Survival of Hedge Funds: Frailty vs Contagion

Darolles, S., Gagliardini, P., Gourieroux, C.
1. INTRODUCTION
Hedge Funds vs Mutual Funds: The Differences

Mutual Funds: regulated investment vehicles with explicit reference to a benchmark.

Hedge Funds: unregulated offshore investment vehicles:

i) For qualified investors only.

ii) No reference to a benchmark.

iii) Specific investment techniques (leverage, short selling, ...).
Hedge Funds Analysis: The Difficulties

i) Only private/partial information (style, historical data, holdings, ....) available through commercial databases.

ii) Non Gaussian marginal distribution of returns implied by both dynamic investment strategies and fee structures (Darolles, Gourieroux (2010)).

iii) Complex serial correlation structure between returns (state dependency).

iv) High liquidation risk.
Liquidation

The majority of hedge funds have rather short lifetimes, with a median of 6-7 years.

However, their mortality rates, as well as the dependence among them, vary considerably according to the year and management style (between 4% and 30% on the period 1994-2003).
Different **reasons for liquidating** an Hedge Fund:

i) The hedge fund return is not attractive for the investor.

ii) The incentive for the fund manager is not sufficient (she does not reach frequently the high water mark performance objective and prefers to create a new fund with more attractive management fees).

iii) There is a need of liquid assets by the shareholders (especially in a situation of liquidity crisis).

iv) A consequence of operational risk including fraud, unauthorized trading, inadequate resources, misappropriation of investor funds.
Liquidation risk (1/2)

From the investor’s point of view, liquidation is a risk, which concerns both the **timing of the payoffs** and the **value of the fund** at liquidation time.

It is partially at the discretion of the fund manager, who can slow down, or accelerate liquidation by an appropriate management of gates for instance.

The liquidation risk is **similar to default risk**, prepayment risk, or lapse risk encountered on corporate bonds, credit derivatives, or life insurance contracts, respectively.
Liquidation risk (2/2)

Liquidation risk is especially important for:

(i) **The institutional investors** : endowments, foundations, pension funds (if they have a minimal capital) invest on a long term basis and want to avoid the consequences of a short term liquidity crisis.

(ii) **The funds of funds** can be very sensitive to risk dependencies, in particular to liquidation correlation.

Moreover, an inappropriate understanding of liquidation risk and liquidation correlation can bias the performance analysis of funds and fund portfolios, the so-called survivorship bias (censored data).
HF lifetimes analysis is generally based on duration models.

Can be assumed parametric, semi-parametric, nonparametric.

Can include individual (and individual time dependent) HF characteristics, time dependent market characteristics.

However, assume the independence between the individual liquidation risks given the selected explanatory variables.
This paper introduces new models to analyze liquidation risk dependence and distinguishes:

i) the effect of common unobserved exogenous shocks (also called frailty, or systematic risk factor).

ii) the contagion effect when an endogenous shock on one fund has an impact on the other ones.
i) Example of exogenous shock

The withdrawal of some prime broker (amplified by the leverage effect), or the outflow of a category of investors.

Effect by means of the liability component of the balance sheet.

ii) Example of endogenous shock

The HF are often invested in illiquid assets.

A fund can be obliged to sell a rather large volume of such an asset (especially when close to liquidation). This will decrease the "market price" of this asset due to the lack of liquidity and the NAV of the other funds invested in this illiquid asset.

Effect by means of the asset component of the balance sheet.
2. THE MODEL
The microscopic model (or binomial model)

At a given date $t$, the HF are classified:

by type $k = 1, \ldots, K$ [management style, registration country, ...]

by age $h = 1, \ldots, H$.

Number of HF at the beginning of the period: $n_{k,h,t}$

Number of liquidated HF during the period: $Y_{k,h,t}$

Underlying exogenous common shock (frailty): $F_t$
The model specifies the conditional distribution of the numbers of liquidated HF by type and age given the path of the dynamic frailty $F_t$ and the past histories $Y_{t-1}$.

**Assumption A.1:** Homogeneity by categories: type $\times$ age

\[ Y_{k,h,t}, k, h \text{ varying, are independent,} \]
\[ Y_{k,h,t} \sim \mathcal{B}[n_{k,h,t}, p_{k,h,t}(\theta)]. \]

**Assumption A.2:** Specification of the mortality intensity

\[ \tilde{\lambda}_{k,h,t}(\theta) = -\log[1 - p_{k,h,t}(\theta)] \]
\[ = a_{k,h}(\theta) + b_{k,h}(\theta)'F_t + c_{k,h}(\theta)'Y_{t-1}^*, \]

where $Y_{t-1}^*$ = (lagged observed mortality rates in the different categories at the source of possible contagion).
Assumption A.3: The common frailty is unobservable. This is a Markov process with transition density:

\[ g(f_t | f_{t-1}; \beta). \]

The two sources of liquidation risk dependence are:

- the **common frailty**: Vaupel, Manton, Stallard (1979). Since it is unobservable, it has to be integrated out to deduce the joint distribution of counts \((Y_{k,h,t})\). This creates a first dependence.

- the **effect of lagged counts** \(Y^*_{t-1}\), which introduces jumps in the mortality intensity at each HF liquidation [Freund (1961)]. This captures the multiple contagion effects.
The model is:

- flexible enough to test for contagion and/or frailty effects by analyzing the significance of the regressors in the mortality intensity:
  \[ \iff \text{considering if } b_{k,h}(\theta) = 0, \text{ or } c_{k,h}(\theta) = 0 \]

- But complicated since the likelihood function involves multiple integrals with large dimension due to the integration w.r.t. the unobserved dynamic frailty path.
• The effects of frailty and contagion can be significantly different [Gagliardini, Gourieroux (2011)]. With a single type and considering the conditional overdispersion:

\[ \tau(Y_{t-1}) = \frac{V(Y_t | Y_{t-1})}{E(Y_t | Y_{t-1})} \]

An increase of frailty \( F \rightarrow \) positive drift on \( \tau \)
An increase of contagion \( \rightarrow \) change in the slope of \( \tau \).

• Thus, it is theoretically possible to disentangle frailty and contagion effects.

• However, this might be difficult empirically. Indeed, these effects cannot be identified in a static framework: the reflection problem [Manski (1993)].
The macroscopic model (or Poisson approximation)

Is it possible to circumvent the numerical difficulties created by the multiple integrals?

Partly by considering the limiting model obtained for large size categories: \( n_k, h, t \sim \gamma_k, h, t n \), for \( n \to \infty \).

Let us consider the Poisson approximation of the Binomial distribution:

\[
\lim_{n \to \infty} np_{k, h, t}(\theta) = \lambda_{k, h, t}(\theta) \geq 0.
\]

Then:

\[
Y_{k, h, t} \sim \mathcal{P}[\gamma_{k, h, t} \lambda_{k, h, t}(\theta)],
\]

\[
\lambda_{k, h, t}(\theta) = a_{k, h}(\theta) + b_{k, h}(\theta)' F_t + c_{k, h}(\theta)' Y_{t-1}^*.
\]
The advantage of the macroscopic model

The conditional log-Laplace transform

$$\log \psi_t(u) = \log E[\exp(-uY_t)|F_t, Y_{t-1}]$$

is affine in $F_t, Y_{t-1}$.

By choosing an affine process for the dynamic frailty, for instance a time discretized Cox-Ingersoll-Ross process,

$$E[\exp(-uF_t)|F_{t-1}, Y_{t-1}] = \exp[-\alpha(u)F_{t-1} - \beta(u)],$$

we get a joint affine process for $(Y_t, F_t)$.

$\rightarrow$ explicit recursive formulas for nonlinear predictions at any horizon;

$\rightarrow$ tractable nonlinear filter to derive the maximum likelihood estimators [Bates (2009)].
3. DYNAMIC PROPERTIES OF THE MODEL
A simulation study

2 types ($K = 2$), a single frailty factor Autoregressive Gamma (ARG) process

Similar subpopulation sizes: $n_{k,h,t} = 20, \forall k, h, t$.

Contagion effect by means of $Y_{k,t-1}^* = \sum_{h=1}^{H} Y_{k,h,t-1}, \ k = 1, 2$.

**Model 1:** Mortality intensity independent of age

$$\lambda_{k,h,t} = a_k + b_k F_t + c_k,1 Y_{1,t-1}^* + c_k,2 Y_{2,t-1}^*.$$  

**Model 2:** Mortality intensity with proportional hazard

$$\lambda_{k,h,t} = [1 + (\mu_{0,k})^h(1 + \mu_{1,k} h)](a_k + b_k F_t + c_k' Y_{t-1}^*),$$

with a bump in the baseline intensity.
A simulation study with Model 1

We set the parameters to be able to describe various frailty and contagion effects.

Set F.1: $\eta = 0$ implies a static factor.
Set F.2: $\eta = 0.99$, that is $\rho = 0.33$ implies a persistence of the factor.

The sensitivity parameters of the frailty are set to the following values:

Set SF.1: $b_1 = b_2 = 0$, which means no frailty effect.
Set SF.2: $b_1 = b_2 = 0.5$, to capture a common frailty.
A simulation study with Model 1

The contagion parameters are set to:

Set C.1: \[ C^* = \begin{pmatrix} c_{11}^* & c_{12}^* \\ c_{21}^* & c_{22}^* \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \] which corresponds to no contagion within types or between types.

Set C.2: \[ C^* = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \] which captures a contagion within types and no contagion between types.

Set C.3: \[ C^* = \begin{pmatrix} 0 & 0.0 \\ 0.2 & 0 \end{pmatrix}, \] which is a recursive model of contagion.
F.1-SF.1-C.1: No Frailty No Contagion

- Frailty with $\eta = 0, \delta = 2, \nu = 0.5$
- Mortality Count Type 1 with $a_1 = 1, b_1 = 0, c_{1,1} = 0, c_{1,2} = 0$
- Mortality Count Type 2 with $a_2 = 1, b_2 = 0, c_{2,1} = 0, c_{2,2} = 0$
F.1-SF.2-C.1: Only Frailty with Uncorrelated Factor Effect
F.2-SF.2-C.1: Only Frailty with Correlated Factor Effect
F.2-SF.2-C.2: Frailty with Correlated Factor Effect and Direct Contagion
F.2-SF.2-C.3: Frailty with Correlated Factor Effect and Recursive Contagion
<table>
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<th>F.1-SF.1-C.1</th>
<th>F.1-SF.2-C.1</th>
<th>F.2-SF.2-C.1</th>
<th>F.2-SF.2-C.2</th>
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<td>-0.057</td>
<td>0.3521</td>
<td>0.3521</td>
<td>0.3521</td>
</tr>
</tbody>
</table>

| Mortality Count Type 1 |              |              |              |              |              |
| Mean                  | 1.0304       | 1.972        | 2.0164       | 2.4692       | 2.0164       |
| Variance              | 1.0275       | 2.4706       | 2.6044       | 3.3064       | 2.6044       |
| Dispersion            | 0.9972       | 1.2528       | 1.2916       | 1.339        | 1.2916       |
| Correlation           | 0.0416       | -0.0193      | 0.0629       | 0.2834       | 0.0629       |

| Mortality Count Type 2 |              |              |              |              |              |
| Mean                  | 0.9784       | 1.9724       | 2.3988       | 2.5224       | 2.3988       |
| Variance              | 0.9495       | 2.6095       | 2.4508       | 3.3068       | 3.1314       |
| Dispersion            | 0.9705       | 1.323        | 1.2403       | 1.311        | 1.3054       |
| Correlation           | 0.0438       | -0.0247      | 0.0811       | 0.2604       | 0.0774       |

| Cross Correlation between Mortality Counts |              |              |              |              |              |
| Correlation                       | 0.0279       | 0.2253       | 0.1789       | 0.2359       | 0.2129       |
**Figure 1**: no Frailty, no contagion
Two count white noises with Poisson distribution
mean=variance, overdispersion = 1, serial correlation = 0,
Cross correlation = 0

**Figure 2**: Frailty, no contagion
increase of overdispersion, serial correlation = 0,
positive cross-correlation, common peaks

**Figure 3**: Correlated Factor, frailty, no contagion
increased serial correlation, some clustering in frailty
not necessarily visible on counts.

**Figure 4**: plus direct contagion
different clustering effects on both counts
large increase in persistency

**Figure 5**: plus recursive contagion
Observe the magnitude of the main peak on count 2 coming from
a direct impact of the frailty plus the indirect impact of count 1.
Conditional dispersion with Frailty and Contagion

- □ frailty, no contagion
- * frailty and contagion

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4. EMPIRICAL RESULTS
The Data

- The age is measured since the inception date (we do not take into account the incubation period).
- The "Live" and "Graveyard" databases are considered together. Dead funds are in the Graveyard, but also in the "Live" database (when they no longer report their performance during a sufficiently long time).
- The funds of funds are eliminated.
- To apply the Poisson approximation within the management styles, we select the styles with a sufficient size.
Long/Short Equity : 38%
Event Driven : 10%
Managed Futures : 11%
Equity Market Neutral : 7%
Fixed Income Arbitrage : 5%
Global Macro : 7%
Emerging Markets : 8%
Multi-Strategy : 10%
Convertible Arbitrage : 4%
Number of Funds by strategy

- Convertible Arbitrage
- Emerging Markets
- Equity Market Neutral
- Event Driven
- Fixed Income Arbitrage
- Global Macro
- Long/Short Equity Hedge
- Managed Futures
- Multi-Strategy

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Marginal effects of time and age

We report the frequency counts of defunct HF:

by age

by date

to detect the marginal effects of these two notions of time for funds in each category.
Mortality Intensity per age
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<th>Maximum</th>
<th>Right Boundary</th>
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<td>Multi-Strategy</td>
<td>0.052</td>
<td>0.119</td>
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</table>
Frequency counts:
- not adjusted for the cohort size
- common effect of the crisis, except for Global Macro
- the effect of LTCM failure in 1998 for ”Emerging Markets”

Mortality intensity:
- A bump around 4 years
- similar patterns
- but these patterns are not parallel: no proportional hazard
Cross-effects of time and age

Difficult to observe with marginal analysis

Lexis diagrams for each HF strategy give some insight on the joint effect of time and age:

The date of death is given in the $x$–axis,

The age at death is on the $y$–axis.

The diagonals characterize the different cohorts.
Lexis Diagram
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Emerging Market:
- age effect around 20 months
- effect of the crisis
- regular mortality for the cohorts starting in 1993

Multi-Strategy:
- Essentially an age effect

Managed Future:
- Two time effects around 2003 and the crisis
Estimation of a model with contagion only:
This is a Poisson regression model, with lagged counts as regressors

\[ Y_{k,h,t} \sim \mathcal{P} \left( \gamma_{k,h,t} \left( a_k + \sum_{k' = 1}^{K} c_{k,k'} \frac{Y_{k',t-1}^*}{\gamma_{k',t-1}^*} \right) \right), \]

with two adjustments for cohort sizes

- intercept parameters
- contagion matrix

Possibility to aggregate size adjusted data w.r.t. age
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The contagion scheme

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Estimation of a model with both contagion and frailty

Much more difficult, but the parameters of interest to understand direct and indirect dependence can be calibrated from second-order moment conditions:

\[
V(Y_t^*) = CV(Y_t^*)C' + diag[E(Y_t^*)]
+ \sigma^2 bb' + \sigma^2 \rho C(Id - \rho C)^{-1} bb' + \sigma^2 \rho bb'(Id - \rho C')^{-1} C',
\]

\[
\text{Cov}(Y_t^*, Y_{t-1}^*) = \rho^2 \sigma bb'(Id - \rho C')^{-1} + CV(Y_t^*),
\]

with : \( \rho = \eta \nu, \sigma^2 = 1/\delta. \)
This is enough to estimate the parameters of interest:

- to test the presence of contagion and/or frailty
- to analyze the decomposition of serial correlation.

\[
\begin{align*}
\text{Cov}(Y^*_t, Y^*_{t-1}) V(Y^*_{t-1})^{-1} \\
= C + \sigma^2 \rho bb'(I_\theta - \rho C')^{-1} V(Y^*_t)^{-1}
\end{align*}
\]

for each management style.
5. CONCLUSION
The aim of this paper is to disentangle the two sources of dependence between mortality risks:

- the exogenous effects, described by the unobserved frailty,
- the endogenous contagion effects, corresponding to the impact of lagged counts.

Such analysis is a preliminary step before measuring and managing systematic risk in the hedge fund market.