Systemic risk in derivative markets: a graph-theory analysis

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Objectives

• Empirical study on systemic risk in derivative markets
• Integration as a necessary condition for systemic risk to appear
• Previous literature on integration
• Three-dimensional approach
  - Observation time
  - Spatial integration
  - Maturity
• Influence of physical as well as derivative markets
Selected markets

• Choice directed by:
  - Concerns about speculation in commodities
    Energy products
  - Development of bio fuels
    Agricultural products
  - Portfolio management / Commodities as a new class of assets
    Financial instruments
  - Markets with the highest transaction volumes

• 14 markets ( > 655 000 daily futures prices (settlement))
Methodology

• Huge volume of data
• 3 dimensional analysis: complex evolving system
• Use of methods originated from statistical physics
• Graph-theory
• Full connected graph:
  all possible connections between N nodes (time series of price returns) with N-1 links
• Filtered graph: Minimum Spanning Trees (MST)

1. Construction of MST
2. Topology of the filtered networks
3. Evolution of the MST over time

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1. Minimum spanning trees

- Synchronous correlation coefficients $\rho$ of prices returns $r$:

$$r_i = \frac{(\ln F_i(t) - \ln F_i(t - \Delta t))}{\Delta t}$$

$$\rho_{ij}(t) = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{\langle r_i^2 \rangle - \langle r_i \rangle^2} \sqrt{\langle r_j^2 \rangle - \langle r_j \rangle^2}}$$

- With: $F(t)$, futures prices at $t$
- Correlation matrix $C$, $(N \times N)$, symmetric

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From correlation to distances

- Non linear transformation
- Distances $d$ between two nodes defined as follows:

$$d_{ij} = \sqrt{2 \left(1 - \rho_{ij}\right)}$$

- Distance matrix $D$, $(N \times N)$
- Full connected graph
  - represents all the possible connections between $N$ nodes (vertices)
  - can be weighted (by the distances)
Minimum spanning tree

- All the nodes of the graph are spanned
- No loops
- Result: links of the MST are a subset of the initial graph
- The information space is reduced from $(N(N-1)/2)$ to $(N-1)$
- In this study: shortest path linking all nodes
  
  Easiest path for the transmission of prices shocks
2. Topology of the MST

- Maturity, Spatial, 3-D
- Allometric properties of the MST:
  - Quantifying the degree of randomness in the tree
  - The root is the node with the highest connectivity
  - Starting from this root, two coefficients $A_i$ and $B_i$ are assigned to each node $i$:

$$A_i = \sum_j A_j + 1 \quad B_i = \sum_j B_j + A_i$$

$$B \sim A^\eta$$

Where $\eta$ is the allometric exponent

$\eta$ stands between 1+ (star-like) and 2- (chain-like)
Maturity dimension

Heating oil – Month 1 to 18

Samuelson effect

$\eta = 1.9$

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Spatial dimension: $\eta = 1.493$

Agriculture
- Wheat
- Corn
- Soy Bean
- Soy Oil

Finance
- Interest rates
- Exchange rates
- Gold
- Crude, US
- Crude, UK
- Heating oil, US
- Gas oil, UK
- Nat. Gas, US
- Nat. Gas, UK

Energy
- S&P 500
- Crude, US
- Crude, UK
Three dimensions: $\eta = 1.757$

- Agriculture
- Energy
- Finance
<table>
<thead>
<tr>
<th>Name</th>
<th>Static coefficient</th>
<th>Dynamical coefficient</th>
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<tr>
<td>IED</td>
<td>1.927 ± 0.056</td>
<td>1.913 ± 0.011</td>
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<tr>
<td>LNG</td>
<td>1.874 ± 0.002</td>
<td>1.886 ± 0.059</td>
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<tr>
<td>LLE</td>
<td>1.88 ± 0.003</td>
<td>1.943 ± 0.02</td>
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<td>NNG</td>
<td>1.75 ± 0.037</td>
<td>1.774 ± 0.018</td>
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<tr>
<td>LLC</td>
<td>1.889 ± 0.003</td>
<td>1.904 ± 0.095</td>
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<td>NCL</td>
<td>1.994 ± 0.045</td>
<td>1.906 ± 0.013</td>
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<tr>
<td>NGC</td>
<td>1.732 ± 0.092</td>
<td>1.908 ± 0.013</td>
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<tr>
<td>CBO</td>
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<td>1.886 ± 0.032</td>
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<tr>
<td>CS</td>
<td>1.848 ± 0.095</td>
<td>1.822 ± 0.095</td>
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<tr>
<td>CW</td>
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<td>1.761 ± 0.125</td>
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<tr>
<td>CC</td>
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<td>1.834 ± 0.024</td>
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<tr>
<td>Spatial</td>
<td>1.493 ± 0.056</td>
<td>1.621 ± 0.024</td>
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<tr>
<td>3-D</td>
<td>1.757 ± 0.023</td>
<td>1.85 ± 0.009</td>
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3. Dynamical studies

3.1. Correlation coefficients
3.2. Node’s strength
3.3. Normalized tree’s length
3.4. Pruning the trees

Rolling time windows : 480 trading days
3.1. Mean correlations and their variances (3-D)
3.2. Node’s strength

- Full connected graph
- The node’s strength $S_i$ indicates the closeness of one node $i$ to all others:

$$S_i = \sum_{i \neq j} \frac{1}{d_{ij}}$$
3.3. Normalized tree’s length

- Sum of the lengths of the links belonging to the MST:

\[ L(t) = \frac{1}{N-1} \sum_{(i,j) \in MST} d_{ij} \]

- The more the length shortens, the more integrated the system is.

Spatial dimension
3.4. Survival ratios

• Robustness of the topology over time
• The survival ratio $S_R$ refers to the fractions of edges in the MST, that survives between two consecutive trading days:

$$S_R(t) = \frac{1}{N-1} \left| E(t) \bigcap E(t-1) \right|$$

$E(t)$ : set of edges at date t
Survival ratios in the spatial dimension
3.4. Pruning the trees

- Analysis of inter-market and inter-sectors reorganizations
- Consider only the links between markets, whatever the maturity is considered
- Survival ratios on the basis of market links, in the MST
  - Robustness of the topology over time
  - The survival ratio $S_R$ refers to the fraction of links that survives between two consecutive trading days:

$$S_R(t) = \frac{1}{N-1} \left| \frac{E(t) \cap E(t-1)}{E(t)} \right|$$

$E(t)$ : set of links at date $t$
Pruned trees, survival ratios

\[ S_R \]

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Links responsible for reorganizations

(a)

(b)
Main results & conclusions

1. Topology
   - Chain-like trees in the maturity dimension
   - Star-like trees in the spatial and 3-D dimensions

2. Emerging taxonomy
   - Trees organized around the three sectors of activity
   - Center of the graph: two crude oils

3. Integration
   - Increases in all dimensions (spatial, maturity, 3D)
   - Progresses at the heart of the system

4. Next
   - Analysis of price’s shocks
   - Causality and directed graphs
   - Adding new information in the graph