News and correlations: an impulse response analysis

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Abstract

We proceed to an impulse response analysis on the conditional correlations between three stock indices returns: the Nikkei, the FTSE 100 and the S&P 500. As a first step, we estimate an extension of the general asymmetric dynamic conditional correlation (GADCC) model proposed by Cappiello, Engle and Sheppard (2006) to model the possible interactions between conditional correlations. In a second step, we apply the definition of the impulse response function in nonlinear models of Koop, Pesaran and Potter (1996) to the conditional correlations matrix. The estimates of the GADCC model are used to estimate the impact of an innovation on the conditional correlations for different forecast horizons through bootstrapped simulations. For each forecast horizon, we estimate the density of the impulse response function with non parametric kernel estimator. These densities show that the impacts of shocks on conditional correlation are most often asymmetric and depend on history. They disappear as the forecast horizon increases. In a first step, we estimate the unconditional correlation impulse response functions with random shocks and histories. In a second step, we estimate these impulse response function with the observed history for several recent dates. We end by computing the impulse response function for an observed shock and history.

JEL Classification: C22, C32, G15, E17

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1 Introduction

It is well known, at least since Longin and Solnik (1995) that it is hard to consider correlations between markets as constant through time. If these correlations are time-varying, it is tempting to consider the impact of shocks on them, at least because correlations are crucial ingredients for any portfolio selection exercise.

Literature on the impact of shocks on moments of the second order or higher\(^1\) can be divided into two groups. Contributions as Gallant et al. (1993), Koop et al. (1996), Lin (1997) or Hafner and Herwartz (2006) are interested in the definition of the impulse response function in nonlinear models. This implies to define what kind of shock should be considered, what should be the baseline against which to measure the impact of a shock on the variable of interest.

Other contributions are more interested in the measure of the impact of well identified shocks, as macroeconomic announcements, on the volatility of the time series. This second category includes, among others, Balduzzi et al. (2001), Jones et al. (1998), Li and Engle (1998) for some studies in a parametric setting, and Andersen et al. (2003, 2007), Faust et al. (2003) for papers using nonparametric ultra-high frequency. The literature on the impact of shocks on cross-moments is also fruitful with previous contributions as Kroner and Ng (1998) and some recent extensions as Cappiello et al. (2006) and Scruggs and Glabadanidis (2003). In addition to these papers, which as for volatility rely on a parametric specification, recent contributions have measured the impact of news on covariance and correlation either using GARCH like model (Christiansen, 2000) or intraday data and nonparametric estimators (Christiansen and Ranaldo, 2007).

Our paper concerns the impact of a shock on cross-moments of a vector of three stock index returns. We focus on correlation between stock markets for different reasons. First, the correlation is a main input in portfolio selection. Portfolios that contain stocks from different markets have proven to be good diversification tools, particularly when assets from emerging markets are included, because different markets have different risk-return characteristics. Nevertheless, when markets are contaminated one from another, i.e. they all are falling at the same time beyond what fundamentals would indicate, the diversification effect may be decreased. This is also true from stock-bond relation (see Flavin and Panopoulou, 2009). The time-varying correlation between bonds and stocks has been explored in Cappiello et al. (2006), De Goeij and Marquering (2004, 2009), Brière et al. (2008), Li and Zou (2008) among others. This issue goes further than the general analysis of correlation within asset classes. and investigates the conditional correlation between classes.\(^2\) Second, the level of correlations between markets may be an indication of market

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\(^1\)See Jondeau and Rockinger (2009) for a recent analysis of the impact of shocks on skewness and kurtosis in a parametric setting.

\(^2\)The issue of flight-to-quality has recently been reexamined in Beber et al. (2009). Authors show that moving
integration. This has implications for policymakers who may be interested in the success or not of their policy dedicated to integrate some markets. Third, conclusions about expectations investors form about growth differentials can be drawn from the analysis of correlations.

The analysis of response of conditional correlation using multivariate GARCH modeling is not new. Cappiello et al. (2006), Christiansen (2000) and more recently Brenner et al. (2009), among others, have used some multivariate GARCH models to investigate the response of covariances and correlations to macroeconomic news or simple innovations (represented by their last period’s standardized return residuals). From a technical point, we also rely on a multivariate GARCH model. As in Cappiello et al. (2006) we use a modified version of the Engle’s (2002) dynamic conditional correlation (hence DCC) model allowing for asymmetry in the conditional correlation specification as well as in the univariate GARCH specifications. Because the number of series we aim to model is limited to 2 or 3 series, we choose a full version of the model allowing for non zero off-diagonal coefficients in the matrices of parameters.

A non constant correlation has been observed in many papers and it may appears, at first sight, that this observation lacks some theoretical foundations. A number of papers provided some possible explanations. Dumas et al. (2003) propose an approach in terms of national output to justify the change in correlation between national markets. Karolyi and Stulz (1996) argue that shocks are mainly responsible of correlation variations across time. In a series of paper, Veronesi (1999), Ribeiro and Veronesi (2002), Barlevy and Veronesi (2000, 2003) propose and find some extensions to a model based on rational expectations which is able to describe some stylized facts. The authors provide some theoretical support to the well-known increase in correlation during bear markets or crisis periods.

The present paper may also be related to the very active strand of economic literature dealing with contagion. Among others, Forbes and Rigobon (2002), Fleming et al. (1998), Dungey and Martin (2007), Campbell et al. (2008) and Chiang et al. (2007) have investigated this issue, leading to the conclusion that markets move beyond fundamentals. In all these papers, the analysis of cross-moments is a support for a better determination of periods where contagion occurs or not. The analysis of the covariance dynamics is also investigated in a number of papers dedicated to the determination of the optimal hedge ratio. Motivated by the computation of the optimal hedge ratio, Meneu and Torró (2003) examine the asymmetric dynamics of the covariance between spot and futures markets. The authors are interested in determining the response of volatility when computed using an asymmetric multivariate GARCH model allowing for asymmetry in both univariate

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from one market to another may sometimes be motivated by liquidity considerations. For an earlier analysis of the impact of news on bond volatility see Jones et al. (1998).

For a survey see Pericoli and Sbracia (2003) and Dungey et al. (2005). Note that Campbell et al. (2008) and Chiang et al. (2007) use some multivariate GARCH models as we do. Hartmann et al. (2004) rely on extreme value theory.
variance dynamics and covariance dynamics.

The paper contributes to the existing literature at least in two respects. First, we investigate for the first time the impact of news on correlation with a temporal perspective. Previous papers, beginning with Kroner and Ng (1998), have focused on the reaction of correlation one-step-ahead to different values of innovations. Differently, we investigate the reaction of correlation to a given (orthogonal) shock, drawn from the empirical distribution of errors, during a number of periods ahead. This is an original methodology, in the spirit of the original paper by Sims (1980), dedicated to a better understanding of the response of correlation to shocks of different amplitudes. Because our DCC model is mean-reverting to unconditional correlation, the issue of interest is the estimation of the period necessary for the impact of the shock to vanish. Second, it proposes a multivariate GARCH model with a great flexibility where asymmetry is permitted in the dynamics of volatilities and correlation as well as in the distribution. Using this flexible model and the methodology of impulse-response adapted to the multivariate GARCH framework, we examine empirically different kind of shocks (random, non-random) when history is assumed to be given of random as well.

The paper continues as follows. The next section presents the data and some descriptive statistics. Section 3 develops the GADCC model and the way it is estimated. The definition of the conditional correlation impulse response functions is given in section 4. Section 5 provides our empirical results for the different frameworks mentioned above. The last section reports some potential extensions to the present work.

2 Data

We consider the S&P 500, the FTSE 100 and the Nikkei indices, that is to say the main stock indices of respectively the New York, the London and the Tokyo Stock Exchanges. The values of the indices are taken from DataStream. We use weekly log returns for a period spanning January 31, 1978 to February 3, 2009, which give us a sample of 1618 returns. This long period includes bull and bear markets as well as more or less turbulent phases.

The three indices are displayed on Figure 1. The similarity between the British and the US stock indices is particularly striking. The behavior of the Nikkei is, as has been widely documented, quite different from the two others. Weekly returns are depicted in Figure 2. For convenience, each graph in this Figure has the same scale. The three indices exhibits volatility clustering commonly observed for financial time series. and the Japanese index

Note that Li and Zou (2008) choose to only allow for dynamics in the distribution but not in the volatility dynamics. This is not equivalent as is shown in the present paper.

As is well-known, volatility clustering in the data provides some indication for a GARCH approach, which accommodates some serial correlation in volatility and consequently a kurtosis higher than for the normal
is much more volatile than the two others. Some usual descriptive statistics for the weekly returns are provided in Table 1. Each series is negatively skewed and the excess kurtosis is high, particularly the FTSE 100 and the S&P 500. As is expected, the Jarque-Bera statistic strongly rejects the null hypothesis of Gaussianity for our three series. Finally, Table 2 displays sample correlation coefficients computed over the full sample of data. The FTSE 100 and the S&P 500 returns show the highest level of correlation, while the lowest is obtained for the correlation between the Nikkei and the FTSE 100. Figure 3 display the estimates of the three correlations estimated on a rolling window of 200 observations. These estimates show evidence of a time varying profile of correlations which support our use of a DCC model.

The Box-Pierce test for autocorrelation show that only the S&P returns exhibits significant serial correlation. An AR(4) is necessary to remove this autocorrelation.

3 The GADCC model

3.1 Presentation

We denote \( r_t \) the vector of the \( N = 3 \) stock indices returns. We define the \( ith \) component of the vector \( r_t \) as \( r_{i,t} = \log(I_{i,t}) - \log(I_{i,t-1}) \) where \( I_{i,t} \) denotes country \( i \) stock index at time \( t \). As a first step, we model separately the conditional mean of each return. The conditional means are modeled as a constant for the Nikkei and the FTSE returns and an AR(4) for the S&P. We obtain the \( N \times 1 \) vector of errors \( e_t \) for each date \( t \). This vector of errors is such that \( E(e_t | \psi_{t-1}) = 0 \) and \( Var(e_t | \psi_{t-1}) = H_t \) where \( H_t \) is the conditional variance covariance matrix at date \( t \) and \( \psi_{t-1} \) is the observed history up to \( t - 1 \). As our focus is on the impact of shocks on the expected conditional correlation, we apply an extended version of the DCC multivariate GARCH model of Engle (2002) as previously done by Cappiello, Engle and Sheppard (2006). This DCC model is based on the following decomposition of the conditional covariance matrix:

\[
H_t = D_t P_t D_t
\]

where \( D_t \) is the \( N \times N \) diagonal matrix of time-varying standard deviations estimated from the univariate GARCH models with the \( i^{th} \) return conditional standard deviation \( \sqrt{h_{it}} \) on the \( i^{th} \) position, and \( P_t \) is the time varying correlation matrix. The estimation of the DCC parameters follows a two steps process. As a first step, we estimate univariate volatility models. Once the univariate GARCH models are estimated, we use the standardized residuals, \( e_{it} = e_{it}/\sqrt{h_{it}} \) to estimate the parameters of the DCC model. The dynamics of the conditional correlation matrix is modelled as follows:

distribution.
\[ Q_t = (\bar{P} - A' \bar{P}A - B' \bar{P}B - G' \bar{N}G) + A' \epsilon_{t-1} \epsilon_{t-1}' A + G' \nu_{t-1} \nu_{t-1}' G + B' Q_{t-1} B \]

with

\[ P_t = Q_t^{-1} Q_t Q_t^{-1} \]

\( A, B, \) and \( G \) are \( N \times N \) matrices of parameters. \( \nu_{t-1} = I[\epsilon_t < 0] \circ \epsilon_t \) is the vector of negative standardized residuals. \( \bar{P} = E[\epsilon_t \epsilon_t'] \) and \( \bar{N} = E[\nu_t \nu_t'] \) are respectively the unconditional covariance matrices of \( \epsilon_t \) and \( \nu_t \). \( Q_t^* = [q^*_{it}] = [\sqrt{q^*_{it}}] \) is a diagonal matrix with the square root of the \( i \)th diagonal element of \( Q_t \) on its \( i \)th diagonal position. A sufficient condition for \( Q_t \) to be positive definite for all possible realizations is that the intercept, \( \bar{P} - A' \bar{P}A - B' \bar{P}B - G' \bar{N}G \), is positive semi-definite and the initial covariance matrix \( Q_0 \) is positive definite.

The assumptions we make on the matrices \( A, G \) and \( B \) differ from those of the initial DCC model of Engle (2002) or the model estimated by Cappiello, Engle and Sheppard (2006). In the standard DCC model of Engle (2002), these matrices were scalar while Cappiello, Engle and Sheppard (2006) estimated diagonal matrices. In the former case, the impact of a shock is supposed to be the same on each conditional correlation, while it can differ in the DCC model estimated by Cappiello, Engle and Sheppard (2006). However, in this model, the conditional correlation between two returns is only affected by the product of the shocks on these returns. The possible effect of a shock on a third market is excluded by assumption. To avoid this restriction, we impose no zero constraint on the matrices \( A, G \) and \( B \). The curse of dimensionality is the price we have to pay to get this more precise representation of the dynamics of shocks. The number of parameters to estimate is of order \( N^2 \) when the number of returns is \( N \).

Another concern is the assumption on the distribution of the standardized residuals \( \epsilon_{it} \). As it will be shown below, these standardized residuals do not follow a Gaussian distribution. However, we put aside this difficulty and estimate the univariate as well as the GADCC model with the QML estimator.

### 3.2 Estimation

In order to give support to the estimation of a dynamic conditional correlation model, we apply Tse (2000) test for constant correlations. This test is a Lagrange multiplier test which only requires the estimation of a Constant Conditional Correlation model. As the p-value to the LM statistics is equal to 0, the null hypothesis of constant conditional correlation is rejected which justifies the estimation of a DCC model.

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6 See Engle and Sheppard (2001) and Nakatani and Teräsvirta (2008) for further details on the positivity constraint in the class of DCC models.

7 For the sake of space, we do not display the estimate of the CCC model.
We use a two step estimation procedure. In the first step, a Glosten et al. (1993) asymmetric GARCH model is estimated for each returns and conditional variances are computed from these estimated models. Estimates of each of these three models are displayed in table 3 and estimated conditional variances are shown in figures 4. In a second step, we compute the standardized residuals and use them to obtain estimates of the GADCC model. The estimated parameters of the GADCC model are displayed in Table 5. These estimates show that off-diagonal terms are significant for each matrix. Many parameters of the matrix $G$ are also significant showing that there are asymmetric effects in the dynamics of conditional correlations.

The conditional covariances and correlations for the Nikkei and FTSE (left panel), the Nikkei and S&P (middle) and the FTSE and S&P (right panel) are displayed on Figures 5 and 6, respectively.

4 Conditional correlation impulse response function

4.1 Impulse response functions in nonlinear models

Once the GADCC model is estimated, our next step will be to quantify the impact of shocks on these conditional correlations. To do so, we will resort to the impulse response analysis, put forward by Sims (1980). We must stress here that our focus is on the impact of shocks on conditional correlation between returns and not their conditional mean, which makes a first significant difference with the traditional impulse response analysis applied to VAR models for instance.

Another difference is that, while Sims analyses impulse response in linear models, we apply it to a nonlinear one. The extension of the impulse response analysis to nonlinear models was provided by Gallant, Rossi and Tauchen (1993) (GRT hereafter) and Koop, Pesaran and Potter (1996) (KPP herafter). These authors propose two different definitions of impulse response in nonlinear models. GRT define a baseline and evaluate the impact of a deterministic shock added to the initial condition. Their “conditional moment profile” is the difference between the “shocked” and the baseline trajectories. This shock is supposed to be either observable or estimated.

KPP define the generalized impulse response function (GIRF) as the difference between the mean response of the variable of interest, here conditional correlation, conditional on both history and a shock and the mean response conditioned on history only.

Despite these differences, these authors shows that the properties of impulse response functions in a nonlinear model are very different from their properties in a linear model. To sum up briefly their results, they show that in nonlinear models, impulse response function are history dependent, non homogenous of any degree, non symmetrical and that
most often an analytical expression is not available. Therefore, these impulse response functions must be estimated through simulations.

Several papers have previously use the competing approaches of GRT and KPP to the analysis and estimation of the impulse response function of shocks on conditional variance. Lin (1997) applies GRT’s methodology to multivariate GARCH models while Hafner and Herwartz (2006) follow KPP and define the volatility impulse response function (VIRF). Hafner and Herwartz (2006) show that the two competing definitions of the impulse response function of GRT’s and KPP lead to substantial differences in the estimation of the impact of a shock. In trying to explain these differences, they argue that finding out a deterministic and realistic shock is far from reachable, all the more so that we are using high frequency data, the links of causality of which may be much intricate. Devising a baseline scenario is also quite difficult. For instance, a zero shock baseline scenario will produce an artificially increase in volatility for every kind of shock. This difficulty is used as an argument against GRT’s definition of the impulse response function. For these reasons, Hafner and Herwartz (2006) propose to use random shocks drawn from the estimated data generating process and not to include a baseline scenario. In this paper, we follow the definition of impulse response functions proposed by Koop et al. (1996) and Hafner and Herwartz (2006).

### 4.2 Definition of Correlation Impulse Response function

We use the definition of impulse response function in nonlinear models of KPP. In their paper, KPP define the Generalized Impulse Response Function (GI). We apply their framework to define the Correlation Impulse Response Function (CIRF) as follows:

\[
C_h(\xi_t, \psi_{t-1}) = E[\text{vech}(P_{t+h}) \mid \xi_t, \psi_{t-1}] - E[\text{vech}(P_{t+h}) \mid \psi_{t-1}]
\]

where \(\xi_t\) is a shock hitting the system at date \(t\) and \(\psi_{t-1}\) is the observed history at \(t - 1\). \(\psi_{t-1}\) is the set of past residuals, conditional variances and correlations up to time \(t - 1\): \(\psi_{t-1} = \{e_j, P_j, H_j, j = t - 1, t - 2, \ldots\}\). The index \(h\) represents the forecast horizon. \(C_t(\xi_h)\) is the \((N(N + 1)/2)\) vector of the impact of the shock on the \(h\)-ahead conditional correlation matrix components. The CIRF is therefore the difference between the \(h\)-ahead expected conditional covariance matrix given a shock and history and the expectation given history only.

KPP show that the above definition is a realization of the GI defined as the following random variable:

\[
C_h(\Xi_t, \Psi_{t-1}) = E[\text{vech}(P_{t+h}) \mid \Xi_t, \Psi_{t-1}] - E[\text{vech}(P_{t+h}) \mid \Psi_{t-1}]
\]

\(^8\)So far, VIRF methodology has only been applied in Shields et al. (2005) for macroeconomic purpose and Hafner and Herwartz (2006) on exchange rates.
where the shock $\Xi_t$ and the history $\Psi_{t-1}$ are random variables. KPP indicate that the impulse response function can be estimated for different combinations of observed or random shocks and histories. We follow KPP methodology and use a bootstrap procedure to estimate the correlation impulse response function. We will consider different case according to the assumptions we make on shock and history. The maximum forecast horizon is set to 10 weeks.

4.3 Identification of independent shocks

Finding out realistic shocks is crucial for the impulse response analysis in a multivariate framework. The vector of errors $\epsilon_t$ shows contemporaneous correlation and therefore one cannot shock one of its component without taking into account the changes in the others. A current method to solve this difficulty is to use a Cholesky decomposition of the conditional covariance matrix $H_t = P_t P_t'$, where $P_t$ is a lower triangular matrix, and to infer from this a random vector $\xi_t = P_t^{-1} \epsilon_t$ with independent components, zero mean, and identity covariance matrix. However, Cholesky decomposition makes $\xi_t$ depend on the ordering of the components of $\epsilon_t$. Another solution would be to impose some a priori structure of causality based for instance on economic theory. This method is hard to apply to our financial data or other high frequency data because the links of causation are rather unclear.

Hafner and Herwartz (2006) propose to use a Jordan decomposition of $H_t$ in order to obtain independent and identically defined (hence i.i.d.) innovations. The symmetric matrix $H_t^{1/2}$ is defined as:

$$H_t^{1/2} = \Gamma_t \Lambda_t^{1/2} \Gamma_t'$$

where $\Lambda_t = diag(\lambda_{1t}, \lambda_{2t}, ..., \lambda_{NT})$ is the diagonal matrix whose components $\{\lambda_{i,t}\}_{i=1}^N$ are the eigenvalues of $H_t$, $\Gamma_t = (\gamma_{1t}, ..., \gamma_{Nt})$ is the matrix $N \times N$ of the corresponding eigenvectors. A vector of independent shocks is defined as $\xi_t = H_t^{-1/2} \epsilon_t$. Hafner and Herwartz show that under the hypothesis of a non Gaussian distribution, $\xi_t$ is uniquely defined. This vector of innovation is treated as news, that is to say some independent perturbations unpredictable from the past that affect each markets. Summary statistics on the standardized residuals estimated from the GJR models are displayed in table 4 and show that, even after the GJR estimation, the standardized residuals still display negative skewness and excess kurtosis not compatible with a normal distribution. These statistics therefore give support to our resort of the Jordan decomposition to obtain a vector of orthogonalized residuals.
5 Empirical results

5.1 Estimation of CIRF: random shock and random history

As a first step we estimate the CIRF function under the assumption that the shocks and the histories are random variables, that is to say we estimate the random CIRF:

\[ C_h(\xi_t, \psi_{t-1}) = E[vech(P_{t+h}) \mid \Xi_t, \Psi_{t-1}] - E[vech(P_{t+h}) \mid \Psi_{t-1}] \]

We use the total number of 1593 histories at our disposal\(^9\). Note that we do not resample history as we consider we have enough observed history at our disposal. For each history, we bootstrap 10 shocks. For each of this 10 shocks, we bootstrap 100 future trajectories with and without this initial shock to compute the CIRF for the chosen forecast horizon. The average of these 100 CIRF measures the impact of the given initial shock. At the end of the simulation, we obtain 15930 estimates of the CIRF functions. We use these estimated CIRF to fit non-parametrically the densities of the CIRF functions at the forecast horizons \( h=1 \), \( h=5 \) and \( h=10 \). The CIRF are divided by each corresponding conditional correlation at the date the shock occurs.

The fitted densities are displayed on Figure 7 for the forecast horizons \( h = 1 \) (top panel), \( h = 5 \) (middle panel) and \( h = 10 \) (bottom panel). These graphs show what kind of information the estimation of the CIRF density at each forecast horizon can bring. A striking feature of the CIRF function at forecast horizon \( h = 1 \) is that the fitted densities are asymmetric for the correlations between the Nikkei and the FTSE and the Nikkei and the S & P. The mode of these two distributions is also positive around 50%. The shape of the density of the CIRF between the Japanese and the British returns appears to be bimodal with two modes of opposite signs. These features could not have been shown by a single indicator such as the average impulse response function. Furthermore, the construction of confidence intervals will have to integrate the information contained in these fitted densities. As the forecast horizon increases, the shape of the densities get closer to a single peak distribution around zero which shows the impact of the shock disappears through time.

5.2 Estimation of CIRF with random shocks conditional to a given history

In this section, we estimate the CIRF for four selected histories which represent situations when the financial markets are "bear" or "bull" markets or when the volatility is high or low. These histories were selected by looking at the past events on the New York Stock Exchange, and more precisely the trend and the volatility of the S&P index. From a technical point of view, the CIRF function we want to estimate as the following expression:

\[ C_h(\Xi_t, \psi_{t-1}) = E[vech(P_{t+h}) \mid \Xi_t, \psi_{t-1}] - E[vech(P_{t+h}) \mid \psi_{t-1}] \]

The past history is considered as the outcome of a random variable but the shocks that hit the system at that date are

\(^9\)The bootstrapping procedure is extensively described in KPP(1996).
random. As previously, shocks are bootstrapped from the sample of the observed residuals (properly orthogonalized).

5.2.1 CIRF in bull market

We consider a first week beginning at $t_1 = 05/13/2003$ where the S&P is rising and the level of volatility is rather low. The CIRF for this date are depicted on Figure 8 for the three forecast horizons $h=1$, $h=5$ and $h=10$. Each of the three fitted densities is asymmetric. The range of values of the CIRF has narrowed compared to the preceding case. The CIRF for the correlation between the Nikkei and the FTSE is bimodal with a positive global mode and a negative local one. These fitted densities tell us that the probability to observe an increase in correlation after a shock is slightly greater than the probability to observe a decrease, especially for the conditional correlation between the FTSE and the S&P.

The second date we consider is the week of $t_2 = 07/31/07$ at the beginning of the subprimes crisis. The S&P index has not yet begin to go down but the level of volatility has risen up. The CIRF are depicted for the three forecast horizons $h=1$, $h=5$ and $h=10$ on Figure 9. There is no big difference with the previous graphs for the correlations between the nikkei return and the other two. We can however that the fitted density for the FTSE and the S&P is more concentrated on positive values. The probability to observe an increase of the correlation between these two returns seems to be greater at this date than with the previous one. Perhaps this increase can be linked to the highest level of volatility.

5.2.2 CIRF in bear market

The third date is $t_3 = 01/15/08$ at the beginning of the subprimes crisis. At that time, the level of volatility on the NYSE was not very low. The CIRF function for the three conditional correlations are depicted on Figure 10. These fitted densities for the forecast horizon $h=1$ are once again asymmetric. The mode is positive and slightly greater than 50% but a noticeable part of the estimated densities is located under under zero. The fitted density is still bimodal for the conditional correlation between the Nikkei and the FTSE. A further step in our work will be to quantify the effect of a positive or a negative shock which will perhaps explain this bimodality. As the forecast horizon increases from 1 to 5 and 10 weeks, one can see that the impact of the shocks on the conditional correlations weakens.

The fourth date we consider is $t_4 = 07/10/08$ when the level of volatility is particularly high. The CIRF are shown on Figure 11. As there is no major difference with the CIRF for $t_3$, it seems that the level of volatility has no effect on the impulse response function when the market is bear.
5.3 Estimation of CIRF with a real shock conditional to a given history: the Lehman bankrupt

Finally, we examine the impact on the conditional correlation of a real shock, namely the fall of Lehman Brothers on September 15, 2008. This situation departs from the ones investigated in the previous sections as we do not simulate the initial shock but use and observed one. We also use the observed history to estimate the impulse response function. Impulse-response functions for conditional correlations are depicted in Figure 12.\(^{10}\)

Following the Lehman event, all three indices are sharply falling. However, we observe very different profiles for the three CIRF. The short-term impact on the correlation between the Nikkei and the two others indices is negative: the decrease is of 50\% with the FTSE and of 140\% with the S&P. Conversely, we observe an increase in the correlation between the FTSE and the S&P of around 25\% (the maximum of the impact is for the period \(h = 2\)).

The impact of this identified shock vanishes more slowly for this latter correlation than for the two others. In this case, benefits from diversification are removed for a longer period. Another conclusion is that this shock had a decoupling effect between the Nikkei and the two other indices. The de-correlation has a positive effect on a portfolio which would include the three indices.\(^{11}\)

6 Conclusion

This paper empirically extends the notion of IRF to conditional correlations. This issue is worthwhile because of the importance of correlation in many financial applications. Our results provide evidence of a different pattern for shocks depending on the period where the shock occurred and the amplitude of the shock. Our estimation show that the impulse response function are asymmetric and could be bimodal. The quantification of the impact of the negative shock of the Lehman Brothers’ bankruptcy has produced an increase in the correlation between the FTSE and the S&P. This increase is in line with the literature which highlights an increase in correlation following a significant unexpected decrease in returns. This effect is larger and more long-standing in an environment where volatility is higher (turbulent periods, see, e.g., Longin and Solnik (2001) on the differential impact of shocks in bull or bear periods). The negative impact on the correlations between the Nikkei returns and the other two returns show however that this increase in correlation is not global. The explanation of this difference requires a more careful analysis of the interdependencies between national stock exchanges.

The CIRF may be used in a context of contagion analysis. As soon as relevant common

\(^{10}\)At this date, the vector of residuals is \(e_0 = (-0.066169, -0.07612, -0.015359)\) and conditional volatilities and correlations are \((7.0791, 7.904, 13.221) \times 10^{-4}\) and \((0.5102, 0.4560, 0.7080)\), respectively.

\(^{11}\)We do not investigate the economic value of considering change in the correlation. For such an analysis, see Engle and Colacito (2006).
factors are considered (which is indeed a hard task as commonly emphasized in the literature), the impact of shocks on correlation should be insignificant under the null of no contagion (see Chiang et al. (2007) for a recent discussion of this issue using DCC model). Therefore, CIRF could be used to assess existence of contagion and the expected form it would take: long-term vs. short-term impact, etc.

On a more methodological side, it should be noted that, in this paper, we do not address the issue of parameter uncertainty. As noted in Koop (1996, p. 135-6): “[...] when an impulse response function is presented, it should reflect both parameter uncertainty and the uncertainty inherent in the fact that the shocks hitting the system are unknown. In an empirical exercise, these two sources of randomness must first be disentangled in order to understand the effect of shocks on the series.” To alleviate the parameter uncertainty problem, Koop (1996) proposes a Bayesian approach which treats parameters as random variable thus allowing to disentangle uncertainty due to parameter uncertainty and uncertainty coming from the random feature of the shock.
## Tables

### Table 1
Descriptive statistics for Nikkei, FTSE 100 and S&P 500 returns.

<table>
<thead>
<tr>
<th></th>
<th>Nikkei</th>
<th>FTSE 100</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean:</td>
<td>0.0003</td>
<td>0.0014</td>
<td>0.0014</td>
</tr>
<tr>
<td>Standard Deviation:</td>
<td>0.0278</td>
<td>0.0240</td>
<td>0.0235</td>
</tr>
<tr>
<td>Skewness:</td>
<td>-0.4015</td>
<td>-1.1196</td>
<td>-1.4467</td>
</tr>
<tr>
<td>Excess Kurtosis:</td>
<td>5.1016</td>
<td>15.1220</td>
<td>16.0030</td>
</tr>
<tr>
<td>Bera-Jarque chi(2):</td>
<td>1798.0448</td>
<td>15754.4780</td>
<td>17829.4500</td>
</tr>
<tr>
<td>p-value:</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>1st Autocorrelation:</td>
<td>-0.0311</td>
<td>-0.0392</td>
<td>-0.0926</td>
</tr>
</tbody>
</table>

### Table 2
Sample correlation between the Nikkei, FTSE 100 and S&P 500 returns.

<table>
<thead>
<tr>
<th></th>
<th>Nikkei</th>
<th>FTSE 100</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>0.3840</td>
<td>0.4049</td>
<td></td>
</tr>
<tr>
<td>0.3840</td>
<td>1.0000</td>
<td>0.5476</td>
<td></td>
</tr>
<tr>
<td>0.4049</td>
<td>0.5476</td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>
Table 3
QML estimates of the univariate GJR model for Nikkei, FTSE 100 and S&P 500 returns.

<table>
<thead>
<tr>
<th></th>
<th>Nikkei</th>
<th>FTSE 100</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>$2.7868 \times 10^{-4}$</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>$(7.4078 \times 10^{-9})$</td>
<td>$(1.919 \times 10^{-8})$</td>
<td>$(9.61 \times 10^{-9})$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0409</td>
<td>0</td>
<td>0.0086</td>
</tr>
<tr>
<td></td>
<td>$(0.0009)$</td>
<td>$(0.0176)$</td>
<td>$(0.0015)$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3784</td>
<td>0.1796</td>
<td>0.5280</td>
</tr>
<tr>
<td></td>
<td>$(0.0072)$</td>
<td>$(0.0210)$</td>
<td>$(0.0389)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.4594</td>
<td>0.5026</td>
<td>0.3713</td>
</tr>
<tr>
<td></td>
<td>$(0.0148)$</td>
<td>$(0.0840)$</td>
<td>$(0.0342)$</td>
</tr>
<tr>
<td>LL</td>
<td>3591.1</td>
<td>3770.2</td>
<td>3856.2</td>
</tr>
</tbody>
</table>

Table 4
Descriptive statistics for the standardized residuals from the GJR (Glosten et al., 1993) univariate models. From left to right: Nikkei, FTSE 100 and S&P 500 returns.

<table>
<thead>
<tr>
<th></th>
<th>Nikkei</th>
<th>FTSE 100</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean:</td>
<td>-0.0023</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
<tr>
<td>Standard Deviation:</td>
<td>0.9633</td>
<td>0.9874</td>
<td>0.9702</td>
</tr>
<tr>
<td>Skewness:</td>
<td>-0.5726</td>
<td>-1.5681</td>
<td>-1.6079</td>
</tr>
<tr>
<td>Excess Kurtosis:</td>
<td>4.1060</td>
<td>19.0168</td>
<td>16.9378</td>
</tr>
<tr>
<td>Bera-Jarque chi(2):</td>
<td>1221.9727</td>
<td>24981.7232</td>
<td>19988.7783</td>
</tr>
<tr>
<td>p-value:</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>1st Autocorrelation:</td>
<td>0.0023</td>
<td>-0.0244</td>
<td>0.0109</td>
</tr>
</tbody>
</table>

Table 5
Estimates of the GADCC model for Nikkei, FTSE 100 and S&P 500 returns.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>C</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1555 -0.0608 -0.0361</td>
<td>0.0506 0.0449 0.0881</td>
<td>0.8501 -0.0440 -0.0393</td>
</tr>
<tr>
<td></td>
<td>(0.0187) (0.0025) (0.0109)</td>
<td>(0.0089) (0.0017) (0.0014)</td>
<td>(0.0254) (0.0259) (0.0090)</td>
</tr>
<tr>
<td></td>
<td>0.0181 0.1151 -0.0102</td>
<td>-0.0427 0.2094 0.0143</td>
<td>0.0014 0.9786 -0.0054</td>
</tr>
<tr>
<td></td>
<td>(0.0062) (0.0026) (0.0094)</td>
<td>(0.0001) (0.0003) (0.0002)</td>
<td>(0.0061) (0.0478) (0.0101)</td>
</tr>
<tr>
<td></td>
<td>0.0722 -0.0041 0.0004</td>
<td>-0.0068 -0.1511 0.0200</td>
<td>0.0529 0.0240 1.0090</td>
</tr>
<tr>
<td></td>
<td>(0.0076) (0.0106) (0.0036)</td>
<td>(0.0017) (0.0003) (0.0005)</td>
<td>(0.0049) (0.0270) (0.0052)</td>
</tr>
<tr>
<td>Log-likelihood:</td>
<td>3775.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

14
Figures

Figure 1
Nikkei, FTSE 100 and S&P 500 stock indexes

Figure 2
Nikkei, FTSE 100 and S&P 500 returns.

Figure 3
Sample correlation using a rolling window of 200 observations for Nikkei, FTSE 100 and S&P 500 returns.
Figure 4
Estimated conditional variance from the univariate GJR-GARCH model (Glosten et al., 1993): Nikkei, FTSE 100, S&P 500.

Figure 5
Estimated conditional covariance from the full GA-DCC model: Nikkei and FTSE 100, Nikkei and S&P 500, FTSE 100 and S&P 500.

Figure 6
Estimated conditional correlation from the full GA-DCC model: Nikkei and FTSE 100, Nikkei and S&P 500, FTSE 100 and S&P 500.
Figure 7
Unconditional CIRF distribution for the horizon $h=1,5,10$. From left to right, Nikkei and FTSE 100, Nikkei and S&P 500, FTSE 100 and S&P 500.

CIRF for the horizon $h=1$

CIRF for the horizon $h=5$

CIRF for the horizon $h=10$
Figure 8
CIRF distribution conditional to history for $t_1$: 05/13/2003 for the horizon $h=1,5,10$.
From left to right, Nikkei and FTSE 100, Nikkei and S&P 500, FTSE 100 and S&P 500.

CIRF for the horizon $h=1$

CIRF for the horizon $h=5$

CIRF for the horizon $h=10$
Figure 9
CIRF distribution conditional to history for $t_2$: 07/31/2007 for the horizon $h=1, 5, 10$. From left to right, Nikkei and FTSE 100, Nikkei and S&P 500, FTSE 100 and S&P 500.
Figure 10
CIRF distribution conditional to history for $t_0$: 01/15/2007 for the horizon $h=1,5,10$.
From left to right, Nikkei and FTSE 100, Nikkei and S&P 500, FTSE 100 and S&P 500.

CIRF for the horizon $h=1$

CIRF for the horizon $h=5$

CIRF for the horizon $h=10$
Figure 11
CIRF distribution conditional to history for $t_4$: 10/07/2008 for the horizon $h=1,5,10$.
From left to right, Nikkei and FTSE 100, Nikkei and S&P 500, FTSE 100 and S&P 500.

CIRF for the horizon $h=1$

CIRF for the horizon $h=5$

CIRF for the horizon $h=10$
Figure 12
CIRF following the Lehman Brothers fall (September 16, 2008). The conditional correlations plotted are between Nikkei and FTSE, Nikkei and S&P and FTSE and S&P.
References


Scruggs, J.T., 1998. Resolving the puzzling intertemporal relation between the market risk premium and condi-


