Should more ambiguity averse agents exert more effort?

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Following the pioneering work by Ellsberg (1961), ample evidence in favor of ambiguity aversion has been accrued[1]. It suggests that agents systematically violate Savage's "Sure Thing Principle". More precisely, it seems that how we evaluate uncertainty depends on how precise our information about the underlying probabilities are - as opposed to the linearity of expected utilities in beliefs. The Stern Review (Stern (2007)) put a tremendous pressure for acting quickly and heavily against global warming while the Copenhagen Consensus (Lomborg (2004)) put top priority to preventive effort on health and development. In both cases, individuals may act in order to reduce the probability that the risk is realized at some cost. They may also act in order to reduce the loss in case the risk is realized. Then agents have to choose their self protection and self-insurance level as define by Erlhich and Becker (1972).

While risks on health and or development are well-known (relatively) the future effects of global warming are quite ambiguous. The IPCC had defined 4 families of possible emission scenarii. All of them predict an increase of average temperature however the distribution of the possible increases are not the same. Moreover individuals do not know exactly which one is the true one. One main objective of several policies is to induce economic agents to exert an effort in their greenhouse gas emission. Then a natural question is does ambiguity aversion reduce or increase preventive efforts. Several experimental contributions address this question. Following Shogren (1990), Di Mauro and Maffioletti (1996) investigate how individuals value risk reduction when the probability of loss is ambiguous. We choose to address this question from a theoretical point of

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[1]The *Ellsberg-Paradox* refers to the outcome of an experiment (Ellsberg (1961)), where a majority of participants' choices were inconsistent with expected utility theory. In an urn containing 90 balls, there were 30 red balls and the remaining were either black or yellow in unknown proportions. If participants won a bet, they received $100. A large group preferred to bet on drawing *red* vs. betting on *black*. However, in a second stage they preferred to bet on *not* drawing *red* vs. betting on *not* drawing *black*. Within an expected utility model, one cannot find a set of beliefs compatible with such preferences. Note, that betting on (or against) *red* is indeed an unambiguous act with well-defined winning probabilities, while betting on (or against) *black* is not. For a survey on the literature consult e.g. Camerer and Weber (1992).

view. We consider individuals with "smooth ambiguity preferences" as proposed by Klibanoff, Marinacci and Mukerji (2005, 2007). Our paper is technically related to Gollier (2006). In a finance context, he investigates comparative statics results of an increase in ambiguity aversion on the demand for risky assets. He shows that, in general, omitting ambiguity aversion cannot be corrected for by assuming a higher degree of risk aversion. Treich (2008) also consider the effect of ambiguity aversion on self protection. He investigates the effect of ambiguity aversion on the value of statistical life in a model of state dependant utility.

We show in this paper that ambiguity aversion may reduce the preventive effort an agent may undertake. Indeed, ambiguity averse agents behave as an expected utility maximizer with a distorted belief on the scenarii. They overweight the worse scenario. Under the condition that marginal expected utility and expected utility are anti-commonotonic, the marginal expected benefit is lower in the worse scenario that in the most favorable scenario. The individuals valuate less the preventive effort. The same effect exists when we consider self insurance rather than self protection.

The remainder of the paper is organized as follows. Section 2 introduces the basic model. In Section 3 we characterize the effect of ambiguity aversion on self protection. Section 4 investigates the effect of ambiguity aversion on self insurance. Section 5 concludes.

1 The model

We consider an agent whose preferences are describe by an increasing and concave utility function $u(x)$. She faces a risky wealth $\widetilde{\omega}=\omega-\widetilde{l}$ which is ambiguous, ω being the initial wealth and $\widetilde{l}$ the potential loss. We assume that there are S possible states of the world and the probability of each state $s\in\{1;...;S\}$ equals p_s, with $\sum_{s=1}^{S}p_s=1$. The distribution of probability p over these S states of nature is a function of a parameter θ that can take value $1,2,...,n$ with probability $q_1,...,q_n$, respectively. The agent has the opportunity to invest in self protecting activities $i.e.$activities that may modify the probabilities of loss. In order to capture these opportunities, we assume that the probabilities depend on an effort e such that $p_\theta\equiv p_\theta(e)$.[2] This investment has a non monetary cost $c(e)$[3] with $c'(e)>0$ and $c''(e)>0$. We assume that the consumer is ambiguity averse in the sense of Klibanoff, Marinacci, Mukerje (2005).

The agent chooses e in order to maximize her welfare:

$$\max_e V(e)=\sum_{\theta=1}^{n}q_\theta\Phi\left[U_\theta(e)-c(e)\right]=E\Phi\left[F(\theta,e)\right]$$

[2] The effort reduces the probability $p_{h\theta}$ with $p'_{h\theta}(e)<0$ for all $h=\{1,...,S-1\}$ and $p''_{h\theta}(e)>0$ $\forall\theta=1,2,...,n$. Then $p'_{S\theta}(e)$ is positive since $\sum_{s=1}^{S}p'_{\theta s}(e)=0$

[3] Our results remain true if the cost of effort is monetary and ambiguous.

with $U_\theta(e) = \sum_{s=1}^{S} p_{\theta s}(e)\, u(\omega_s)$.

The first order condition is

$$\sum_{\theta=1}^{n} q_\theta \Phi' \left[U_\theta(e) - c(e)\right] \left[\sum_{s=1}^{S} p'_{\theta s}(e)\, u(\omega_s) - c'(e)\right] = 0 \qquad (1)$$

We can rewrite this condition as

$$\sum_{\theta=1}^{n} \widehat{q}_\theta U'_\theta(e) = c'(e)$$

with $U'_\theta(e) = \left[\sum_{s=1}^{S} p'_{\theta s}(e)\, u(\omega_s)\right]$ and

$$\widehat{q}_\theta = \frac{q_\theta \Phi' \left[U_\theta(e) - c(e)\right]}{\sum_{\theta=1}^{n} q_\theta \Phi' \left[U_\theta(e) - c(e)\right]}$$

Again the agent behaves as a expected utility maximizer with a distorted probability distribution of the initials beliefs q_θ.

Assume now that the agent is ambiguity neutral: Φ' is constant. The first order condition rewrites :

$$\sum_{\theta=1}^{n} q_\theta U'_\theta(e) - c'(e) = 0 \qquad (2)$$

The optimal self-protecting activity e^* is as usual such that the marginal benefit is equal to the marginal cost of effort.

2 The effect of ambiguity aversion on self-protection and self-insurance

2.1 Self protection

In order to study the effect of ambiguity aversion on the optimal self protection we consider equation **??** evaluated at e^*.

$$V'(e^*) = \sum_{\theta=1}^{n} \widehat{q}_\theta U'_\theta(e) = c'(e^*)$$

By definition of e^*, we know that

$$\sum_{\theta=1}^{n} q_\theta U'_\theta(e) = c'(e^*)$$

then

$$V'(e^*) = \sum_{\theta=1}^{n} \widehat{q}_\theta U'_\theta(e) - \sum_{\theta=1}^{n} q_\theta U'_\theta(e) \qquad ((2))$$

In order to analyze the sign of $V'(e^*)$ we will make use of the following concept.

Definition 1 *Two vectors $(x_1, ..., x_n)$ and $(y_1, ..., y_n)$ are said to be* ***anti-commonotonic*** *if, for all $(i, j) \in \{1, n\}^2$, $x_i \leq x_j$ implies $y_i \geq y_j$.*

Take the case where the entries x_θ of the vector are ranked and ordered such that they are decreasing in θ. Such a vector is anti-commonotonic with any vector of the same dimension whose entries y_θ are increasing in θ.[4]

Proposition 2 *If the self protection has the same effect in each scenarii, ambiguity aversion has no effect on the optimal preventive activity chosen by the agent.*
If the marginal expected utility $U'_\theta(e)$ and the expected utility $U_\theta(e)$ are anti-commonotonic then an ambiguity averse agent chooses a lower self protecting effort than an ambiguity neutral one.

Proof. Without loss of generality, we consider that the values $U_\theta(e)$ are ranked and ordered such that they are increasing in θ. Since Φ is concave, we know that $\hat{q}_\theta$ is dominated by q_θ in the sense of the monotone likelihood ratio[5]. Moreover, considering that $U'_\theta(e)$ and $U_\theta(e)$ are anti-commonotonic implies that $U'_\theta(e)$ is decreasing in θ. Then we obtain that $V'(e^*)$ is negative. From the second order condition we know that $V'(e^*)$ is decreasing in e implying that the optimal effort for an ambiguity averse agent $\widehat{e}$ is lower than e^* the ambiguity neutral agent's optimal effort. ■

The intuition is the following. Due to ambiguity aversion, the agent behaves as an expected utility maximizer with a distorted belief on the scenarii. Indeed she overweights the worse scenario with respect to the true distribution of scenarii. Then if the marginal benefit of effort is increasing with the quality of scenarii then the agent valuates the preventive effort less than an ambiguity neutral agent. Since the marginal cost of effort is the same whatever is the ambiguity aversion, ambiguity averse agents choose a lower preventive effort.

2.2 Self insurance

Consider now, an agent that faces a risk of loss (or accident) and can engage in effort of self-insurance rather than self-protection. For a level e of effort, his wealth is $\omega - l_s(e)$ with probability $p_{\theta s}$ in each state $s \in \{0; 1; ...; S\}$. Note that $p_{s\theta}$ no longer depends on the effort e but still depends on θ a parameter of ambiguity that can take value $1, 2, ..., n$ with probability $q_1, ..., q_n$, respectively. As before, the function $c(e)$ can be thought of as the cost of effort. The function

[4] The definition is by Gollier and Gierlinger [2007].
[5] For a formal proof see Gollier [2006].

$l(e)$ is the loss, and ω is the initial wealth. We assume that the loss is positive: $l_s(e) > 0$. Moreover, we assume that $l_s(e)$ is decreasing. Then, the effort decrease the loss rather than the probability of loss.

Thus the expected utility of the agent is

$$V(e) = \sum_{\theta=1}^{n} q_\theta \Phi \left[U_\theta(e) - c(e) \right]$$

with $U_\theta(e) = \sum_{s=1}^{S} p_{\theta s} u(\omega - l_s(e))$.

The optimal action e maximizes the expected utility $V(e)$. Therefore $\widehat{e}$ is solution of the following condition

$$\sum_{\theta=1}^{n} q_\theta \Phi' \left[U_\theta(e) - c(e) \right] \left[U'_\theta(e) - c'(e) \right] = 0$$

with $U'_\theta(e) = \sum_{s=1}^{S} p_{\theta s} \left[-u' \left[\omega - l_s(e) \right] l'_s(e) \right]$

The above condition is equivalent to

$$V'(e) = \sum_{\theta=1}^{n} \widehat{q}_\theta U'_\theta(e) - c'(e) = 0$$

with

$$\widehat{q}_\theta = \frac{q_\theta \Phi' \left[U_\theta(e) - c(e) \right]}{\sum_{\theta=1}^{n} q_\theta \Phi' \left[U_\theta(e) - c(e) \right]}$$

Notice that this condition is equivalent to the one we obtain in the case of self protection. Again an ambiguity neutral agent chooses e such that

$$\sum_{\theta=1}^{n} q_\theta U'_\theta(e) = c'(e)$$

Let us denote e^* this self insurance investment. Then we obtain that $V'(e^*) = \sum_{\theta=1}^{n} \widehat{q}_\theta U'_\theta(e^*) - \sum_{\theta=1}^{n} q_\theta U'_\theta(e^*)$. Again this condition is exactly the condition (2).

It follows that proposition 2 is also true when we consider self insurance rather than self protection.

3 Self protection against both risk and ambiguity

We assume now that the action undertake by the agent does not only modify the probability of the state of the world but also the probability of the scenario

θ. In order to capture this effect, we assume that q_θ is a function of e the self protecting effort with $q'_{j\theta}(e) < 0$ and $q''_{j\theta}(e) > 0$.

The agent chooses e in order to maximize her welfare:

$$\max_e V(e) = \sum_{\theta=1}^{n} q_\theta(e)\, \Phi\left[U_\theta(e) - c(e)\right]$$

with $U_\theta(e) = \sum_{s=1}^{S} p_{\theta s}(e)\, u(\omega_s)$.

The first order condition is

$$\sum_{\theta=1}^{n} q_\theta(e)\, \Phi'\left[U_\theta(e) - c(e)\right] \left[\sum_{s=1}^{S} p'_{\theta s}(e)\, u(\omega_s) - c'(e)\right] + \sum_{\theta=1}^{n} q'_\theta(e)\, \Phi\left[U_\theta(e) - c(e)\right] = 0 \tag{3}$$

Let us define $\widehat{q}_\theta(e) = \frac{q_\theta(e)\Phi'[U_\theta(e)-c(e)]}{\sum_{\theta=1}^{n} q_\theta \Phi'[U_\theta(e)-c(e)]}$.

Then, we can rewrite the above condition as

$$\sum_{\theta=1}^{n} \widehat{q}_\theta(e)\, U'_\theta(e) + \sum_{\theta=1}^{n} \frac{q'_\theta(e)\, \Phi\left[U_\theta(e) - c(e)\right]}{\sum_{\theta=1}^{n} q_\theta(e) \Phi'\left[U_\theta(e) - c(e)\right]} = c'(e)$$

The first order condition in this setting is the similar to the one obtained in the previous section with an extra term $\sum_{\theta=1}^{n} q'_\theta(e)\, \Phi\left[U_\theta(e) - c(e)\right]$ which can be interpreted as the marginal benefit of decreasing ambiguity.

An ambiguity neutral agent chooses her optimal effort e^* such that :

$$\sum_{\theta=1}^{n} q_\theta U'_\theta(e) + \sum_{\theta=1}^{n} q'_\theta(e)\left[U_\theta(e) - c(e)\right] = c'(e^*)$$

Ambiguity aversion has an effect on the optimal effort which can be highlight by considering the ambiguity averse agent first order condition around e^*. By definition of e*, we obtain that

$$\begin{aligned} V'(e^*) &= \sum_{\theta=1}^{n} \widehat{q}_\theta U'_\theta(e) + \sum_{\theta=1}^{n} q'_\theta(e)\, \Phi\left[U_\theta(e) - c(e)\right] - c'(e^*) \\ &= \sum_{\theta=1}^{n} \widehat{q}_\theta U'_\theta(e) - \sum_{\theta=1}^{n} q_\theta U'_\theta(e) + \sum_{\theta=1}^{n} \frac{q'_\theta(e)\, \Phi\left[U_\theta(e) - c(e)\right]}{\sum_{\theta=1}^{n} q_\theta \Phi'\left[U_\theta(e) - c(e)\right]} - \sum_{\theta=1}^{n} q'_\theta(e)\left[U_\theta(e) - c(e)\right] \end{aligned}$$

The first part of $V'(e^*)$ is the same as in the previous section. The second part is the difference between the marginal benefits from the reduction of ambiguity.

The sign of the effect is not clear then it could induce an increase or a decrease of the optimal preventive effort leading to an increase or a decrease of ambiguity.

4 Conclusion

The effect of global warming are quite ambiguous meaning that there are several possible distributions of probability on the state of the world. There are a growing number of experimental studies that highlight the fact that agents are ambiguity averse (even actuaries and insurers). In this paper, we show that due to ambiguity aversion, the agent behaves as an expected utility maximizer with a distorted belief on the possible scenarii. Indeed she overweights the worse scenario with respect to the true distribution of scenarii. Then if the marginal benefit of effort is increasing with the quality of scenarii then the agent valuates the preventive effort less than an ambiguity neutral agent. Since the marginal cost of effort is the same whatever is the ambiguity aversion, ambiguity averse agents choose a lower preventive effort.

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