Legal Institutions, Innovation and Growth*

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Abstract. We analyze the relationship between legal institutions, innovation and growth. We compare a rigid (law set ex-ante) legal system and a flexible one (law set after observing current technology). The flexible system dominates in terms of welfare, amount of innovation and output growth at intermediate stages of technological development — periods when legal change is needed. The rigid system is preferable at early stages of technological development, when (lack of) commitment problems are severe. For mature technologies the two legal systems are equivalent. We find that rigid legal systems may induce excessive (greater than first-best) R&D investment and output growth.

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1. Introduction

Technology and legal rules are bound to coevolve. Early examples of technology that impacted the legal system are steamboats and railroads. They brought up a variety of unprecedented cases and placed novel demands on the law. Steamboats proved risky because of fires from sparks and boilers explosions, leading eventually to the responsibility for steamboat owners and captains to prove non negligent behavior in case of litigation (Khan, 2004). Liability rules were also challenged by railroads because of sparks on crop and incredibly numerous injuries and fatalities (Ely, 2001).

The American system of copyright law system also experienced changes because of technological innovation: “Copyright decisions illustrate how adjudication by analogy economized on the costs of technological transitions. Still, many of the technological innovations of the nineteenth century (photography) were sufficiently different from existing technologies as to make judicial analogies somewhat strained, and ultimately required accommodation by the legislature instead” (Khan, 2004, p. 20). Even more recent innovations posed similar worries, for instance in the 80’s it was not clear whether computer source code could be considered literary works for the purposes of copyright law (Bennett Moses, 2003).\(^1\)

Other technological advances that also demanded legal innovation include medicine (e.g., in vitro fertilization and genetic testing), automobiles, computing, and communication (e.g., telegraphy and, more recently, the internet).\(^2\)

This paper analyzes the link between legal institutions, innovation and growth. In particular, it investigates how legal institutions deal with the challenges presented by technological innovation. In its relationship with technological change, a legal system faces (at least) two challenges. First, since new technologies may require different legal rules, a legal system must adapt to changing conditions. Second, to the extent that the legal framework affects the investment climate, a legal system should also be judged by its capacity to provide incentives to innovate.\(^3\) For instance, an adaptable legal system is of no use in the absence

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1. The development of the right to privacy is another interesting illustration of a new legal concept meant to cope with the (negative) externalities following the adoption of new technologies such as photography (Khan, 2004).

2. See Khan (2004) and Friedman (2002) for a discussion of how the US legal institutions responded to various examples of technological innovation.

3. The importance of the legal and regulatory frameworks on investment (and innovation) strongly emerges from the Investment Climate Surveys recently launched by the World Bank.
of change. To address these issues, we analyze a stylized model of endogenous technological progress. More specifically, as in Aghion and Howitt (1992) we consider a model where innovations improve the quality of existing products and make old products obsolete. In the context of our model, the amount of R&D investment (and, consequently, the probability that new technologies are discovered) depends on the law expected in the new technological environment.

We study two legal systems: a rigid regime and a flexible regime. The two regimes differ with respect to their ability to adapt to changing conditions. The reason why we focus on this particular distinction is twofold. First, at least since Posner (1973) flexibility is usually regarded as a key feature that differentiates Common Law from Civil Law. Second, recent empirical work (namely, Beck et al., 2003) has provided some evidence that the adaptability of Case Law partly explains some of the benefits of Common Law for financial and other variables.

This paper assumes that in the rigid legal system, courts or regulators have no discretion and are bound to enforce the existing rule (statute or administrative regulation). Statutes and regulations are written ex-ante, before knowing the technology that will prevail. Following the incomplete contract literature (e.g., Grossman and Hart, 1986, and Hart and Moore, 1990) we assume that ex ante it is not possible to describe accurately future contingencies, so that in the rigid regime the existing rule is assumed to be non-contingent. In other words, the same law is applied in all technological environments. In our model of the rigid regime we also assume that the legislator (or regulator) understands how the law affects the incentives to innovate and knows the payoff consequences of the law in each technological environment.

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4In the growth literature, quality-improving innovations are known as “vertical” innovations. Aghion and Howitt (1992) and Grossman and Helpman (1991) are the seminal papers on growth with vertical innovation. See Romer (1987, 1990) for growth models where innovation is horizontal (i.e., innovators expand the variety of available goods).

5As argued in Beck and Levine (2005), “legal systems that embrace case law and judicial discretion tend to adapt more efficiently to changing conditions than legal systems that adhere rigidly to formalistic procedures and that rely more strictly on judgements based narrowly on statutory law.”

6Beck et al. (2003) construct a measure of adaptability of the legal system that takes into account whether judicial decisions are based on previous court decisions and on principles of equity rather than on statutory law. Their measure uses data from Djankov et al. (2003) and La Porta et al. (2004).

7In Section 4.4 below we weaken this assumption and consider the possibility that the statute or regulation can be changed at a cost.

8Similarly, the incomplete contract literature assumes that the contracting parties cannot write a fully-contingent contract but they correctly anticipate the consequences of their actions in all future states of the
In the flexible legal regime courts and regulators are assumed to have discretion and choose the law ex post, after observing the current state of the technology. The law is therefore state-contingent.

Leaving discretion ex post seems a sensible choice, especially in periods of rapid and ongoing technological change. Are there instances where the lack of flexibility of the rigid regime is preferable? The answer is “yes” since in our model law-makers suffer from credibility problems: the ex-ante optimal law, which is the law that provides better incentives to innovate, is not always optimal ex-post, once innovation has taken place. The rigid regime does not suffer from commitment problems because rules, which law-enforcers are bound to follow, are written ex ante. However, as discussed above, the drawback of the rigid regime is that it is ill-suited to changing conditions because the statute (or regulation) prescribes the same law in all contingencies.

For example, in their survey of the development of intellectual property institutions in the US, Khan and Sokoloff (2001) hint at the role played by the legal system in reinforcing the effectiveness of the patent system. They provide a clear example underlying the possible advantages of (ex ante) certainty and lack of flexibility in shaping the incentives to innovate.\(^9\)

It is then apparent that the choice between the two legal systems involves a trade-off between commitment and flexibility. In this paper, we argue that the terms of this trade-off change over time as technology matures. Consequently, legal institutions that are appropriate in the early stages of technological development may no longer be preferable.

Needless to say, the trade-off between commitment and flexibility (in other words, between rules and discretion) has long been studied in macroeconomics. However, we want to emphasize one important point of distinction from the rule-versus-discretion literature. This literature assumes that the degree of uncertainty, which is the crucial parameter to evaluate the trade-off, is exogenous.\(^10\) Instead, in this paper the degree of uncertainty (which is related

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\(^9\)Specifically, treating the introduction of the 1836 Patent Act they write that: “The previous registration system, modeled on British practices, had left issues of novelty and validity or appropriate scope in patent applications to be resolved through civil actions, which proved to be an inefficient way of resolving competing claims. [...] The change led to a substantial increase in the potential returns to inventive activity.” (Khan and Sokoloff 2001, p. 236).

\(^10\)For example, Rogoff (1985) compares rigid targeting systems and flexible monetary regimes. The key parameter in his comparison is the variance of aggregate productivity shocks: intuitively, rigid regimes are preferable if uncertainty is low. More recently, Amador et al. (2006) study the optimal trade-off between
to the speed of technological change) is *endogenous* (via R&D investment) and depends on the chosen rule.

The assumption that legal rules (laws) are incomplete (that is, not contingent on the realized state) and that the underlying uncertainty is endogenous have important implications in our model of the rigid regime. For example, consider the problem of a legislator who has to write a non-contingent law before knowing whether or not the status-quo technology will be replaced by a more advanced technology. When the likelihood of discovering the new technology is either very low or very high, the incompleteness constraint that the legislator faces in the rigid regime matters less: in either case, the legislator will simply select the rule that optimally regulates the most likely state. Since the probability of replacing the status-quo technology depends on the law that is selected ex ante and since a rigid system has a comparative advantage in a certain environment (where the incompleteness constraint matters less), the legislator has an incentive to choose a law that reduces the underlying uncertainty in the economy. In particular, he may end up selecting a rule that either discourages or, more surprisingly, strongly encourages R&D investment. The result is that the rate of growth in a rule-based system is either very low or excessive (greater than first-best). Conversely, overinvestment in R&D never occurs when legal institutions are flexible.

The goal of this paper is to study which legal system is better suited to maximize welfare. In the context of a simple model with only two technological states, we show that flexible legal systems dominate (in terms of welfare, amount of innovation and output growth) in economies at intermediate stages of technological development, these are periods when legal change is needed. Instead, rigid legal systems are preferable at the early stages of technological development, when commitment problems are more severe. Indeed, at the early stages of development considerations about consumers’ health and safety are more likely to matter. Since R&D firms correctly foresee that in the flexible regime law-makers will heavily regulate ex-post, investment in research is suboptimally low and the old, inefficient, technology is likely to survive. Finally, we show that when technology is mature, the two legal systems lead to the same economic outcomes.

commitment and flexibility in an intertemporal consumption/savings model with time inconsistent preferences and show that the optimal amount of flexibility depends negatively on the degree of disagreement (which measures the severity of the commitment problem) relative to the dispersion of taste shocks. In both papers the degree of uncertainty is exogenously given.
The rest of the paper is organized as follows. Section 2 briefly discuss the related literature. In Section 3 we present the basic model with two possible technologies and characterize the optimal laws for each technology. Section 4 compares the rigid and flexible regimes. Section 5 analyzes a dynamic model where technology undergoes continuous change and derive similar results to the ones obtained in the basic model. In the stationary equilibrium of the rigid regime the speed of technological change is either very low or very high. In the flexible regime, because of lack of commitment, investment in R&D is especially low at the early stages of technological development. Section 6 concludes. For ease of exposition all proofs are in the Appendix.

2. Related Literature

To the best of our knowledge, Anderlini et al. (2011) is the first paper to consider time-inconsistency problems in judicial decision making. That paper considers a model of Case Law in which the judges suffer from an ex-post temptation to be excessively lenient that stems from the fact that all economic decisions are sunk by the time the parties go to court. In this set up, there is a specific role for the rule of precedent (stare decisis).

There is a sequence of cases, each considered by a forward-looking court in a parallel sequence. Precedents, with some probability, bind the decisions of future courts, thus mitigating their tendency towards excessive leniency. Since each court can affect the state of precedents via its current decision, this creates an incentive for the current court, even though it rules ex-post, to avoid ex-ante inefficiently lenient decisions. The thrust of Anderlini et al. (2011) is to characterize the optimal trade-off created by these incentives. The state variable there is the current “state of precedents,” while here it is the current state of technology.

Kaplow (1992) is a fundamental and wide-scoped work on the economics of “rules versus standards” rooted in the scholarly tradition of law. A rule is a law with an ex-ante prescription (it has ex-ante “content”) while a standard only acquires “content” ex-post.\footnote{As a example, a rule might prescribe that is it forbidden to drive “over 55 miles per hour,” while a standard would forbid “excessive speed.” See Kaplow (1992), p. 560.} The back-bone of the analysis in Kaplow (1992) is the study of the trade-offs that (normatively) drive the choice between rules and standards as they apply to the economic sphere. While he explores many variations and extensions of the basic set-up, the main trade-off he identifies is due to the fact that rules are more expensive to formulate ex-ante, while standards are more
expensive to interpret (and hence enforce) ex-post. As a result, an important consideration in the choice of a rule versus a standard is the frequency with which it will be invoked, and the heterogeneity of the pool of situations to be considered.

Comin and Hobijn (2009) analyze a model of lobbying and technology adoption and argue that countries where the legislative authorities have more flexibility, the judicial system is not effective, or the regime is not very democratic, new technologies replace old technologies more slowly. This happens because rigidity in lawmaking makes lobbying for protecting the old technology more difficult. The mechanism that explains why in their paper a rigid system may favor technological progress relatively to a flexible system is completely different from ours. In our model, the channel is twofold. First, flexibility may harm technological progress because of time consistency problems. This explains why law-makers in a flexible system may choose ex-post a law that is less favorable to inventors than the one in the first-best solution. Second, for the reasons explained above rigid systems may choose a law that is more favorable to investors compared to the first-best solution.

Similarly to this paper, Acemoglu et al. (2006) argue that the policies that increase growth in the early stages of development may be suboptimal at later stages. In particular, they formalize the Gerschenkron’s (1962) view that relatively backward economies should pursue an investment-based strategy, which relies on long-term (hence, rigid) relationships between entrepreneurs and financiers and on a less competitive environment. However, as the economy approaches the world technology frontier, they argue that countries should switch to an innovation-based strategy, which requires more short-term (hence, more flexible) relationships, better selection of firms and managers and more competitive policies.

Acemoglu et al. (2007) study the relationship between contractual incompleteness, technological complementarities, and technology adoption. In their model, a firm chooses its technology and investment levels in contractible activities with suppliers of intermediate inputs. Suppliers then choose investments in noncontractible activities, anticipating payoffs from an ex post bargaining game. Their paper argues that greater contractual incompleteness leads to the adoption of less advanced technologies, and that the impact of contractual incompleteness is more pronounced when there is greater complementarity among the intermediate inputs.\footnote{See also Acemoglu (2009, p. 801) for a discussion of the possibility of an hold-up problem in technology.
Finally, Immordino et al. (2011) analyze optimal policies when firms’ research activity leads to innovations that may be socially harmful. Public intervention, affecting the expected profitability of innovation, may both thwart the incentives to undertake research and guide the use of each innovation. In our setting we abstract from the enforcement problem, and we judge the optimality of a legal system by studying the trade off between its adaptability to technological change and its capacity to provide incentives to innovate.

Before moving on, we briefly discuss the large legal literature that has studied the interaction between law and technology. More specifically, various legal scholars have investigated how legislation (a relatively inflexible system) and common law adjudication (a relatively flexible system) deal with technological change. One of the main findings of this literature is that the main limitation of legislation is that rules, which are set in advance, likely suffer from either overinclusiveness or underinclusiveness. Conversely, being more gradual, common law adjudication may benefit from society’s experience with a technology. However, the literature has also pointed out that legislation has several merits. Legislatures have greater democratic legitimacy. Moreover, in drafting the law they can take a broader perspective since, unlike courts, they do not focus on the case at hand. Furthermore, legislation can act in advance and does not have to wait until the issue is litigated. As a result, it can act at a stage where technology is still capable of being shaped and technology is not irrevocably set.

3. The Basic Model

As in Aghion and Howitt (1992), we consider a model of endogenous technological change where new products provide greater quality than existing goods. The economy consists of three sectors: the R&D sector, the intermediate good sector and the final good sector. As discussed below, we focus on the law that disciplines the production process of the intermediate good. To keep our setting tractable and focus attention on the interaction between legal systems and innovation, our model of technological change is simplified along various adoption.\footnote{For instance, see Tribe (1973), Furrow (1982), Jasanoff (1995), Dworkin (1996), and Bennett Moses (2003).} It bears mentioning that the distinction between legislation and common law is often blurred: common law has become less flexible over time and, at the same time, legislatures are increasingly delegating to administrative agencies in order to enhance flexibility (Calabresi, 1982).
dimensions: for instance, the input prices in the R&D sector and in the intermediate good sector are assumed to be exogenously given.

3.1. Technology and Market Structure

The final good is produced competitively using the intermediate good. The production function of the final good is assumed to be Cobb-Douglas:

\[ y(i) = A(i) x(i)^{\frac{1}{2}}, \]  

(1)

where \( x(i) \) is the intermediate good and \( A(i) \) is a parameter that measures the productivity of the intermediate good. To keep matters simple, we assume that the output elasticity with respect to \( x(i) \) is \( 1/2 \).\(^{15}\) The index \( i \in \{0, 1\} \) denotes the state of technology sophistication of the intermediate good which is available in the current period.\(^{16}\) Technology 0 is assumed to be strictly less productive than technology 1: that is, \( A(1) = \gamma_A A(0) \), with \( \gamma_A > 1 \). The invention process (which will be described shortly) is stochastic. In particular, technology 1 is available only if the investment in R&D is successful. If R&D investment succeeds, the old technology becomes obsolete. In other words, we assume that the innovation is drastic.\(^{17}\)

The intermediate good sector is assumed to be a monopoly. Monopoly power derives from intellectual property: the intermediate good firm purchased the relevant patent from the R&D firm. We assume that the production of the intermediate good firm only uses (inelastically supplied) labor and that its marginal cost does not change with \( x(i) \) and is decreasing in \( a \),

\[ MC(a) = \frac{1}{a}, \]  

(2)

where \( a \) is the activity regulated by the law. We assume \( a \in [\underline{a}, \overline{a}] \), with \( \overline{a} > \underline{a} > 0 \). For instance, \( a \) can be thought as inversely related to the level of caution used in the production process. When \( a \) is high, the firm is not cautious and, consequently, its marginal cost is low.

\(^{15}\) Under this assumption, the indirect utility of the representative agent in the economy has a very simple form (see Subsection 3.6). This will allow us to obtain closed form solutions for the equilibrium laws in the two legal regimes. The main thrust of our results, however, does not change in a more general specification.

\(^{16}\) In Section 5 we take \( i \) to have values \( i = 0, 1, \ldots, \infty \).

\(^{17}\) Innovation is nondrastic if and only if the firm that uses the status-quo technology can make positive profits when the firm that produces the most advanced technology is charging the monopolistic price. As in Aghion and Howitt (1992) (Section V) innovations are drastic if \( \gamma_A \) is sufficiently high.
To abide by the law, the intermediate good firm must choose the activity level that the law prescribes. We also assume that \( a \) is observable at no cost, so that the law can be perfectly enforced.\(^{18}\) The price of the intermediate good (relatively to the final good) is denoted by \( p(i) \).

### 3.2. Research

The R&D firm chooses how much to invest in research. The amount of investment affects the probability of discovering the new technology for the intermediate good. Denoting by \( z \) the amount of investment by the R&D firm, we assume that the new technology is discovered with probability \( \theta z \), where \( \theta > 0 \) (the probability is equal to one if \( z \geq 1/\theta \)). The patent of the new technology is sold to a firm in the intermediate good sector. With probability \( 1 - \theta z \) there is no innovation and the old technology survives.

### 3.3. Preferences

The utility of the representative agent of this economy is

\[
 u(c(i), a, i) = c(i) - \lambda(i) a. \tag{3}
\]

Utility depends linearly on the consumption of the final good \( c(i) \) and, due to a production externality from the intermediate good firm, on the activity level \( a \). Note that this externality is reduced if the intermediate good firm is more cautious (that is \( a \) is close to \( a \)). To motivate (3) consider the case where the final good, for instance biscuits, is produced with genetically modified corn and \( a \) is inversely related to amount of regulation in the intermediate good sector. The emissions of sparks and cinder caused by railroads is another classic example of externality.\(^{19}\) We assume that \( \lambda(1) = \gamma(0) \lambda(0) \) where \( \gamma \geq 0 \). For simplicity, we normalize \( \lambda(0) \) to 1. If \( \gamma > 1 \), the consumer faces a more dangerous innovation. In this case, the innovation makes it more costly for the consumer to have a more permissive legislation. If instead \( 0 \leq \gamma < 1 \), the negative externality from production is less severe under the new technology.

\(^{18}\)We abstract from the enforcement issue in the belief that the type of legal regime has little impact on it.

\(^{19}\)See Grady (1988) and Ely (2001) for an account of early cases that addressed these issues. At the end of the 19th century, for instance, typical allegations of negligence included the failure to have a spark arrester, to keep it functioning, to use the appropriate type of fuel, to keep the roadway free of weeds, or the failure to build fire guards on the edge of the roadway.
3.4. The Maximization Problem of the Intermediate Good Firm

We denote by \( \pi(a,i) \) the profit function of the monopolist that produces the intermediate good according to technology \( i \),

\[
\pi(a,i) = \max_{x(i) \geq 0} [p(i) - MC(a)] x(i). \tag{4}
\]

Since the final-good producer is competitive, the inverse demand of the intermediate good is

\[
p(i) = \frac{1}{2} A(i) x(i)^{-\frac{1}{2}}. \tag{5}
\]

That is, \( p(i) \) is equal to the marginal product of the intermediate good. The monopolist consequently chooses to produce

\[
x(i) = \left[ \frac{A(i)}{4MC(a)} \right]^2. \tag{6}
\]

After substituting (6) into (4), we obtain

\[
\pi(a,i) = a\Phi(i), \tag{7}
\]

where

\[
\Phi(i) = \left[ \frac{A(i)}{4} \right]^2. \tag{8}
\]

Note that \( \Phi(i) \) does not depend on the level of activity \( a \), but only on the state of the technology \( i \). More importantly, notice from (7) that profits are increasing in \( a \).

3.5. Optimal Investment in Research

We assume that the R&D firm that discovers the new technology has all the bargaining power and can sell its patent for a price equal to \( \pi(a,1) \). Therefore, this firm’s optimal choice of \( z \) solves the following problem:

\[
\max_{z \in [0,\frac{1}{\theta}]} \left[ \theta z \pi(a,1) - \frac{1}{2} z^2 \right], \tag{9}
\]
where, for simplicity, we take the cost of $z$ to be quadratic.\(^{20}\)

The optimal choice of $z$ is then:

$$\tilde{z} = \theta a \Phi(1), \quad (10)$$

which is increasing in $a$. The amount of investment $\tilde{z}$ in (10) is not, in general, socially optimal because the R&D firm chooses $z$ in order to maximize profits, ignoring consumer surplus.\(^{21}\)

The decision problem of the R&D firm highlights the mechanism through which the law affects the probability of successful innovation in our model: a pro-business law (which allows higher levels of activity $a$) increases the profits of the intermediate good firm and makes R&D investment more profitable, thereby increasing the probability of discovering the more productive technology.

It is important to notice that in order to determine the amount of R&D investment what matters is the law the R&D firm expects it will prevail under technology 1. Indeed, the law that is enforced under the status-quo technology does not enter in (9) and, consequently, does not affect the decision of the R&D firm.

We are now able to compute the expected rate of output growth of the economy using (1), (6), (2) and (10):

$$g = \theta \tilde{z} \left[ \frac{y(1) - y(0)}{y(0)} \right] = (\gamma_A^2 - 1) \theta^2 \Phi(1) a. \quad (11)$$

Clearly the more permissive the regulation (the higher is $a$), the higher the rate of growth in the economy.\(^{22}\)

### 3.6. Ex-Post Optimal Laws

As discussed above, $a$ can be interpreted as an inverse index of regulatory strictness embodied in the law. Law-makers are benevolent in the sense that they choose the law in order to maximize the utility of the representative consumer. In order to solve the legislator’s problem,

\(^{20}\)Notice that since the probability of successful innovation has constant returns to scale, the number of firms is indeterminate. Throughout this section, we assume that there is a single R&D firm.

\(^{21}\)This is the standard appropriability effect emphasized by the literature on innovation.

\(^{22}\)See Gray (1987) for an empirical analysis of the negative consequences of regulation on productivity in the US manufacturing industry.
we derive the indirect utility of the representative consumer in each state $i$. Using (1), (3), (6) and the equilibrium condition $c(i) = y(i)$, we obtain that the indirect utility is linear in $a$:

$$u(a, i) = a \vartheta(i),$$  \hspace{1cm} (12)

where

$$\vartheta(i) = \frac{1}{4} A(i)^2 - \lambda(i)$$ \hspace{1cm} (13)

From (12) note that an increase of $a$ has two effects on utility. First, it has a direct (and negative) effect due to the externality it creates. The higher $\lambda(i)$, the higher is this effect. Second, a higher $a$ decreases the marginal cost of the intermediate good producer and increases the production of the final good. A more pro-business law has then an indirect (and positive) effect on utility because consumption increases; the higher $A(i)$, the higher the marginal benefit of increasing $a$ due to this second effect. Since $A(i)$ and $\lambda(i)$ both depend on $i$, the law that optimally solves the trade-off between the two effects is potentially different under the two technologies.

We now compute the law that law-makers would choose ex-post (that is, after observing the current technological state). We shall refer to this law as the ex-post optimal law and denote it $a^*(i)$.\footnote{As we will see in Section 4, this law may not coincide with the law that law-makers would choose ex-ante, when the uncertainty about the technology has not yet been resolved.} Given the linearity of (12), it is straightforward to find for each technological environment the level of $a \in [a, \bar{a}]$ that maximizes (12). In particular, we get

$$a^*(i) = \begin{cases} \bar{a} & \text{if } \vartheta(i) \geq 0 \\ a & \text{if } \vartheta(i) < 0 \end{cases}$$ \hspace{1cm} (14)

From (13) notice that $\vartheta(i) \geq 0$ when $A(i)$, the productivity of the intermediate good, is relatively high compared to its externality $\lambda(i)$. In this case the ex-post optimal law will be a pro-business law (a law that minimizes the marginal cost of the intermediate good firm). When instead $\vartheta(i) < 0$, the direct effect of $a$ on consumers’ utility is relatively large. In this case the ex-post optimal law will be punitive for the intermediate good firm.

Throughout the paper we assume that innovation, besides increasing the productivity of the intermediate good, is welfare-improving. In other words:
Assumption 1: Innovation increases consumers’ utility: $\vartheta(1) > \vartheta(0)$.

An implication of Assumption 1 is that the ex-post optimal law is weakly increasing in $i$. That is, the ex-post optimal law is (weakly) more favorable to the firm producing the intermediate good after the innovation than before.

As discussed above, the ex-post optimal law is either $\underline{a}$ or $\overline{a}$ depending on the value of $A(i)$ relatively to $\lambda(i)$. Notice that if $A(0)$ is sufficiently low (in relative terms), both $a^*(0)$ and $a^*(1)$ are equal to $a$. If instead the productivity of the status-quo technology is relatively high, both $a^*(0)$ and $a^*(1)$ are equal to $\overline{a}$. A feature common to both cases is that the ex-post optimal laws under the two technologies coincide. When instead the starting value of $A(0)$ belongs to an intermediate range, we have that $a^*(0) = \underline{a}$ while $a^*(1) = \overline{a}$.

In the early stages of their life cycle, most technologies are likely to be characterized by low productivity and sizable consequences on consumers’ safety. As technologies develop, we expect productivity to increase and the negative externality on consumers to matter less. As a result we can postulate the following classification:

Definition 1: Suppose that Assumption 1 is satisfied. Technology is said to be at an early stage of development when $\vartheta(0) < \vartheta(1) < 0$. This occurs when the productivity of the status-quo technology is sufficiently low:

$$A(0) < \frac{2 \sqrt{\gamma_A}}{\gamma_A}. \quad (15)$$

Technology is said to be at an intermediate stage of development when $\vartheta(1) \geq 0 > \vartheta(0)$. This occurs when

$$\frac{2 \sqrt{\gamma_A}}{\gamma_A} \leq A(0) < 2. \quad (16)$$

Finally, technology is said to be mature when $\vartheta(1) > \vartheta(0) \geq 0$. This occurs when

$$A(0) \geq 2. \quad (17)$$

It is important to stress that we are not saying that in the early stages technological innovation is not welfare improving. By Assumption 1 innovation always increases consumers’ utility. Our point is that at early stages considerations concerning consumers’ protection
matter relatively more: that is, the marginal benefit from a more permissive law is lower than its marginal cost. The opposite holds true when technology is mature.

As discussed in the previous section, the probability of a successful innovation depends on the law that will be enforced if the new technology is discovered. Since innovation is welfare improving, law-makers want to promote innovation beyond the suboptimal level chosen by the R&D firm. Law-makers have an effective instrument to promote research: choosing a pro-business law. This increases the profit of the intermediate good firm, raises the price of a patent and provides stronger incentives to invest in R&D. The goal of fostering innovation, however, is not the only goal that law-makers pursue. The law must also optimally regulate the technological environment. As we will see in the next section, the two goals often do not coincide.

4. Commitment vs. Flexibility with Endogenous Uncertainty

In this section we finally compare our two legal regimes. First, consider a flexible regime (denoted by $F$) where the law-maker chooses the law ex-post, after knowing the current state of the technology. In this case, it is easy to find out the law that is implemented in each state $i$: it coincides with $a^*(i)$, the ex-post optimal law in that state. Consider next a rigid regime (denoted by $C$, for commitment) where the law is chosen ex-ante, before knowing the current state of the technology. In the rigid regime, law-makers are bound to enforce ex-post the law that was chosen ex-ante. We crucially assume that in the rigid regime, the law cannot be made contingent on the technological environment. To justify this, one may assume that the two environments are difficult to describe ex-ante. However, as is standard in the incomplete contracting literature, we also assume that law-makers understand how the law affects the probability of successful innovation and knows the payoff consequences of the law in the two technological states. This assumption is necessary to make the legislator in the rigid regime able to optimally write the law before uncertainty is realized. Let $a_C$ denote the law that will be enforced under both technologies in the rigid regime.

Notice that, in general, and for different reasons, the two legal systems that we have just described are both bounded away from efficiency. On the one hand, the flexible regime is adaptable but it lacks commitment. As a result, it may not provide sufficient incentives to innovate. To see this, assume for instance that we are at an early stage (that is, $\vartheta(1) < 0$). In this case, the R&D firm correctly foresees that ex-post the law in the flexible legal regime
will be costly for the intermediate good firm. As a result, discovering the new technology is not very profitable. This depresses investment and reduces the probability that welfare-improving innovation occurs. On the other hand, in the rigid regime the law-maker is able to commit but is bound to choose a single law and, consequently, he cannot adapt to changing conditions. The incompleteness of the law is then the source of inefficiency of the rigid regime.

Under the rigid regime, the timing is as follows. First, the legislator chooses the value of $a$, say $a_C$. Then, the R&D firm chooses the investment level. Investment is either a success or a failure. Regardless of the current state of the technology, the intermediate good firm exerts caution in the amount $a_C$. Finally, the production of the intermediate good and of the final good take place. The legislator chooses $a_C$ in order to maximize the expected utility of the representative agent. Using (10), welfare in the rigid regime can be written as:

$$W_C = \max_{a_C \in [a, \bar{a}]} \left[ \theta^2 \Phi(1)a_C \right] a_C \vartheta(1) + \left[ 1 - \theta^2 \Phi(1)a_C \right] a_C \vartheta(0).$$

(18)

Notice that ex-ante (before uncertainty is realized) the law has another effect on consumers’ utility, besides the ones discussed in the previous section, it affects the probabilities of the two technological states. It is then apparent that law-makers have one instrument (namely $a_C$) to pursue two goals: to provide incentives to innovate and to optimally regulate the new technological environment. In general, the legislator cannot achieve both goals with a single instrument and welfare in the rigid regime is then suboptimal.

Under the flexible regime, the timing is as follows. First, R&D firms choose how much to invest. In making this choice, they correctly foresee the choice that law-makers will make ex-post. Investment is either a success or a failure. Law-makers observe the current technological environment and have discretion to choose the law. As discussed above, in each state $i$, law-makers choose $a^\ast(i)$, the law that maximizes consumers’ ex-post welfare in that state. Finally, the production of the intermediate good and of the final good take place. Welfare in the flexible regime can then be written as:

$$W_F = \left[ \theta^2 \Phi(1)a^\ast(1) \right] a^\ast(1) \vartheta(1) + \left[ 1 - \theta^2 \Phi(1)a^\ast(1) \right] a^\ast(0) \vartheta(0).$$

(19)

Notice that the probability of a successful innovation depends on $a^\ast(1)$ because the R&D firm
correctly expects $a^*(1)$ to be enforced in state 1.

4.1. **Exogenous Innovation**

In comparing the rigid and the flexible regimes, it is instructive to begin by considering the benchmark case where the probability of successful innovation is exogenous. Let $\iota$ denote the probability that innovation occurs.

When technological innovation is exogenous, the law cannot (obviously) provide incentives to innovate. Therefore, legal systems differ only with respect to their ability to choose the best law for each technology. Given this premise, it is entirely straightforward to conclude that when innovation is exogenous the flexible regime weakly dominates the rigid one. The two regimes are equivalent only in two cases: when there is no uncertainty and when the ex-post optimal laws are the same under both technologies.\(^{24}\) This occurs because in both instances the incompleteness constraint is not binding.

Let $W_j(\iota)$ denote maximized welfare when the probability of innovation is equal to $\iota$, where $j = C, F$. We state the following without proof.

**Proposition 1. Exogenous Innovation:** When technology is either at an early stage or is mature, for all $\iota \in [0, 1]$ we have $W_F(\iota) = W_C(\iota)$. When instead technology is at an intermediate stage, $W_F(\iota) > W_C(\iota)$ for all $\iota \in (0, 1)$ and $W_F(\iota) = W_C(\iota)$ when $\iota = 0, 1$.

It is instructive to compute $W_F(\iota)$ and $W_C(\iota)$ when technology is at an intermediate stage. This is clearly the most interesting case since when the signs of $\vartheta(0)$ and $\vartheta(1)$ coincide, we know from Proposition 1 that the two legal systems yield the same outcomes. Recall that in the flexible regime law-makers choose ex-post the optimal laws (14), we then have

$$W_F(\iota) = \iota \bar{a} \vartheta(1) + (1 - \iota) \underline{a} \vartheta(0).$$ (20)

Consider now the rigid regime. It is easy to verify that in the rigid regime the legislator chooses $\underline{a}$ (resp. $\bar{a}$) when $\iota$ is below (resp. above) a certain threshold. To understand this result, recall that the legislator must choose a single (non-contingent) law. Therefore, he will choose the law that better regulates the status-quo technology (which is equal to $\underline{a}$, when

\(^{24}\)The latter possibility arises when the economy is at an early or at an advanced stage of development
technology is at an intermediate stage) if and only if successful innovation is not very likely. One can verify that this threshold, which is denoted by $\tau$, is given by

$$\tau = \frac{(a-a) \vartheta(0)}{(a-a)(\vartheta(1) - \vartheta(0))}. \quad (21)$$

We, therefore, obtain

$$W_C(\iota) = \begin{cases} \iota a \vartheta(1) + (1-\iota) a \vartheta(0) & \text{if } \iota \leq \tau \\ \iota \pi \vartheta(1) + (1-\iota) \pi \vartheta(0) & \text{otherwise} \end{cases} \quad (22)$$

In Figure 1 below, we draw (20) and (22). Both $W_F(\iota)$ and $W_C(\iota)$ are increasing in $\iota$ by Assumption 1. It is important to notice that the welfare loss of the rigid regime vis-à-vis the flexible one is relatively large for intermediate values of $\iota$. In this region of parameters it is relatively more costly to have an non-contingent law (the Lagrange multiplier associated with the incompleteness constraint is high). The convexity of $W_C(\iota)$ helps explain why in a model where R&D investment is endogenous, the legislator in the rigid regime may have an incentive to select a law that leads to either overinvestment or underinvestment in R&D.

![Figure 1: Welfare levels with exogenous innovation for $\vartheta(0) < 0 < \vartheta(1)$.](image-url)
4.2. Endogenous R&D investment

When R&D investment is endogenously chosen, the probability of discovering the new technology depends on the law. This assumption has two important implications. First, it does matter now whether or not law-makers are able to commit. Credibility problems arise because the first order conditions in the ex-ante problem (before R&D investment is chosen) and in the ex-post problem (after knowing whether R&D investment was a success) differ. This is because at the ex-ante stage law-makers do take into account the effect of the law on the incentives to invest — see (10) above — but do not do so at the ex-post stage. In other words, because of credibility problems, in some cases committing to a rule (choosing the rigid regime) is preferable to leaving ex-post discretion to law-makers (choosing the flexible regime). A second implication is that the legislator in the rigid regime may now have an incentive to select a rule that reduces the underlying uncertainty in the economy. As shown below, this result can be achieved by either strongly encouraging or strongly discouraging R&D investment.

To determine the law in the rigid regime, we rewrite (18) as:

\[ W_C = \max_{a_C \in [a, \bar{a}]} \vartheta(0) a_C + \theta^2 \Phi(1) a_C^2 \left[ \vartheta(1) - \vartheta(0) \right]. \] (23)

Notice that, by Assumption 1, \( \vartheta(1) - \vartheta(0) > 0 \). The objective function is then convex in \( a_C \). This implies that (23) yields a bang-bang solution: the chosen law \( a_C \) is either \( a \) or \( \bar{a} \). As a result, the probability of discovering the new technology is either the lowest or the highest possible one.

\[ a_C = \begin{cases} \bar{a} & \text{if } \frac{\vartheta(0)}{\vartheta(1) - \vartheta(0)} + \theta^2 \Phi(1) (\bar{a} + a) \geq 0 \\ a & \text{otherwise} \end{cases} \] (24)

We now discuss the optimal law chosen by the legislator in each of the three stages of technological development.

Mature stage. When \( \vartheta(0) \geq 0 \), using (24) we obtain \( a_C = \bar{a} \).\(^{25}\) Choosing law \( \bar{a} \) achieves two goals at the same time: it provides the right incentive to conduct research and it optimally

\(^{25}\)Indeed, from (23), welfare in the rigid regime is increasing in \( a_C \) when \( \vartheta(0) \geq 0 \).
regulates the two technological environments we may observe ex post.

**Intermediate Stage.** As always, the goal of favoring innovation is achieved by selecting $\pi$. However, when technology is at an intermediate stage, $\pi$ is ex-post optimal when innovation is successful but is suboptimal when innovation is not successful. As from (24) the legislator chooses $\pi$ when $\vartheta(1) - \vartheta(0)$ is small and, consequently, it is not valuable to provide incentives to innovate. We also expect $\pi$ to be selected when $\vartheta(0) << 0$. In this case, it would be too risky to choose $\pi$. A pro-business law is extremely inefficient if the new intermediate good is not discovered. Finally, if the probability of a successful innovation can be made sufficiently close to 1, then the choice $\pi$ dominates.

![Figure 2: Utility of the representative consumer in the rigid regime when $\vartheta(1)$ and $\vartheta(0) < 0$](image)

**Early Stage.** In this environment providing incentives (choosing $\pi$) is suboptimal when R&D investment fails but also when it succeeds. However, (24) implies that in some cases the legislator will select $\pi$. To understand why consider Figure 2 above. It depicts the indirect utility of the representative consumer, which was defined in (12), as a function of the law for both technological states. Given that $\vartheta(0)$ and $\vartheta(1)$ negative, both utilities are decreasing.
in the law, \( a \). Points A and B (resp. points C and D) indicate the agent’s utility associated to law \( a \) (resp. \( \pi \)) in state 1 and 0. Since at this technological stage we have that in both states law \( a \) is ex-post optimal, from a welfare view point A dominates C and B dominates D. However, to see why the legislator may sometimes choose \( \pi \) notice that, from (18), even if A dominates C and B dominates D, the weighted sum of A and B may be smaller than the weighted sum of C and D, since the weights are endogenous. This may occur when the choice \( \pi \) raises the probability of state 1 by a considerable amount.\(^{26}\)

After deriving the laws that are enforced in the two regimes, it is straightforward to compare the two legal institutions. Proposition 2 establishes that, in contrast to Section 3, when R&D investments are endogenous the flexible regime is not necessarily optimal in all circumstances. In particular, when technology is at an early stage we have that the rigid regime may actually dominate the flexible regime because of its ability to provide better incentives to innovate. At the early stages of technological development the flexible regime selects a law that protects public safety and provides weak incentives to innovate. Moreover, by choosing \( a_C = a \) the legislator in the rigid regime can achieve the same welfare that is obtained in the flexible regime. Hence, the legislator might choose \( a_C > a \) in order to provide incentives to innovate. This possibility is not available in the flexible regime and this explains why we obtain that the commitment regime weakly dominates the flexible one. When technology is mature, the two systems yield the same outcomes. Finally, in economies at an intermediate stages of development — periods when legal change is needed and there are no commitment problems — the flexible regime is strictly better than the rigid one because of its ability to choose the best law for each technology.

Let \( g_i \), with \( i = F, C \), denote the rate of output growth under legal regime \( i \). The next proposition states the main result of this section.

**Proposition 2. Welfare Comparison:** (i) When technology is mature, we have that \( W_C = W_F \) and \( g_C = g_F \). (ii) When technology is at an intermediate stage of development, we have that \( W_C < W_F \) and \( g_C \leq g_F \). (iii) When technology is at an early stage of development, we have that \( W_C \geq W_F \) and \( g_C \geq g_F \).

A natural question is whether the theoretical predictions of Proposition 2 are validated

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\(^{26}\)Notice that a necessary condition to choose \( \pi \) is to have \( \pi \theta (1) \geq \theta (0) \). In Figure 2 this implies that utility associated with C must be greater than the one associated with B.
by the data. While a full investigation is clearly beyond the scope of the present paper, we pause to point out that such an empirical analysis would have to overcome several challenging obstacles. To begin with, one would need cross-country historical data on technological innovation. Moreover, in order to empirically distinguish among the three stages of technological development discussed in this paper, one would need measures of (possibly, industry-level) productivity and also measures of the externality caused by technological innovation. \footnote{A useful starting point might be the dataset on technology adoption compiled by Comin and Hobijn (2004). They classify technologies according to whether they have a previous competing technology. Whether or not a technology has a predecessor may be related to its stage of development.}

4.3. Rigidity and Overinvestment

As a benchmark, we now define and derive the law in the \textit{first-best} environment. Similarly to the flexible regime, the first-best law specifies a law for each technological environment and, similarly to the rigid regime, the first-best law is specified ex ante under full commitment. Let $a_{i}^{FB}$ denote the first-best law that will be enforced under technology $i = 0, 1$. Welfare in the first-best world, denoted $W^*$, is equal to

$$W^{FB} = \max_{a_{FB}^0, a_{FB}^1 \in [\underline{a}, \overline{a}]} \left[ \theta^2 \Phi(1) a_{FB}^1 \right] a_{FB}^1 \vartheta(1) + \left[ 1 - \theta^2 \Phi(1) a_{FB}^1 \right] a_{FB}^0 \vartheta(0).$$

To compute the first-best law, first notice that $a_{FB}^0$ has no effect on the amount of R&D investment. Then, we have that $a_{FB}^0$ is equal to $a^*(0)$, the ex-post optimal law in state 0. To find $a_{FB}^1$, two cases must be considered. First, assume that $\vartheta(1) \geq 0$. In this case, we have that $a_{FB}^1 = a^*(1) = \overline{a}$. To see this, notice that this choice of law fosters innovation and at the same time optimally regulates the more advanced technological environment. Second, when $\vartheta(1) < 0$, it is immediate to verify that the objective is concave in $a_{FB}^1$ on the interval $[\underline{a}, \overline{a}]$. Therefore, to find $a_{FB}^1$ we have to study the sign of the derivative at $\underline{a}$ and $\overline{a}$. We obtain that $a_{FB}^1 = \underline{a}$ if and only if $2\vartheta(1) - \vartheta(0) \leq 0$, while $a_{FB}^1 = \overline{a}$ if and only if $2\overline{a} \vartheta(1) - a \vartheta(0) \geq 0$. In the remaining cases, $a_{FB}^1$ is an interior solution. The interpretation is quite straightforward. When $\vartheta(1) < 0$ and $\vartheta(1) - \vartheta(0)$ is small, innovation does not increase welfare by much and, consequently, $a_{FB}^1$ coincides with $\underline{a}$, the ex-post optimal law at this stage.

In what follows, we compare the probability of a successful innovation under the first-best law with the one obtained under the rigid and the flexible regimes. Surprisingly, we find that
in some cases $a_C > a^{FB}_1$. This implies that in the rigid regime there may be overinvestment in R&D (hence, too much growth) compared to what would be socially optimal.\footnote{Usually, the literature on incomplete contracts has focused on the possibility of underinvestment due to ex-post exploitation (see Grout, 1984). The overinvestment result is also obtained in the incomplete contract literature: see for instance, Chung (1995). The underlying reason is somewhat different from ours: in that literature, some parties may overinvest to strategically affect their bargaining power ex-post.}

It is easy to verify that the possibility of overinvestment occurs only when technology is at the early stage.\footnote{When instead $\vartheta(1) \geq 0$ we obtained $a^{FB}_1 = \bar{a}$. Then, it is not possible to observe $a_C > a^{FB}_1$.} At this stage, two reasons push the legislator in the rigid regime to choose a pro-business law: to increase the probability that welfare improving innovation occurs and to reduce the probability of staying with the status-quo technology and suffering from inefficient regulation. Notice in fact that at an early stage a high $a_C$ is always suboptimal but is relatively more inefficient under the old than under the new technology (see Figure 2 which shows that $C > D$). In the first-best problem, the second reason is not present because the law is state-contingent and, consequently, the status-quo technology is optimally regulated. This is why rigid legal system may induce overinvestment in R&D compared to the optimal level.

As summarized in the following proposition, in the rigid legal regime we may have either overinvestment (if $a_C = \bar{a}$ while $a^{FB}_1 < \bar{a}$) or underinvestment (when $a_C = \underline{a}$ while $a^{FB}_1 > \underline{a}$). In the flexible legal regime investment is never larger than the efficient one.

**Proposition 3.** *The Possibility of Overinvestment:* The rate of output growth in the flexible regime is always smaller or equal than first-best. In the early stage of technological development, economies adopting the rigid regime may grow faster than first-best.\footnote{When instead $\vartheta(1) \geq 0$ we obtained $a^{FB}_1 = \bar{a}$. Then, it is not possible to observe $a_C > a^{FB}_1$.}

At first glance, it may seem paradoxical that the rate of output growth in the rigid regime may be inefficiently high. However, recall that welfare depends on the consumption of final good but also on the activity level of the intermediate good firm. A high rate of growth may be suboptimal when it is obtained by committing to a high $a$, which implies a low use of precaution in the intermediate good sector.

### 4.4. Costly Change of the Law

We now assume that the law in the rigid regime can be changed by incurring an exogenous cost $\kappa > 0$ after knowing which technology is available. Let $W_C(\kappa)$ denote welfare in the
rigid regime when the cost of changing the statute is equal to \( \kappa \). Clearly, \( W_C(\infty) = W_C \) and \( W_C(0) = W_F \) where \( W_C \) and \( W_F \) are defined in (18) and (19) above, respectively.

The timing is as follows. At the beginning, the legislator selects the law \( a_C \). The R&D firm chooses the amount of investment. In making this decision the R&D firm understands the legislator’s incentives to change the law ex-post. After knowing the current technological environment, the legislator decides whether to enforce the existing law \( a_C \) or to change it by incurring the cost \( \kappa \). Finally, production and consumption take place.

We obtain two key results. First, we show that for all \( \kappa > 0 \) the possibility of changing the statute does not alter the welfare rankings that we have established in Proposition 2 above. Second, we show that \( W_C(\kappa) \) varies (in a possibly non-monotonic way) with \( \kappa \).

**Proposition 4. Costly Change:** (i) When technology is mature, \( W_C(\kappa) \) is constant in \( \kappa \) and for all \( \kappa > 0 \) we have \( W_C(\kappa) = W_F \); (ii) When technology is at an intermediate stage, \( W_C(\kappa) \) is decreasing in \( \kappa \) and for all \( \kappa > 0 \) we have \( W_C(\kappa) < W_F \). (iii) When technology is at an early stage, \( W_C(\kappa) \) is not necessarily monotone in \( \kappa \) and for all \( \kappa > 0 \) we have \( W_C(\kappa) \geq W_F \).

To understand part (i) of Proposition 4, notice that when technology is mature both legal regimes attain the first-best by selecting \( \bar{a} \). Since it is never optimal to change the law ex-post, \( \kappa \) does not affect welfare. When technology is at an intermediate stage, flexibility is needed and a pro-business law is also credible: the rigid regime would then benefit from a low \( \kappa \). As long as \( \kappa \) is strictly positive, however, the flexible regime remains superior. In the early stage, \( W_C(\kappa) \) weakly dominates the flexible regime because the rigid regime has the possibility of selecting \( \bar{a} \) and reproducing the flexible regime.

Interestingly, Proposition 4 also states that in the early stage of technological development, \( W_C(\kappa) \) may not be monotone in \( \kappa \). In particular, for some finite \( \kappa \) it may happen that

\[
W_C(\kappa) > W_C(\infty) \geq W_C(0).
\]

(26)

In other words, choosing a positive, but finite, cost of changing the law would further improve welfare in the rigid regime. The second inequality in (26) follows from part (iii) of Proposition 2. We now prove that the first inequality is verified for some parameter values. For instance, assume that when \( \kappa = \infty \) the parameters in (24) are such that the optimal law in the fully
rigid regime is \( \bar{a} \). Moreover, pick a \( \kappa \) that satisfies the following two inequalities:

\[
\vartheta (0) \bar{a} < -\kappa + \vartheta (0) a, \tag{27}
\]

\[
\vartheta (1) \bar{a} \geq -\kappa + \vartheta (1) a. \tag{28}
\]

This partially rigid legal system provides credible incentives to innovate since inequality (28) establishes that \( \bar{a} \) is not changed ex-post in state 1. Moreover, given that \( \kappa \) satisfies (27), the law \( \bar{a} \) will be changed ex-post in the case innovation fails (an event occurring with strictly positive probability). This indicates that an intermediate value of \( \kappa \) would provide flexibility and also commitment. Therefore, it would be preferable to having either \( \kappa = \infty \) or \( \kappa = 0 \).

5. The Dynamic Model

We now consider a dynamic model. We assume that time, which is indexed by \( t \geq 0 \), is continuous and unbounded. Moreover, we postulate that the number of feasible innovations is infinite. In other words, technological progress never settles. Let \( i \) denote the current technological state. The productivity of the intermediate good increases as follows: \( A(i + 1) = \gamma_A A(i) \), with \( \gamma_A > 1 \). As before, we assume that innovations are drastic: if the more productive intermediate good \( i + 1 \) is discovered, the intermediate good \( i \) becomes obsolete. Flow production of consumption good is given by (1). To simplify the algebra, we assume that the externality from the intermediate good sector is constant: \( \lambda(i + 1) = \lambda(i) = \lambda \).

The representative consumer has the following intertemporal preferences

\[
U(c, a) = \int_0^\infty e^{-rt} (c_t - \lambda a_t) \, dt, \tag{29}
\]

where \( r \) is the constant rate of time preference, also equal to the interest rate.\(^{31}\) We denote by \( c_t \) and \( a_t \) the time-\( t \) consumption and intermediate good producer’s activity, respectively. Flow consumption at time \( t \) is stochastic since it depends on the technological state that is

\(^{30}\)By looking at Figure 2, it is easy to see that such a \( \kappa \) always exists. Just notice that \( \vartheta (0) \bar{a} \) corresponds to point \( D \), \( \vartheta (0) a \) to point \( B \), \( \vartheta (1) \bar{a} \) to point \( C \) and \( \vartheta (1) a \) to point \( A \). A value of \( \kappa \) that satisfies (27) and (28) always exists because \( B - D > A - C \).

\(^{31}\)This is because the marginal utility of consumption is constant. The linearity of the utility removes any incentive to either save or borrow for consumption-smoothing or risk-sharing purposes. Then, \( c(i) = y(i) \).
available at that time. The economy is similar to the one analyzed in Section 3. As above, marginal cost is affected by the law according to equation (2).

Using (1), (3), (6) and the equilibrium condition \( c(i) = y(i) \), we obtain that when the technological state is \( i \), the instantaneous utility of the representative agent is equal to \( a_i \vartheta(i) \), where

\[
\vartheta(i) = \frac{1}{4} A(0)^2 \gamma_i A - \lambda, \tag{30}
\]

and \( a_i \) is the law which is enforced under technology \( i \).

As in Section 3, innovation is welfare improving. Indeed, \( \vartheta(i + 1) > \vartheta(i) \). It is also important to notice that in the long-run, after a possibly long sequence of innovations, \( \vartheta(i) \) will become positive and, consequently, \( \bar{a} \) will be the ex-post optimal law. Let \( N \geq 0 \) denote the lowest technological state where \( \vartheta(i) \) is weakly positive: that is, \( \vartheta(N) \geq 0 \) but \( \vartheta(N - 1) < 0 \). \( N = 0 \) if the initial \( A(0) \) is sufficiently high and \( \lambda \) is sufficiently low.

The innovation process is assumed to be a continuous time Markov Chain, as in Aghion and Howitt (1992). The economy moves through a sequence of states \( 0 \rightarrow 1 \rightarrow \ldots \) staying with innovation \( k \) for a sojourn time \( X_k \) with density \( \theta z(k+1)e^{-\theta z(k+1)} \), where \( z(k+1) \) are the R&D expenditures that are incurred in order to discover the intermediate product of quality \( k + 1 \).\(^3\) We assume \( 0 \leq z(k + 1)\theta < \infty \).

Throughout this section, the term transition will denote the interval of time from 0 to the time when innovation \( N \) is discovered. Knowing the equilibrium investment levels, we compute the expected duration of a transition. Since sojourn times are independent, this is given by

\[
\sum_{i=1}^{N} \frac{1}{\theta z(i)}. \tag{31}
\]

From (31) it is clear that transition is fast when R&D expenditures and \( \theta \) are high and when \( N \) is small.

In the dynamic model, the intermediate good firm owns a life-time patent. However, since innovations are assumed to be drastic, the intermediate good firm stops making profits as soon as a more advanced technology is discovered. Hence, the value of the patent of invention

\(^3\)This stochastic process is known in the statistical literature as a pure birth process. See Feller (1966). Notice that a pure birth process is a generalization of a Poisson process in which the arrival rate is not constant but is allowed to depend on the current state (in our case, the current value \( i \)).
\[ \Pi(i) = \frac{\pi(a_i, i)}{r + \theta z(i + 1)}. \]  

(32)

This is the expected present value of the flow of monopoly profits \( \pi(a_i, i) \) generated by the \( i \)-product over an interval of time that is exponentially distributed with parameter \( \theta z(i + 1) \). Notice that \( \Pi(i) \) is decreasing in future R&D expenditures, since higher values for \( z(i + 1) \) shortens the expected tenure of the monopolist producing the \( i \)-intermediate product. In deriving (32) we assume perfect foresight: at each \( t \) all agents correctly foresee the R&D expenditures that will be incurred and the laws that will be enforced in all subsequent technological environments.

As before, we consider two legal regimes. In the flexible regime, at any instant courts and regulators select the ex-post optimal law.\(^{33}\) Notice that the law enforced in the flexible regime does not depend on \( t \), but only on \( i \). This occurs because the trade-off that law-makers face only depends on \( i \). In the rigid regime, the law \( a_c \) is chosen at \( t = 0 \) and is never changed. In this case, regardless of the current technological state and of the current time, courts and regulators are bound to enforce \( a_c \).

Since innovations increase the productivity of the intermediate good, we have from (7) that profits \( \pi(a_i, i) \) are increasing in \( i \). This implies that over time R&D firms have stronger incentive to invest. In order to have a balanced growth path for R&D investment, we modify (9) as follows:

\[
\max_{z \geq 0} \left[ \frac{\theta z \Pi(i) - \Omega(i) \frac{1}{2} z^2}{\Omega(i)} \right],
\]

where \( \Omega(i + 1) = \gamma \omega \Omega(i) \), with \( \gamma \omega > 1 \). That is, we implicitly assume that the cost of hiring researchers is increasing as technology matures (possibly due to increasing wages). Therefore, the optimal amount of R&D investment needed to discover the intermediate good \( i \), where \( i > 0 \), is

\[
\widetilde{z}(i) = \frac{\theta \pi(a_i, i)}{\Omega(i)(r + \theta \widetilde{z}(i + 1))}.
\]

(34)

Since \( \widetilde{z}(i) \) is strictly decreasing in \( \widetilde{z}(i + 1) \), (34) clearly illustrates the negative dependency between current and future research. It is also important to notice that R&D investment for the \( i^{th} \) invention depends on future laws. The mechanism through which this occurs is

\(^{33}\)This is because even in this dynamic model law-makers in the flexible regime solve a static problem since their choice only affects the current payoff.
twofold. On the one hand, the law that is enforced in the technological state $i$ affects flow profits; on the other hand, the law that regulates the technological state $i+1$ changes the expected duration of the monopoly for the intermediate good firm.

We now make the following assumption in order to have closed form solutions for maximized welfare in the rigid regime.

**Assumption 2:** Let $\gamma_\omega = \gamma_A^2$.

Under Assumption 2, using (7) and the laws of motion of $\Omega(i)$ and $A(i)$, it is immediate that the ratio between $\pi(a, i)$ and $\Omega(i)$ is constant for all $i$.

### 5.1. Dynamics: Rigid Regime

We now characterize the rigid regime. For any given law $a_C$, we identify the equilibrium R&D investment for innovation $i$. As in Aghion and Howitt (1992), there exist many sequences $\{\bar{z}(i)\}_{i=1}^\infty$ satisfying (34). Among these we focus on the unique stationary equilibrium. Using Assumption 2, the stationary R&D investment $\bar{z}$ must satisfy the functional relationship

$$\bar{z} = \frac{A(0)^2 a_C \theta}{4^3 \Omega(0) (r + \theta \bar{z})}. \quad (35)$$

One can verify that $\bar{z}$ is increasing in $a_C$ and $A(0)$ and decreasing in $\Omega(0)$.

In our stationary equilibrium, the expected number of innovations per unit interval is then constant and equal to $\theta \bar{z}$. Moreover, the probability that there will be exactly $i$ innovations from time 0 to time $t$ is given by

$$\frac{(\theta \bar{z}t)^i e^{-\theta \bar{z}t}}{i!}. \quad (36)$$

In other words, in a stationary equilibrium the number of innovations up to time $t$ is a random variable following a Poisson distribution of constant rate $\theta \bar{z}$.

Expected welfare in the rigid regime can be easily computed:

$$W_C = \max_{a_C \in [a, b]} \int_0^\infty e^{-\theta t} \sum_{i=0}^\infty \frac{e^{-\theta \bar{z}t} (\theta \bar{z}t)^i}{i!} a_C \vartheta(i) dt. \quad (37)$$

Since Assumption 2 implies that $\bar{z}$ does not depend on $i$, using (30) one can write
Notice that the law $a_C$ enters (39) twice. The law has a direct effect on welfare: it multiplies the term in square brackets. Also, the law has an indirect effect through $z$; in particular, recall that $z$ is increasing in $a_C$.34

Next, we identify the optimal law in the rigid regime. It is obvious that when $\vartheta(0) \geq 0$, the legislator will choose $a_C = \bar{a}$ since this law is ex-post optimal for all $i$. When $\vartheta(0) < 0$ the trade-off is less trivial: choosing a pro-business law is costly along the transition but optimal in the long-run. We can show (see the proof of Proposition 5 in the Appendix) that if $\gamma_A^2 > 2$ the objective in (38) is convex in $a_C$. In the remainder of this paper, we assume that this condition is met.35 As in the static model, we obtain a bang-bang solution: $a_C$ is either $\bar{a}$ or $a$.  

The next proposition provides sufficient conditions that guarantee that the law in the rigid regime is equal to $\bar{a}$.

**Proposition 5: (Rigid Regime: Dynamic Model)** The law in the rigid regime is equal to $\bar{a}$ when either $\theta$, or $\gamma_A$ or $A(0)$ is sufficiently high.

The intuition why, when $\gamma_A$ and $\theta$ are sufficiently high, the legislator selects $\bar{a}$ is the following. First, high values of $\gamma_A$ and $\theta$ give the legislator more incentives to use the law to increase R&D investment. Second, when $\gamma_A$ and $\theta$ are high, the transition will likely be short.

It is important to emphasize that the fact that the speed of technological change is endogenous gives the legislator an additional incentive to choose a pro-business law. By providing strong incentives to innovate the length of the transition becomes shorter, thereby reducing

---

34For the welfare to have an upper bound, the following condition must be met: for all $a_C \in [\underline{a}, \bar{a}]$ we need $r - (\gamma_A^2 - 1) \theta (\bar{a}) > 0$. If this is not the case, criteria for evaluating infinite utility streams (such as the over-taking criterion) must be used.

35This condition, which is sufficient but not necessary for the objective in (38) to be convex, is defendable in the context of our model: a low $\gamma_A$ might be inconsistent with the assumption that innovations are drastic. See footnote 17 above.
the cost of committing, via a rigid statute, to a pro-business law. As in Subsection 4.3, this suggests that rigid regimes may sometimes grow at a rate greater than the optimal one.

Finally, from (39) it is also intuitive that the lower $A(0)$, the weaker the incentives to choose a pro-business law. The reason is simply that if the initial productivity is low, the transition from state 0 to state $N$ is longer.

5.2. Dynamics: Flexible Regime

Consider now the flexible regime. In contrast to the commitment regime, the enforced law in the flexible regime depends on the current technological state. In particular, the law is $\bar{a}$ for all $i \geq N$ and $\underline{a}$ for all $i < N$.

Knowing the laws that are enforced for all $i$, we can determine $\tilde{z}(i)$ using (34). We proceed backwards. For all $i \geq N$ we solve for the stationary solution which solves (35) from $N$ onwards, where in (35) $a_C$ is replaced by $\bar{a}$. Next, we determine $\tilde{z}(N - 1)$. We expect $\tilde{z}(N - 1)$ to be low. The reason is twofold. First, R&D investment for invention $N - 1$ is low because the law that is enforced in the case invention $N - 1$ occurs is equal to $\underline{a}$, which is costly for the intermediate good firm. The second reason for the lower investment in this state is that the monopoly power of the producer of the $N - 1$ intermediate good is expected to be short-lived. This is because the law in state $N$ will be $\bar{a}$ and, consequently, the R&D investment for innovation $N$ is expected to be high.\footnote{This suggests that growth in the flexible regime may slow down exactly before taking off. A similar “no-growth trap” has been derived in Aghion and Howitt (1992).}

Knowing $\tilde{z}(N - 1)$ and proceeding backwards one can then compute all expenditures in R&D along the transition.

In Figure 3 below, we compute and draw for specific parameter values the sequence $\{\tilde{z}(i)\}_{i=0}^{\infty}$. Notice that R&D investment is constant for all $i \geq N$.\footnote{In Figure 3 we obtain $N = 8$.} In the transition, we observe innovation cycles: low innovation when $i$ is odd stimulates innovation when $i$ is even.

As discussed above, the research investment for product $N - 1$ is especially low.

Given the sequence of equilibrium investment levels, we determine expected welfare in the flexible regime:

$$W_F = \int_0^{\infty} e^{-rt} \sum_{i=0}^{N-1} P_i(t)\bar{a}\vartheta(i)dt + \int_0^{\infty} e^{-rt} \sum_{i=N}^{\infty} P_i(t)\bar{a}\vartheta(i)dt,$$

(40)
where $P_i(t)$ is the probability that at time $t$ the most advanced innovation is exactly $i$. In particular, we have

$$P_0(t) = e^{-\theta \tilde{Z}(1)t}$$

and for $i \geq 1$

$$P_i(t) = \theta \tilde{Z}(i) e^{-\theta \tilde{Z}(i+1)t} \int_0^t e^{\theta \tilde{Z}(i+1)s} P_{i-1}(s) ds.$$  

For example, solving the above recursive relation we obtain (assuming that $\tilde{Z}(1)$ is different from $\tilde{Z}(2)$)

$$P_1(t) = \theta \tilde{Z}(1) \left( \frac{1}{\theta \tilde{Z}(2) - \theta \tilde{Z}(1)} e^{-\theta \tilde{Z}(1)t} + \frac{1}{\theta \tilde{Z}(1) - \theta \tilde{Z}(2)} e^{-\theta \tilde{Z}(2)t} \right),$$

which shows, for instance that $P_1(t)$ is low if $\theta \tilde{Z}(2)$ is high and/or $\theta \tilde{Z}(1)$ is low.\footnote{For the derivation of (41), (42) and (43), see, for instance, Karlin and Taylor (1975, p. 121) and Feller (1966, p. 41).}

5.3. Discussion

We start comparing the speed of technological change in the two legal systems.\footnote{As discussed above, for each legal regime we picked the equilibrium in which the economy eventually moves to a balanced growth path, where all expected rates of growth are constant.} Two cases need to be considered in the rigid regime: when $\alpha_C$ is $\bar{\alpha}$ and when $\alpha_C$ is $\underline{\alpha}$.\footnote{Recall that we assumed that $\gamma_A^2 > 2$. Then, the objective in (38) is convex.} First, assume
that parameters are such that the law in the rigid regime is \( \bar{a} \). In this case, the rate of output growth in the rigid regime will be greater than in the flexible one. If instead the rigid regime chooses \( a \), comparing the rates of output growth during the transition is not straightforward. To see this, look at Figure 3, which shows that along the transition R&D investment in the flexible regime oscillates above and below the level of investment that is observed in the rigid regime when \( \bar{a} \) is chosen. However, as soon as invention \( N \) is discovered, we would have that in the flexible regime the enforced law will switch to \( \bar{a} \) and the economy without commitment will grow at a faster pace.

We now develop some intuition as to some of the main parameters that impact the welfare comparison between the two regimes. For instance, we expect the flexible legal system to be dominated when \( a \) is close to zero and when either \( \theta \) or \( \gamma_A \) is large. To see this, recall that in the flexible regime \( a \) is chosen during the transition. Moreover, note that when \( a \) is close to zero, flow profits are also close to zero. This reduces R&D investment and lengthens the transition in the flexible regime. On the other hand, when either \( \theta \) or \( \gamma_A \) is large, the rigid regime undergoes a rapid transition when \( a_C \) is set to \( \bar{a} \), thereby reducing the static welfare losses of choosing \( \bar{a} \) when \( i \) is less than \( N \).

Along the same lines, when \( N \) is high the rigid regime is likely to dominate. In this case, in fact, credibility problems are more serious and commitment more desirable. Provided that either \( \theta \) or \( \gamma_A \) is sufficiently large, the legislator in the rigid regime selects \( \bar{a} \), which speeds up the transition and make the lack of flexibility of the rigid regime less costly.

6. Conclusion

This paper investigates whether a flexible legal system is preferable to a rigid system in keeping up with technological progress. To answer this question we developed a simple model of endogenous technological change where innovations are vertical (and new products provide greater quality and replace existing ones) and we analyze the two legal regimes.

We argue that the comparison between the two institutions involves a trade-off between commitment and flexibility. In this paper, this trade-off is is far from trivial since the degree of uncertainty, which is a key parameter in the comparison, is not exogenous, as in the rules-versus-discretion literature, but depends on R&D firms’ investment decisions, which are endogenous to the model.
In the context of a model with only two technological states, we show that rigid legal systems are preferable (in terms of welfare and rate of output growth) in the early stages of technological development. In the intermediate stages we obtain that flexible legal systems are preferable: output grows faster and welfare is greater. Finally, when technology is mature, the two legal systems are shown to be equivalent.

The amount of innovation in the rigid regime may be either inefficiently low or, under some conditions, inefficiently high.

The welfare comparison summarized above holds even when we assume that in the rigid regime the statute (or regulation) can be changed ex-post at a cost.

We then extend our analysis to a model where technology undergoes continuous change, we show that similar results to the ones obtained in the simple setting with only two technologies hold. In the stationary equilibrium of the rigid regime we show that the speed of technological change is either very low or very high. In the flexible regime, we find that because of commitment problems technological change is relatively slow in the early stages of technological development.

A natural question would be how our conclusions would change in an economy where R&D investment increases the variety of available goods (for instance as in Romer, 1990). Various results obtained in the current setting would likely survive. However, we expect the legislator in a rigid regime (where the law is not contingent on each variety) to discourage innovation, but not to induce overinvestment. Indeed, contrary to our conclusions, horizontal innovations always increase the complexity of the economy since new varieties coexist with old varieties. Therefore, the legislator in the rigid regime would likely have a bias against such innovations. Everything else being equal, we expect the rigid regime to grow at a slower pace than in the setting we have analyzed here.
Appendix

Proof of Proposition 2: i) Using Definition 1, when technology is mature we have that \( \vartheta(0) \geq 0 \) and \( \vartheta(1) > 0 \). In the flexible regime, we know from (14) that law-makers select \( \pi \) in both states. In the rigid regime, from (24) we conclude that \( a_C = \pi \). This implies that welfare in the two regimes is the same and that \( g_C = g_F \).

ii) Suppose now that technology is at an intermediate stage, as defined in Subsection 3.6. In the flexible regime, from (14) we conclude that the law enforced in state 1 (resp. 0) is \( \pi \) (resp. \( \varrho \)). In the rigid regime, using (24) we know that \( a_C \) is either \( \varrho \) or \( \pi \). First, assume that \( a_C = \varrho \). Given (10), this implies that the probability that state 1 occurs in the rigid regime is lower than in the flexible one. Consequently, from (11) we have that \( g_F > g_C \). Moreover, we also obtain that \( W_F > W_C \). The reason is twofold: because \( \varrho \) does not maximize \( u(a, 1) \) and because state 1 (which by Assumption 1 provides greater utility than state 0) is more likely in the flexible regime. Second, assume that \( a_C = \pi \). In this case, the probability that state 1 occurs in the two regimes is the same: then, \( g_F = g_C \). Since we assumed an interior solution for R&D investment, the probability that state 1 occurs is strictly smaller than one. Then, with some positive probability state 0 occurs. Since \( \pi \) does not maximize \( u(a, 0) \), this implies that when \( a_C = \pi \) we also have that \( W_F > W_C \).

iii) When technology is at an early stage, using (14) we obtain that the ex-post optimal law is \( \varrho \) in both states so that the flexible regime provides weak incentives to innovate. Since the rigid regime can replicate the flexible one by choosing \( a_C = \varrho \) and since the rigid regime can also choose \( a_C = \pi \), it must be the case that \( W_C \geq W_F \) and \( g_C \geq g_F \).

Proof of Proposition 3: We derive \( a_0^{FB} \) and \( a_1^{FB} \). Since \( a_0^{FB} \) does not affect the amount of R&D investment, the first-best law in state 0 is easy to compute:

\[
a_0^{FB} = \begin{cases} 
\varrho & \text{if } \vartheta(0) < 0 \\
\pi & \text{if } \vartheta(0) \geq 0 
\end{cases}
\]  

\( (A.1) \)

To find \( a_1^{FB} \) two cases must be considered. First, assume that \( \vartheta(1) \geq 0 \). In this case \( a_1^{FB} \) is obviously equal to \( \pi \). Second, assume \( \vartheta(1) < 0 \). In this case, since the second derivative is \( 2\vartheta(1) \), the objective is concave in \( a_1^{FB} \). Therefore, one obtains:

\[
a_1^{FB} = \begin{cases} 
\pi & \text{if } 2\pi\vartheta(1) - a_1^{FB}(0)\vartheta(0) \geq 0 \\
\varrho & \text{if } 2\varrho\vartheta(1) - a_1^{FB}(0)\vartheta(0) \leq 0 \\
\varrho \frac{\vartheta(0)}{\vartheta(1)} & \text{otherwise}
\end{cases}
\]  

\( (A.2) \)

To show that R&D investment in the flexible regime cannot be larger than the one under the first-best law, we must show that \( a^*(1) \leq a_1^{FB} \). Two cases are possible. When \( \vartheta(1) \geq 0 \) in the flexible regime as well as under the first-best law we have that the law for the more advanced technology is equal to \( \pi \), so that
investment in the flexible regime is identical to the first-best level. When \( \vartheta(1) < 0 \), law-makers in the flexible regime choose \( a \). The rate of growth under the first-best can only be larger than \( g_F \).

In order to prove that the commitment regime may induce overinvestment in research, we must show that there exists a region of parameter values where \( a_C = \pi \) and at the same time \( a_{FB} < \pi \). To show this, assume that \( \vartheta(1) < 0 \). (When \( \vartheta(1) \geq 0 \) innovation in the first-best is already at the maximum level and the commitment regime can at most grow at the same rate). Moreover, consider the following parameter values: take \( \pi = 1 \) and \( \theta^2 \Phi(1) = 1 \) so that if the law is \( \pi \), state 1 occurs with probability one. In this case, we have

\[
W_C = \max_{a_C \in [\underline{a}, \pi]} (1 - a_C) \vartheta(0) a_C + a_C \vartheta(1) a_C. \tag{A.3}
\]

Using (24), one can show that the law in the rigid regime is 1 if

\[
a > \frac{\vartheta(1)}{\vartheta(0) - \vartheta(1)}. \tag{A.4}
\]

Using (A.1) and (A.2) we know that when \( \vartheta(1) < 0 \) we have that \( a_{FB}(1) < 1 \) if and only if

\[
a < \frac{2\vartheta(1)}{\vartheta(0)}. \tag{A.5}
\]

One can verify that when \( \vartheta(0) < 2\vartheta(1) \) it is always possible to find a value for \( a \), with \( 0 < a < 1 \), such that both (A.4) and (A.5) are satisfied, which proves our claim that at least for some parameter values the rigid regime induces overinvestment. Note that since (A.4) and (A.5) are strict inequalities, the same argument would also go through if \( \theta^2 \Phi(1) \) is strictly below but sufficiently close to one so that, as we assumed in the paper, the probability of state 1 is strictly lower than one. ■

**Proof of Proposition 4:** We introduce some notation. Let \( a_C(\kappa) \) denote the law that is initially chosen in the partially rigid regime. Moreover, we denote by \( a(i, \kappa, a_C(\kappa)) \) the law that is chosen ex-post in state \( i \) given that the cost of changing the law is \( \kappa \) and that \( a_C(\kappa) \) was initially chosen.

i) When \( A(0) \geq 2 \), it is immediate to verify that \( a_C(\kappa) = \pi \) for all \( \kappa \). Moreover, for \( i = 1, 2 \), we have \( a(i, \kappa, \pi) = \pi \). Welfare in the rigid regime does not depend on \( \kappa \) since the law is never changed ex-post. Then we have that \( W_C(\kappa) = W_C(0) = W_F \) for all \( \kappa \).

ii) Suppose now that technology is at an intermediate stage, as defined in Subsection 3.6. In this case, for all \( \kappa \) and all \( a_C(\kappa) \in [\underline{a}, \pi] \) we have that

\[
a(1, \kappa, a_C(\kappa)) = \begin{cases} 
    a_C(\kappa) & \text{if } a_C(\kappa) \geq \frac{\vartheta(1)\pi - \kappa}{\vartheta(1)} \\
    \pi & \text{otherwise}
\end{cases} \tag{A.6}
\]

It is easy to verify that at this stage (since \( \vartheta(1) \geq 0 \)) for any given \( a_C(\kappa) \) we have that \( a(1, \kappa, a_C(\kappa)) \) is weakly decreasing in \( \kappa \). Next, we define the following expressions:
\[ \tilde{W}_1(\kappa, a_C(\kappa)) = \max \left\{ \vartheta(1) a_C; -\kappa + \max_{a'} \vartheta(1) a' \right\} \]  \hspace{1cm} (A.7)

and

\[ \tilde{W}_0(\kappa, a_C(\kappa)) = \max \left\{ \vartheta(0) a_C; -\kappa + \max a' \vartheta(0) a' \right\}. \]  \hspace{1cm} (A.8)

It can be shown that for a given \( a_C(\kappa) \), both \( \tilde{W}_0(\kappa, a_C(\kappa)) \) and \( \tilde{W}_1(\kappa, a_C(\kappa)) \) are weakly decreasing in \( \kappa \).

Finally, we define maximized welfare in the (partially) rigid regime where the cost of changing the law ex-post is \( \kappa \)

\[ W_C(\kappa) = \max_{a_C(\kappa) \in [\bar{\kappa}, \underline{\kappa}]} \theta^2 \Phi(1) a(1, \kappa, a_C(\kappa)) \left[ \tilde{W}_1(\kappa, a_C(\kappa)) - \tilde{W}_0(\kappa, a_C(\kappa)) \right] + \tilde{W}_0(\kappa, a_C(\kappa)) \]  \hspace{1cm} (A.9)

We now show that for all \( \kappa'' > \kappa' > 0 \), we have that \( W_C(\kappa'') \leq W_C(\kappa') \). To see this, note that

\[
W_C(\kappa') \geq \theta^2 \Phi(1) a(1, \kappa', a_C(\kappa'')) \left[ \tilde{W}_1(\kappa', a_C(\kappa'')) - \tilde{W}_0(\kappa', a_C(\kappa'')) \right] + \tilde{W}_0(\kappa', a_C(\kappa'')) \\
= W_C(\kappa'')
\]

The first inequality follows from the fact that \( a_C(\kappa') \) is the optimal strategy when the cost is \( \kappa' \). Hence it cannot provide less utility than choosing \( a_C(\kappa'') \). To understand the second inequality, just recall that for a given law \( a_C(\kappa) \), we have that \( a(1, \kappa, a_C(\kappa)), \tilde{W}_1(\kappa, a_C(\kappa)) \) and \( \tilde{W}_0(\kappa, a_C(\kappa)) \) are all weakly decreasing in \( \kappa \).

We now show that for all \( \kappa > 0 \) we have \( W_C(\kappa) < W_F \). Consider any \( \kappa > 0 \). Two cases are possible: \( a_C(\kappa) = \bar{\kappa} \) or \( a_C(\kappa) < \bar{\kappa} \).

First, assume that \( a_C(\kappa) = \bar{\kappa} \). In this case, \( a(1, \kappa, a_C(\kappa)) = \bar{\kappa} \) and the probability that state 1 occurs when \( \kappa = 0 \) and when \( \kappa > 0 \) is the same. (Recall in fact that when \( \kappa = 0 \) we have \( a(1, 0, a_C(0)) = \bar{\kappa} \).) Then, when \( a_C(\kappa) = \bar{\kappa} \) we obtain that \( \tilde{W}_1(0, a_C(0)) = \tilde{W}_1(\kappa, \bar{\kappa}) \). However, since in state 0, \( \bar{\kappa} \) is suboptimal, we also have that \( \tilde{W}_0(0, a_C(0)) > \tilde{W}_0(\kappa, \bar{\kappa}) \). Since the probability that state 0 occurs is assumed to be strictly positive, from (A.9) we conclude that \( W_C(\kappa) < W_C(0) = W_F \).

Second, assume that \( a_C(\kappa) < \bar{\kappa} \). Two further cases are possible: either \( a(1, \kappa, a_C(\kappa)) = \bar{\kappa} \) or \( a(1, \kappa, a_C(\kappa)) < \bar{\kappa} \). In the former case, the probabilities of state 1 occurring is the same when \( \kappa = 0 \) and when \( \kappa > 0 \). However, it must be that \( \tilde{W}_0(0, a_C(0)) \geq \tilde{W}_0(\kappa, a_C(\kappa)) \) and, because the law is changed in state 1, \( \tilde{W}_1(0, a_C(0)) > \tilde{W}_1(\kappa, a_C(\kappa)) \). This implies that \( W_C(\kappa) < W_F \). Second, assume that we have that \( a(1, \kappa, a_C(\kappa)) < \bar{\kappa} \). Then, \( \tilde{W}_1(0, a_C(0)) > \tilde{W}_1(\kappa, a_C(\kappa)) \). Moreover, the probability that state 1 occurs when \( \kappa = 0 \) is strictly greater than the same probability when \( \kappa > 0 \). Then, we also have that \( W_C(\kappa) < W_F \).

iii) When technology is at an early stage the ex-post optimal law is always \( \underline{\kappa} \) so that the flexible regime does not provide any incentive to innovate. The rigid regime, on the contrary, can choose to provide incentives.
Therefore, exactly as in case iii) of Proposition 2, the rigid regime could replicate the flexible one by picking \( a \), so that it must be the case that \( W_C(\kappa) \geq W_F \). We refer to the main text (see Section 4.4) for an example that shows that \( W_C(\kappa) \) may not be monotone in \( \kappa \) when technology is at the early stage.

**Proof of Proposition 5:** We rewrite the legislator’s problem in (39) as

\[
W_C = \max_{a_C \in [0, A]} f(a_C) a_C,
\]

where

\[
f(a_C) = \frac{A(0)^2}{4 (r - (\gamma_A^2 - 1) \theta \pi(a_C))} - \frac{\lambda}{r}
\]

and \( \pi(a_C) \) is obtained using (35).

**Step 1:** A sufficient (but not necessary) condition to insure that the objective in problem (A.10) is convex is that \( \gamma_A^2 > 2 \).

**Proof:** To show this, since \( f'(a_C) > 0 \) and \( a_C > 0 \), it is enough to show that \( f''(a_C) > 0 \). For simplicity, we denote \((\gamma_A^2 - 1)\) by \( \gamma \).

The first and second derivatives of \( f(a_C) \) are, respectively,

\[
f'(a_C) = \frac{\gamma \theta \pi'(a_C) A(0)^2}{4 (r - \gamma \theta \pi(a_C))^2}
\]

and

\[
f''(a_C) = \left( \frac{\gamma \theta A^2}{4} \right) \frac{\pi''(a_C) (r - \gamma \theta \pi(a_C)) + 2 \gamma \theta (\pi'(a_C))^2}{(r - \gamma \theta \pi(a_C))^3}
\]

Given that in order for welfare to have an upper bound, we assumed \( r - \gamma \theta \pi(a_C) > 0 \) for all laws, we obtain that \( f''(a_C) > 0 \) if and only if

\[
\pi''(a_C) (r - \gamma \theta \pi(a_C)) + 2 \gamma \theta (\pi'(a_C))^2 > 0.
\]

After obtaining \( \pi'(a_C) \) and \( \pi''(a_C) \) from equation (35), inequality (A.14) can be written (after denoting \( A(0) \) and \( \Omega(0) \) simply by \( A \) and \( \Omega \), respectively) as

\[
\frac{\gamma}{32} \theta^4 A^4 \left[ \Omega(4r^2 \Omega + \theta^2 A_{a_c}) \right]^{-1} - \frac{1}{16} \theta^4 A^4 \Omega \left[ \Omega(4r^2 \Omega + \theta^2 A_{a_c}) \right]^{-\frac{3}{2}} (r - \gamma \theta \pi(a_C)) > 0.
\]

Step 2: If \( \gamma_A^2 > 2 \), the law in the rigid regime is equal to \( \pi \) when \( \theta, \gamma_A \) and \( \vartheta(0) \) are sufficiently high.
Proof: First, note that given that the objective in (A.10) is convex, a sufficient condition to insure that the optimal law is \( \pi \) is that

\[
\frac{A(0)^2}{4(r - (\gamma_A^2 - 1)\theta \bar{z}(\bar{\pi}))} - \frac{\lambda}{r} > 0.
\] (A.16)

Since \( \bar{z}(\bar{\pi}) \) does not depend on \( \gamma_A \), it is easy to verify that when \( \gamma_A \) is sufficiently large, (A.16) is satisfied. Therefore, the solution of problem (39) is \( a_C = \pi \). After verifying that \( \pi(a_C) \) is increasing in \( \theta \) and \( A(0) \), a similar argument is used to show that when either \( \theta \) or \( A(0) \) are sufficiently high, we have \( a_C = \pi \). \( \blacksquare \)
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