
<td>0.0361 (0.27)</td>
<td>0.0396 (0.29)</td>
</tr>
<tr>
<td>3 or more</td>
<td>0.429*** (3.29)</td>
<td>0.435*** (3.33)</td>
</tr>
</tbody>
</table>

23
### Sum of IADL limitations (REF: 1)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.691**</td>
<td>(-2.13)</td>
<td>-2.11 ****</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.635***</td>
<td>(5.36)</td>
<td>5.36 ***</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.848***</td>
<td>(6.56)</td>
<td>6.60 ***</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.056***</td>
<td>(7.59)</td>
<td>7.65 ***</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.309***</td>
<td>(9.16)</td>
<td>9.18 ***</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.406***</td>
<td>(8.74)</td>
<td>8.76 ***</td>
<td></td>
</tr>
<tr>
<td>7 or more</td>
<td>1.879***</td>
<td>(12.47)</td>
<td>12.50 ***</td>
<td></td>
</tr>
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</table>

### Highest Degree (Education) (REF: Primary or lower secondary School)

<table>
<thead>
<tr>
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<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Degree</td>
<td>-0.234***</td>
<td>(-2.97)</td>
<td>-2.99 ***</td>
<td></td>
</tr>
<tr>
<td>Upper Secondary</td>
<td>0.0893</td>
<td>(0.81)</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>Post Secondary</td>
<td>0.157</td>
<td>(0.93)</td>
<td>0.97</td>
<td></td>
</tr>
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</table>

### Income (REF: 700 - 1000)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than 700</td>
<td>0.191</td>
<td>(1.43)</td>
<td>1.51</td>
<td></td>
</tr>
<tr>
<td>1000 - 1500</td>
<td>0.0882</td>
<td>(1.02)</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>1500 - 2000</td>
<td>0.328***</td>
<td>(2.67)</td>
<td>2.51 ***</td>
<td></td>
</tr>
<tr>
<td>More than 2000</td>
<td>0.508***</td>
<td>(3.49)</td>
<td>3.40 ***</td>
<td></td>
</tr>
<tr>
<td>Missing</td>
<td>0.299**</td>
<td>(2.33)</td>
<td>2.21</td>
<td></td>
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</tbody>
</table>

### Living place (REF: town with less than 20,000 residents)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td>-0.0886</td>
<td>(-0.78)</td>
<td>-0.73</td>
<td></td>
</tr>
<tr>
<td>20,000 - 100,000</td>
<td>0.0659</td>
<td>(0.53)</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>more than 100,000</td>
<td>-0.217**</td>
<td>(-2.05)</td>
<td>-2.03</td>
<td></td>
</tr>
<tr>
<td>Paris</td>
<td>-0.334**</td>
<td>(-2.41)</td>
<td>-2.37</td>
<td></td>
</tr>
</tbody>
</table>

### Person lives in a DOM (REF: No)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>-0.337**</td>
<td>(-2.46)</td>
<td>-2.48</td>
<td></td>
</tr>
</tbody>
</table>

### Person is female (REF: No)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>0.0218</td>
<td>(0.25)</td>
<td>0.24</td>
<td></td>
</tr>
</tbody>
</table>

### Age (REF: 60 - 65)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>66 - 70</td>
<td>0.113</td>
<td>(0.63)</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>71 - 75</td>
<td>0.377**</td>
<td>(2.46)</td>
<td>2.42</td>
<td></td>
</tr>
<tr>
<td>76 - 80</td>
<td>0.506***</td>
<td>(3.49)</td>
<td>3.47</td>
<td></td>
</tr>
<tr>
<td>81 - 85</td>
<td>0.852***</td>
<td>(5.78)</td>
<td>5.76</td>
<td></td>
</tr>
<tr>
<td>86 - 90</td>
<td>0.833***</td>
<td>(5.44)</td>
<td>5.45</td>
<td></td>
</tr>
<tr>
<td>over 90</td>
<td>1.030***</td>
<td>(5.75)</td>
<td>5.79</td>
<td></td>
</tr>
</tbody>
</table>

### Constant

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.418*</td>
<td>(-1.80)</td>
<td>-1.80</td>
<td></td>
</tr>
</tbody>
</table>

### INFORMAL CARE

### Self reported health status (REF: Good / Very Good)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fairly Good</td>
<td>0.420</td>
<td>(1.62)</td>
<td>1.56</td>
<td></td>
</tr>
<tr>
<td>Bad</td>
<td>0.233</td>
<td>(0.89)</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>Very Bad</td>
<td>0.0530</td>
<td>(0.16)</td>
<td>0.09</td>
<td></td>
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</tbody>
</table>

### Person has Alzheimer (REF: No)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>0.658**</td>
<td>(2.36)</td>
<td>2.18</td>
<td></td>
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</tbody>
</table>

### Sum of ADL limitations (REF: 1)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.184</td>
<td>(-0.86)</td>
<td>-0.57</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.135</td>
<td>(-0.49)</td>
<td>-0.58</td>
<td></td>
</tr>
<tr>
<td>3 or more</td>
<td>0.442*</td>
<td>(1.68)</td>
<td>1.25</td>
<td></td>
</tr>
</tbody>
</table>

### Sum of IADL limitations (REF: 1)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.673**</td>
<td>(-1.97)</td>
<td>-1.89</td>
<td></td>
</tr>
<tr>
<td>Education Level</td>
<td>Log Likelihood</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>----------------</td>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.565**</td>
<td>(2.20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.388***</td>
<td>(4.96)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.128***</td>
<td>(7.75)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.985***</td>
<td>(6.76)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.682***</td>
<td>(8.62)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 or more</td>
<td>2.756***</td>
<td>(8.69)</td>
<td></td>
<td></td>
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</table>

### Highest Degree (Education) (REF: Primary or lower secondary School)

<table>
<thead>
<tr>
<th>Degree Level</th>
<th>Log Likelihood</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No Degree</td>
<td>-0.0855</td>
<td>(−0.54)</td>
</tr>
<tr>
<td>Upper Secondary</td>
<td>-0.621***</td>
<td>(−2.62)</td>
</tr>
<tr>
<td>Post Secondary</td>
<td>-0.749**</td>
<td>(−2.07)</td>
</tr>
</tbody>
</table>

### Living place (REF: town with less than 20,000 residents)

<table>
<thead>
<tr>
<th>Location</th>
<th>Log Likelihood</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td>0.238</td>
<td>(0.99)</td>
</tr>
<tr>
<td>20,000 - 100,000</td>
<td>0.573**</td>
<td>(2.25)</td>
</tr>
<tr>
<td>More than 100,000</td>
<td>0.594***</td>
<td>(2.70)</td>
</tr>
<tr>
<td>Paris</td>
<td>0.165</td>
<td>(0.56)</td>
</tr>
</tbody>
</table>

### Person lives in a DOM (REF: No)

<table>
<thead>
<tr>
<th>Living Place</th>
<th>Log Likelihood</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>0.362</td>
<td>(1.45)</td>
</tr>
<tr>
<td>Paris</td>
<td>0.177***</td>
<td>(5.16)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Age (REF: 60 - 65)</th>
<th>Log Likelihood</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>66 - 70</td>
<td>0.265</td>
<td>(0.74)</td>
</tr>
<tr>
<td>71 - 75</td>
<td>0.0648</td>
<td>(0.21)</td>
</tr>
<tr>
<td>76 - 80</td>
<td>0.0641</td>
<td>(0.21)</td>
</tr>
<tr>
<td>81 - 85</td>
<td>0.201</td>
<td>(0.65)</td>
</tr>
<tr>
<td>86 - 90</td>
<td>0.410</td>
<td>(1.29)</td>
</tr>
<tr>
<td>Over 90</td>
<td>0.775**</td>
<td>(2.10)</td>
</tr>
<tr>
<td>Constant</td>
<td>-2.468***</td>
<td>(5.15)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Log Likelihood</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Likelihood</td>
<td>1.291***</td>
</tr>
<tr>
<td></td>
<td>1.292***</td>
</tr>
<tr>
<td></td>
<td>2.399***</td>
</tr>
<tr>
<td></td>
<td>2.493***</td>
</tr>
<tr>
<td>theta</td>
<td>-0.829*</td>
</tr>
<tr>
<td>alpha1</td>
<td>-0.132**</td>
</tr>
<tr>
<td>alpha2</td>
<td>-0.252*</td>
</tr>
</tbody>
</table>

| Observations    | 1705           | 1705 |

- *p < .1, **p < .05, ***p < .01

Table 6: Likelihood of different Copula Functions
Gumbel divergence ($\theta \to +\infty$)

**Table 7: Marginal Effects: Informal Care -1h**

<table>
<thead>
<tr>
<th>Formal Care</th>
<th>Informal Care less than Median</th>
<th>Informal Care Median or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than Median</td>
<td>3 1</td>
<td>14 4</td>
</tr>
<tr>
<td>Median or more</td>
<td>0 0</td>
<td>0 0</td>
</tr>
</tbody>
</table>

*Note: Median of positive care hours; effect in minutes per week*

**Table 8: Marginal Effects: Formal Care +1h**

<table>
<thead>
<tr>
<th>Formal Care</th>
<th>Informal Care less than Median</th>
<th>Informal Care Median or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than Median</td>
<td>0 -13 -60</td>
<td>0 -19</td>
</tr>
<tr>
<td>Median or more</td>
<td>0 -5 -19</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note: Median of positive care hours; effect in minutes per week*

**Table 9: Marginal Effects: other Variables**

<table>
<thead>
<tr>
<th>Distribution of Effect</th>
<th>1%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alzheimer: No (\rightarrow) Yes</td>
<td>Formal Care</td>
<td>0</td>
<td>0</td>
<td>31</td>
<td>58</td>
</tr>
<tr>
<td>Informal Care</td>
<td>0</td>
<td>0</td>
<td>28</td>
<td>1h 50</td>
<td>10h 33</td>
</tr>
<tr>
<td>ADL: 1 (\rightarrow) 3 or more</td>
<td>Formal Care</td>
<td>0</td>
<td>59</td>
<td>1h 45</td>
<td>2h 56</td>
</tr>
<tr>
<td>Informal Care</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>55</td>
<td>5h 42</td>
</tr>
<tr>
<td>Children: + 1</td>
<td>Formal Care</td>
<td>-34</td>
<td>-7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Informal Care</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>26</td>
<td>3h 12</td>
</tr>
</tbody>
</table>
## B  Precisions on copula functions

A first approach to measure dependency between the error terms would have been to study the Bravais-Pearson’s linear correlation. This indicator is powerful when the dependency between random variables is linear. When it is not obvious, other indicators of dependency exist, based on concordance between independent draws of our variables of interest in the sample. In this study, a non-linear and non-parametric correlation coefficient is used : the Kendall’s Tau (\(\tau\)). This is a global indicator of dependency between random variables. It is defined as the difference between the probability of concordance minus the probability of discordance between two independent draws from the bivariate distribution of our error terms. Like the linear correlation coefficient, its values vary between -1 and 1, 1 meaning a perfect concordance. Thus in our model, a positive dependency means that the unobserved variable explaining the caregiving time of one type of care (formal or informal) change in the same direction as those explaining the other. The copula function allows to measure dependency with a dependency function. Indeed, the dependency indicator (Kendall’s tau) can be defined as a function of the \(\theta\) parameter of the studied copula.

Tail dependency indicators can also be calculated. Tail dependency describes the strength of the association between two random variables when they take extreme values (small or large). According to Nelsen (2006) [17], the left-(resp. right-)tail dependency parameter, \(\lambda_G\) (resp. \(\lambda_D\)), verifies the following equality :

\[
\lambda_G = \lim_{t \to 0^+} \frac{C(t, t)}{t} \quad \lambda_D = \lim_{t \to 1^-} \frac{1 - 2t + C(t, t)}{1 - t}
\]

We limit our study to several simple copula functions: Archimedian copulas. These one are parametric and depend on a unique \(\theta\) parameter. Their characteristics are presented in what follows.

- **Product copula**

  \[C(a, b) = ab\]

  The Product copula do not depends on any \(\theta\) parameter. If \(C\) is the Product copula, \(F_1\) and \(F_2\) are maginal cumulative distribution functions of the studied random variables \(Y_1\) and \(Y_2\), and \(F\) their joint cumulative distribution function, then the Sklar’s theorem (1973) [20] implies :

  \[F(y_1, y_2) = C(F_1(y_1), F_2(y_2)) = F_1(y_1)F_2(y_2)\]

  The Product copula corresponds to the case where the random variables are independent.

- **Frank copula ( \(\theta \in ]-\infty; 0[ \cup ]0; +\infty[\) )**

  \[C_\theta(a, b) = -\frac{1}{\theta} \ln \left[ 1 + \frac{(e^{-\theta a} - 1)(e^{-\theta b} - 1)}{(e^{-\theta} - 1)} \right]\]

  \[\theta \in ]-\infty; 0[ \cup ]0; +\infty[\)
Kendall’sTau: \( \tau = 1 - 4 \frac{1 - D_{1}(\theta)}{\theta} \)

with: \( D_{1}(x) = \frac{1}{2} \int_{0}^{x} \frac{e^{-t} - 1}{t} dt \approx \frac{1}{3} \theta - \frac{1}{90} \theta^3 + \frac{1}{270} \theta^5 - \ldots \) (Taylor’s approximation in 0, for “moderate” values of \( \theta \) (Nelsen, 2006, p.186)

\[ \lambda_{G} = 0 \quad \lambda_{D} = 0 \]

The Frank copula allows every type of dependency (positive or negative). It converges to the Product copula when \( \theta \) converges to 0 (independency). It does not allow any tail dependency.

- Farlie-Gumbel-Morgenstern (FGM) copula (\( \theta \in [-1; 1] \))

\[ C_{\theta}(a, b) = ab[1 + \theta(1 - a)(1 - b)] \]

\[ \tau = \frac{2}{9} \theta \quad \lambda_{G} = 0 \quad \lambda_{D} = 0 \]

The Frank copula allows every type of dependency (positive or negative), as soon as it remains moderate: Its Kendall’sTau varies indeed between \(-\frac{2}{9}\) et \(\frac{2}{9}\).

The FGM copula equals the Product copula when \( \theta \) equals 0. It does not allow any tail dependency.

- Clayton copula (\( \theta \in [0; +\infty[ \))

\[ C_{\theta}(a, b) = (a^{-\theta} + b^{-\theta} - 1)^{-1/\theta} \]

\[ \tau = \frac{\theta}{\theta + 2} \quad \lambda_{G} = 2^{-\frac{1}{\theta}} \quad \lambda_{D} = 0 \]

Clayton copula only allows positive dependency: Its Kendall’s Tau is indeed strictly positive. It converges to the Product copula when \( \theta \) converges to 0 (independency). It captures a left-tail dependency (i.e. for small values of the studied random variables) but no right-tail dependency (for large values of the studied random variables).

- Gumbel copula (\( \theta \in [1; +\infty[ \))

\[ C_{\theta}(a, b) = \exp \left[ - \left( \tilde{a}^{-\theta} + \tilde{b}^{-\theta} \right)^{1/\theta} \right], \text{ avec } \tilde{a} = -\ln(a) \text{ et } \tilde{b} = -\ln(b) \]

\[ \tau = \frac{\theta - 1}{\theta} \quad \lambda_{G} = 0 \quad \lambda_{D} = 2 - 2^{\frac{1}{\theta}} \]

Gumbel copula only allows positive dependency or independency: Its Kendall’sTau is indeed positive or 0. The Gumbel copula equals the Product copula when \( \theta \) equals 0 (independency). It captures a right-tail dependency (i.e. for large values of the studied random variables) but no left-tail dependency (for small values of the studied random variables).
References


