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Abstract

Privately informed owners securitizing assets signal positive information by retaining sufficient interest. Signaling provides social benefits, allowing uninformed investors to insure without fearing adverse selection. Instead of signaling, owners of high value assets may prefer a pooling equilibrium in which they securitize more of the asset, relying on speculators to gather information and bring prices closer to fundamentals. This induces suboptimal risk sharing, since uninformed investors face adverse selection. We analyze privately optimal securitization and the choice between signaling and reliance on speculative markets. In the model, prices are set competitively, with an endogenously informed speculator trading against uninformed hedgers placing rational orders. If a structuring exists providing sufficient speculator gains, her effort is high, mispricing is low, and all/most of the asset is securitized in a pooling equilibrium. Here risky debt and levered equity are optimal, with optimal face value trading off higher unit profits for the speculator against lower hedging demand. Hedgers imperfectly insure, buying only the concave claim, the only source of speculator profits. If risk-aversion is low and/or endowment shocks are small, hedging demand is low, leading to low speculator effort. Here high types sell only safe debt in a separating equilibrium with perfect risk sharing. The owner's incentive to choose the separating equilibrium is weak when risk-aversion is high and/or endowment shocks are large, precisely when efficient risk sharing has high social value.

Advocates of securitization commonly argue structured products improve risk sharing. For example, in his recent testimony before the U.S. Senate, Goldman Sachs investment banker Fabrice Tourre stated: "To the average person, the utility of these products may not be obvious. But they permit sophisticated institutions to customize the exposures they wish to take in order to better

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manage the credit and market risks of their investment holdings." The deeper intellectual argument in support of this benign view of financial innovation is the First Welfare Theorem which states that complete markets achieve Pareto optimal risk sharing (see e.g. Debreu (1959) and Arrow (1963)).

Policymakers have begun to question this positive view of securitization. For example, the Chairman of the U.K. Financial Services Authority recently contended, "the argument that they created great allocative efficiency benefits via market completion was hugely overstated." There is also a deeper intellectual argument buttressing this negative view of financial innovation. Elul (1995) and Cass and Citanna (1998) show that in a symmetric information economy, opening a new security market can change relative prices of consumption goods, making all agents worse off. Dow (1998) shows that even if there is a single consumption good, opening a new security market can make all agents worse off if there is asymmetric information between risk-averse uninformed agents and informed speculators.

While generic examples of welfare destroying financial innovation are intriguing, they do not allow one to take a completely informed view on the pros and cons of securitization, since they do not analyze which type of asset-backed structures will actually be introduced in equilibrium. That is, they do not comprehend the methods and motives for securitization. The purpose of this paper is to develop a theory of privately optimal securitization in a setting permitting welfare analysis. What are the private motives in the choice of securitization structures, and do they conflict with social objectives?

We consider the following setting. There are two periods and one consumption good. There is a single real asset with verifiable cash flows in the second period, so the only publicly traded securities are those backed by this asset. Thus, the securitization structure chosen by the original owner influences risk sharing. The owner has an intrinsic motive to raise funds in the first period, being able to immediately convert each unit of numeraire received from investors into $\beta > 1$ units. However, he retains the option to hold a claim to a portion of the securitized asset's cash flow. There is asymmetric information, with the owner knowing the true asset value, which is high or low, while other agents do not. Prices are set competitively by risk-neutral market-makers in response to market orders placed by rational risk-averse agents (hedgers below) and a risk-neutral speculator who can exert costly effort to increase the precision of the signal she receives regarding asset value.

The baseline model, which is of independent interest, considers the equilibrium securitization structure under the assumption that the owner must sell the entire asset. For example, one may think of this setting as approximating a distressed bank. The baseline model is nearly identical to that of Boot and Thakor (1993) with one important exception: The trading of the uninformed hedgers is endogenous. In addition to facilitating welfare analysis, which is impossible under pure noise-trading, endogeneity of hedge trading leads to novel implications.

If the owner must sell the entire asset, no separating equilibrium exists, and the equilibrium structure is that preferred by the owner of a high quality asset. Consequently, under full securitization the optimal structuring is that which maximizes the speculator's incentive to put in effort, since this reduces expected underpricing for the high type. Three novel implications emerge from endogenous trading by hedgers. First, hedger trading is confined to the most concave claim, e.g. senior debt, with all informed speculator gains confined to that market. Second, two publicly traded claims are always sufficient, since one may combine all but the most concave claim. Finally, the optimal information sensitivity of the concave claim trades off two competing concerns. On one hand, an increase in the information sensitivity of the concave claim increases the per-unit profit of the speculator, increasing her effort. However, this induces an endogenous decline in hedge trading volume, behind which the speculator hides. Under technical conditions, this tradeoff between information sensitivity and hedge trading volume yields an interior optimal face value for the senior debt when the asset is fully securitized.

In the general model, we allow the owner to retain a claim to the asset's future cash flow. This model is similar to that of Fulghieri and Lukin (2001) but differs in three important dimensions. First, they consider a setting with exogenous noise-trading, precluding analysis of risk sharing and welfare. Second, in our extended model, any claim held by the original owner is optimally structured, whereas they assume the retained claim is ordinary equity. Finally, in our general model there exist fully-revealing separating equilibria in addition to pooling equilibria.

When the owner is allowed to retain a claim to future cash flow, as in the general model, the equilibrium set always includes the least-cost separating equilibrium (LCSE) as viewed by the high type. In the LCSE, the low type sells the entire asset in equity form. In contrast, the high type sells only safe debt, retaining all risk on his own books in the form of a levered equity claim. Of course, this LCSE entails a deadweight loss since the high type is not implementing the first-best scale of the short-term production technology which has NPV of $\beta - 1$. The prediction that low types finance with equity and high types finance with debt is standard in signaling models, as is the prediction that high types pass up positive NPV investments (see e.g. Myers and Majluf (1984)). The attractive feature of the LCSE from a social perspective is that it results in first-best risk sharing. This is because full revelation of information allows hedgers to choose the optimal amount of insurance, no longer being concerned with exposure to adverse selection. In a related paper, Gorton and Pennachi (1990) find that riskless debt can be used to achieve efficient risk sharing. However, we argue that confining attention to riskless debt is overly restrictive in that any separating structure achieves efficient risk sharing.

In the general model, there also exist pooling equilibria provided they weakly Pareto-dominate the LCSE from the perspective of both owner types. In pooling equilibria, only risky securities are sold to outside investors, giving the speculator an incentive to acquire information. For example, there may exist pooling equilibria in which the asset is fully securitized, with the optimal structuring then being that defined above in the baseline model. The pooling equilibria involve social benefits and costs. The benefit of pooling equilibria relative to the LCSE is that the expected scale of first-period investment is first-best. However, in pooling equilibria uninformed hedgers are exposed to adverse selection. This causes them to change their trading decisions, resulting in inefficient risk sharing. Further, the speculator puts in costly effort.

The model can be viewed as providing a novel perspective on security design, one in which owners choose between signaling or relying on speculators to generate information. Further, this new perspective on security design highlights a fundamental conflict between private and public incentives in securitization, and in the degree of reliance on speculative activity. Pooling equilibria exist if and only if the speculator can be incentivized to put in sufficient effort under the effort-maximizing structuring. Intuitively, the high type prefers the pooling outcome to the LCSE when he knows trading by the speculator will bring prices close to fundamental value. *Intriguingly, factors that make efficient risk sharing more important from a social perspective increase the likelihood of private owners implementing pooling equilibria cum speculation and inefficient risk sharing.* To see this, note that hedge trader demand stimulates speculator effort, since she hides behind their buy orders. Thus, increases in the magnitude of the endowment shock hitting hedgers and/or increases in their risk-aversion serve to increase speculator effort. This increases the likelihood of the high type "choosing" the pooling equilibrium. Formally, the LCSE payoffs cease to be the unique equilibrium payoffs when endowment shocks are large and/or risk-aversion is high.

In addition to the papers cited above, our paper is closely related to that of Allen and Gale (1988) who also evaluate optimal security design in a setting with endogenously incomplete markets, with the firm(s) having a monopoly on issuing securities due to the need for asset-backing. The two models are predicated upon very different frictions, however. Allen and Gale assume there is symmetric information, but firms incur a cost when introducing a security. Hence, the tradeoffs in the models differ fundamentally.

DeMarzo and Duffie (1999) also analyze optimal security design from the perspective of an issuer who places intrinsic value on immediate liquidity. However, they consider a very different information structure. In their model, the issuer chooses the design of the security before observing the asset's true value. After the structure is locked-in, the issuer observes the true asset value and decides how much of the security to sell. Under technical conditions, e.g. monotonicity, debt is an optimal security since its low information-sensitivity results in low price impact per dollar raised. In contrast, we consider a setting where the issuer knows the asset's value when choosing the securitization structure. Further, we allow for a speculator to acquire information about asset value, alleviating mispricing. Finally, the model of DeMarzo and Duffie is silent on risk sharing since they assume universal risk-neutrality.

Nachman and Noe (1994) analyze a setting, like ours, where the issuer is privately informed at the time the security is designed. In their setting, the scale of investment is fixed, and there is no possibility for separation or informed speculation. Under technical conditions, e.g. monotonicity, they show firms will pool at a debt contract, since debt minimizes the cross-subsidy from high to low types.

A number of security design papers build on the insight of Hirshleifer (1971) that informed speculation can reduce the efficiency of risk sharing. Hennessy (2008) analyzes optimal security design in a setting where the firm is originally owned by a continuum of uninformed investors who rationally trade the various claims on the firm after being hit with a preference shock. For such owners, it can be optimal to promote information acquisition to the extent that more informative prices allow the firm's manager to make better real investment decisions. However, it may also be optimal to deter information acquisition, since this allows uninformed owners to sell their claims without adverse selection. Dang, Gorton and Holmström (2010) analyze optimal security design in a more general setting than Hennessy (2008), but also rule out feedback effects on real decisions. Consequently, the optimal structure in their setting attempts to deter information acquisition to preserve efficient risk sharing. Both papers show debt is optimal for deterring information acquisition.

Price informativeness has been analyzed in other corporate finance settings. Holmström and Tirole (1993) present a model in which the equity float affects information acquisition, price informativeness, and the risk premium paid to managers under optimal contracts. Aghion, Bolton and Tirole (2004) and Faure-Grimaud and Gromb (2004) show that the price informativeness stemming from speculative monitoring can promote effort by insiders ex ante. In addition to the fact that all three models are predicated upon managerial moral hazard, the other critical difference with our approach is that uninformed trading is exogenous.

There is a voluminous literature on the nature of claims that will be introduced by securities exchanges seeking to maximize trading volume and/or profits earned on the bid-ask spread. These models depart from our working assumption that the issuer of the security is privately informed about the value of the underlying asset. Surveys of the entire financial innovation literature are provided by Allen and Gale (1994) and Duffie and Rahi (1995).

The remainder of the paper is as follows. Section I describes the economic setting. Section II abstracts from the security design problem in order to focus on the market-making process. Section III analyzes optimal structuring in the baseline model where the entire asset must be securitized. Section IV considers a generalization of the baseline model in which the original owner may retain a claim on the underlying asset.

I. Economic Setting

This section describes preferences, endowments, and the market-making process.

A. Preferences and Endowments

There are two periods, 1 and 2, with a single nonstorable consumption good available in each period. This consumption good is the numeraire. The asset type τ is either high (H) or low (L), with Owner being the only agent endowed with perfect knowledge of the type. The asset delivers τ units of the good in period 2 with perfect certainty, with $L \in (0, H)$. The uninformed prior probability of the asset being high quality is $q \in (0, 1)$.

Owner possesses the only tangible real asset in the economy. Owner has no endowment other than this real asset. The tangibility of the asset allows courts to verify its value in period 2. In contrast, the endowments of the various agents are not verifiable by courts. Consequently, other agents cannot issue securities and cannot short-sell. Further, there can be no endowment-contingent contracts. Rather, courts can simply enforce real asset-backed payments contingent upon the observed asset value in $\{L, H\}$ in the second period. Since endowments are not verifiable by courts, Arrow securities are not possible and risk sharing may be inefficient. Allen and Gale

(1988) also consider an incomplete markets setting in which the firm has the unique ability to issue securities that can help to improve risk sharing.

There is a measure one continuum of small investors, labeled hedgers, who have may have an insurance motive for purchasing securities delivering consumption in period 2. The hedgers are sufficiently wealthy in the aggregate to buy the entire asset since each hedger has a first period endowment of $y_1 \geq H$. Hedgers face a common endowment shock, with their period 2 endowment being either 0 or $-\phi$, with each realization equally likely.[1] Prior to securities market trading in period 1, hedgers privately observe a perfectly informative signal regarding the size of their period 2 endowment, allowing them to condition trading decisions on this information.

Hedgers are risk-neutral over first period consumption c_1 and risk-averse over second period consumption c_2. They are indexed by the intensity of their risk-aversion as captured by a preference parameter θ. The utility of generic hedger-θ takes the form:

$$U(c_1, c_2; \theta) \equiv c_1 + \min(0, \theta c_2). \tag{1}$$

The preference parameters have compact support $\Theta \equiv [1, \theta^{\max}]$. Throughout, $\theta^{\max}$ is assumed to be sufficiently high such that there is always strictly positive hedging demand for some security.[2] The θ parameters have density f with cumulative density F. This distribution has no atoms, with f being strictly positive and continuously differentiable. As in Dow (1998), second period utility is piecewise linear, and has a concave kink at zero consumption. Since hedgers are averse to negative consumption in period 2, they have an intrinsic insurance motive for buying securities whenever their period 2 endowment is negative.

There is a single speculator S who is risk-neutral and indifferent regarding the timing of consumption having utility equal to $c_1 + c_2$. In the first period, she is endowed $y_1^S \geq H$ units of the numeraire, so she can afford to buy the entire asset. Her second period endowment is irrelevant and normalized at zero.

[1] The commonality of shock size is inconsequential, serving only to simplify the expression for aggregate demand. Alternative correlation structures yield similar results, with more cumbersome algebra.

[2] This avoids the need to continually check upper limits of integration when computing hedging demand.

The speculator is unique in that she receives a noisy signal of asset type. She can exert costly effort to increase the precision of her signal. Letting $s \in \{s_L, s_H\}$ denote the signal and τ the true asset type, S chooses $\sigma \equiv \Pr(s = s_\tau)$ from the feasible set $[1/2, 1]$. Her non-pecuniary effort cost function e is strictly positive, strictly increasing, strictly convex, twice continuously differentiable, and satisfies

$$\begin{aligned}\lim_{\sigma \downarrow \frac{1}{2}} e(\sigma) &= 0 \\ \lim_{\sigma \downarrow \frac{1}{2}} e'(\sigma) &= 0 \\ \lim_{\sigma \uparrow 1} e'(\sigma) &= \infty.\end{aligned}$$

If S puts in any effort, the signal becomes informative since

$$\sigma > \frac{1}{2} \Rightarrow \Pr[\tau = H | s = s_H] = \frac{\Pr[\tau = H \cap s = s_H]}{\Pr[s = s_H]} = \frac{q\sigma}{q\sigma + (1-q)(1-\sigma)} > q. \tag{2}$$

The final set of agents in the economy is a measure one continuum of market-makers. They are risk-neutral and indifferent regarding the timing of consumption having utility equal to $c_1 + c_2$. In the first period each market-maker is endowed with $y_1^{MM} \geq H$ units of the numeraire, so they too can afford to buy the entire asset. Their second period endowment is irrelevant and normalized at zero.

B. The Market-Making Game

We characterize perfect Bayesian equilibria (PBE) of signaling games, requiring: all agents have a belief at each information set; strategies must be sequentially rational given beliefs; and beliefs are determined using Bayes' rule and the equilibrium strategies for all information sets on the equilibrium path.

The market-making game is a signaling game played between the privately informed speculator and market-makers. Aside from Owner, other agents enter the market-making game holding their prior belief that the asset has high quality with probability q. The market-making game starts with S choosing σ at personal cost $e(\sigma)$. Her choice of σ is not observable, but is correctly inferred by other agents in equilibrium. Then S privately observes the signal s. Next, the hedgers observe (the perfect signal regarding) the size of their future endowment $y_2 \in \{-\phi, 0\}$. Finally, market-makers set prices competitively. To do so, in the market-making game they form beliefs regarding the signal received by S.

The market-making process is in the spirit of Kyle (1985) and Glosten and Milgrom (1985). The speculator and hedgers simultaneously submit non-negative market orders. Market-makers then set prices based upon observed aggregate demands in all markets. There is no market segmentation. At this information set, market-makers must have a belief about the signal s for any aggregate demand configuration. Market-makers clear all markets, buying all securities not purchased by hedgers or the speculator.

Since Owner is the only agent capable of issuing claims delivering goods in period 2, market-makers cannot be called upon to take short positions. To this end, we impose the following technical assumption.

$$A1 : \phi \leq \frac{L}{2}.$$

The role of Assumption 1 is as follows. The aggregate demand of hedgers is weakly increasing in ϕ. Therefore, to avoid the possibility of aggregate demand exceeding supply for any security, the endowment shock must be sufficiently small. Sufficiency of Assumption 1 for ensuring no shorting by the market-makers is established below.

II. Market-Making with a Single Security

We set the stage for our subsequent analysis by initially ignoring the security design problem altogether, focusing on how prices would be set by market-makers if the asset were simply sold in its entirety.

Many of the results derived in this section are relevant for cases where the firm bifurcates asset claims into two securities. To handle bifurcation into two claims A and B, let (A_L, A_H) and (B_L, B_H)

denote their respective period 2 payoffs as a function of the verified value in $\{L, H\}$. Security B is treated as the default in the case of only one security being issued. We have:

$$\text{All Equity:} \quad (B_L, B_H) = (L, H).$$

Since she cannot short-sell, the optimal strategy for the speculator is to place a buy order if and only if she receives a positive signal. She attempts hiding her buy orders behind those of hedgers but cannot do so with probability one since she does not know hedgers' endowment shock. The optimal size of her buy order is equal to the size of the aggregate buy order of hedgers when they are hit with a negative endowment shock. This latter quantity is denoted X.

Each hedger conditions his demand on y_2 and his idiosyncratic preference parameter θ. A hedger will not place a buy order if $y_2 = 0$ since the marginal utility of any increase in c_2 is then zero. An individual hedger may place a buy order if $y_2 = -\phi$ since $\theta > 1$ implies there is a strong insurance motive to avoiding negative consumption. However, each hedger is rational, weighing adverse selection costs against insurance motives when choosing optimal demand.

Table 1 lists the possible aggregate demand configurations confronting market-makers.

Table 1: Aggregate Demand Outcomes

Type	Signal	Endowment	Informed Demand	Hedge Demand	Aggregate Demand	Probability
H	s_H	$-\phi$	X	X	$2X$	$\frac{q\sigma}{2}$
H	s_H	0	X	0	X	$\frac{q\sigma}{2}$
H	s_L	$-\phi$	0	X	X	$\frac{q(1-\sigma)}{2}$
H	s_L	0	0	0	0	$\frac{q(1-\sigma)}{2}$
L	s_L	$-\phi$	0	X	X	$\frac{(1-q)\sigma}{2}$
L	s_L	0	0	0	0	$\frac{(1-q)\sigma}{2}$
L	s_H	$-\phi$	X	X	$2X$	$\frac{(1-q)(1-\sigma)}{2}$
L	s_H	0	X	0	X	$\frac{(1-q)(1-\sigma)}{2}$

After observing aggregate demand, market-makers form beliefs regarding the signal received by the speculator based upon the observed aggregate demand D, with:

$$\begin{aligned} \Pr[s &= s_H | D = 2X] = 1 \\ \Pr[s &= s_H | D = X] = 1 - q - \sigma + 2q\sigma \\ \Pr[s &= s_H | D = 0] = 0. \end{aligned} \tag{3}$$

Next note that beliefs over s can be mapped to beliefs over the asset type since:

$$\Pr[\tau = H|D] = \Pr[\tau = H|s = s_H] \Pr[s = s_H|D] + \Pr[\tau = H|s = s_L] \Pr[s = s_L|D] \tag{4}$$

where

$$\begin{aligned} \Pr[\tau &= H|s = s_H] = \frac{q\sigma}{1 - q - \sigma + 2q\sigma} \\ \Pr[\tau &= H|s = s_L] = \frac{q(1-\sigma)}{q + \sigma - 2q\sigma}. \end{aligned} \tag{5}$$

Substituting (5) into (4) one obtains:

$$\begin{aligned} \Pr[\tau &= H|D=2X] = \frac{q\sigma}{1-q-\sigma+2q\sigma} \\ \Pr[\tau &= H|D=X] = q \\ \Pr[\tau &= H|D=0] = \frac{q(1-\sigma)}{q+\sigma-2q\sigma}. \end{aligned} \tag{6}$$

It is readily verified that beliefs regarding τ increase monotonically in aggregate demand in the sense that

$$\Pr[\tau = H|D = 2X] > \Pr[\tau = H|D = X] > \Pr[\tau = H|D = 0]. \tag{7}$$

The market-makers then set the price (P) of equity as follows:

$$P(D) = L + (H - L)\Pr[\tau = H|D] \quad \forall D \in \{0, X, 2X\} \tag{8}$$

$$\Rightarrow P(2X) > P(X) > P(0). \tag{9}$$

To support the PBE conjectured in Table 1 it is sufficient to verify the speculator has no incentive to deviate regardless of the signal she receives. To that end, off the equilibrium path market-makers form adverse beliefs from the perspective of the speculator, setting prices based upon:

$$\Pr[s = s_H|D] = 1 \quad D \notin \{0, X, 2X\}.$$

It is readily verified that the speculator has no incentive to change her signal-contingent trading strategy when confronted with such beliefs. While such beliefs off the equilibrium path are sufficient to support the conjectured PBE of the market-making game, it is worthwhile to briefly discuss their plausibility. Note that any $D \notin \{0, X, 2X\}$ must be due to the speculator placing a strictly positive order. The chosen specification of beliefs off the equilibrium path is predicated on the intuitive notion that market-makers should view any such (positive) order as being placed by S after having observed s_H. After all, if a negative signal is received, the speculator stands to incur a loss from buying securities unless the market-makers form the most favorable beliefs from her perspective, which would entail $\Pr[s = s_H|D] = 0$. Conversely, if a positive signal is received, the speculator stands to make a strictly positive trading gain provided $\Pr[s = s_H|D] < 1$.

A. Expected Revenue

The expected revenue of the owner, conditional upon his having positive information is:

$$E[R|\tau = H] \equiv \overline{R}_H(\sigma) = L + (H-L)\left[\frac{\sigma \Pr[\tau = H|D = 2X]}{2} + \frac{(1-\sigma)\Pr[\tau = H|D = 0]}{2} + \frac{\Pr[\tau = H|D = X]}{2}\right]. \tag{10}$$

Alternatively, one may rewrite (10) as follows:

$$\begin{aligned} \overline{R}_H(\sigma) &= L + (H-L)Z(\sigma) \\ Z(\sigma) &\equiv \frac{1}{2}\left[\frac{q\sigma^2}{1-q-\sigma+2q\sigma} + \frac{q(1-\sigma)^2}{q+\sigma-2q\sigma} + q\right]. \end{aligned} \tag{11}$$

Anticipating, the variable Z plays an important role in the model. It measures the high type's expectation of the market-makers' updated probability of the asset having high value.

Intuition suggests $\overline{R}_H$ and Z are increasing in the precision of the signal received by the speculator, as captured by σ. For a high type, the more precise the signal, the more likely it is that S observes s_H and places a buy order. Since prices are increasing in aggregate demand, as shown in equation (8), expected revenue also increases with signal precision. To verify this conjecture, we compute

$$\begin{aligned} \frac{\partial \overline{R}_H}{\partial \sigma} &= \frac{H-L}{2}[(\Pr(\tau = H|2X) - \Pr(\tau = H|X)) + (\Pr(\tau = H|X) - \Pr(\tau = H|0))] \\ &+ \frac{H-L}{2}\left[\sigma\frac{\partial \Pr(\tau = H|2X)}{\partial \sigma} + \frac{\partial \Pr(\tau = H|X)}{\partial \sigma} + (1-\sigma)\frac{\partial \Pr(\tau = H|0)}{\partial \sigma}\right]. \end{aligned} \tag{12}$$

The first line of equation (12) captures the direct benefit to a high type of an increase in signal precision. To see this, suppose first that the hedgers experience a negative endowment shock, so their aggregate demand is X. If S receives the correct signal, total aggregate demand observed by the market-makers then increases from X to $2X$. Alternatively, if the hedgers do not experience a negative endowment shock, their aggregate demand is 0. If S receives the correct signal, total aggregate demand observed by the market-makers then increases from 0 to X. The second line in equation (12) accounts for the effect of σ on the belief revision process.

Lemma 1 confirms that the owner of a high value asset benefits from the speculator receiving a more precise signal. Most proofs are relegated to the appendix.

Lemma 1 *The expected revenue of the owner of a high value asset is increasing in the precision of the signal received by the speculator.*

From Lemma 1 it follows that Z is increasing in σ, with

$$\begin{aligned} Z(1/2) &= q \\ Z(1) &= \frac{1+q}{2}. \end{aligned} \tag{13}$$

B. Incentive Compatible Information Acquisition

Consider next the incentives of the speculator. From Table 1 one computes that her expected gross trading gain is

$$\begin{aligned} G(\sigma, X) &= X \cdot \left[\begin{array}{c} \left(\frac{q\sigma}{2}\right)[H - P(2X)] + \left(\frac{q\sigma}{2}\right)[H - P(X)] \\ + \left(\frac{(1-q)(1-\sigma)}{2}\right)[L - P(2X)] + \left(\frac{(1-q)(1-\sigma)}{2}\right)[L - P(X)] \end{array} \right] \\ &= \frac{q(1-q)(2\sigma - 1)(B_H - B_L)X}{2} \end{aligned} \tag{14}$$

It is readily verified that the speculator's trading gain increases linearly in each of its arguments, and that the marginal benefit of signal precision is increasing in X, with

$$\begin{aligned} G_1(\sigma, X) &= q(1-q)(B_H - B_L)X > 0 \\ G_2(\sigma, X) &= \frac{q(1-q)(2\sigma - 1)(B_H - B_L)}{2} > 0 \\ G_{11}(\sigma, X) &= G_{22}(\sigma, X) = 0 \\ G_{12}(\sigma, X) &= q(1-q)(B_H - B_L) > 0. \end{aligned} \tag{15}$$

An incentive compatible signal precision, denoted σ_{ic} satisfies.

$$e'(\sigma_{ic}) = q(1-q)(B_H - B_L)X. \tag{16}$$

Define the inverse function of e' as follows

$$\psi \equiv [e']^{-1}.$$

We may rewrite the incentive compatible signal precision as

$$\sigma_{ic} = \psi[q(1-q)(B_H - B_L)X]. \tag{17}$$

From the implicit function theorem and the convexity of the cost function e it follows that:

$$\begin{aligned} \frac{\partial \sigma_{ic}}{\partial X} &= \frac{q(1-q)(B_H - B_L)}{e''(\sigma_{ic})} \geq 0 \\ \frac{\partial \sigma_{ic}}{\partial (B_H - B_L)} &= \frac{Xq(1-q)}{e''(\sigma_{ic})} > 0 \\ \frac{\partial \sigma_{ic}}{\partial q} &= \frac{X(B_H - B_L)(1-2q)}{e''(\sigma_{ic})}. \end{aligned} \tag{18}$$

Since the incentive compatible signal precision plays a critical role, we summarize these findings in the following lemma.

Lemma 2 *The incentive compatible signal precision of the speculator is increasing in the aggregate demand of the hedge traders (X); increasing in the wedge between the value of claim B under high and low types ($B_H - B_L$); increasing in q for $q < 1/2$; and decreasing in q for $q > 1/2$.*

C. Hedging Demand

The next step is to determine aggregate hedging demand (X) for security B in response to a negative endowment shock. Before conducting this analysis it is worth noting that if there were symmetric information regarding the asset's value, hedgers would fully insure. In particular, whenever they faced a negative endowment shock they would submit demand for ϕ/τ units of equity for an asset of type $\tau \in \{L, H\}$. Such a demand would result in $c_2 = 0$, avoiding losses associated with negative consumption.

Letting $x^*(\theta)$ denote the optimal θ-contingent demand, aggregate hedging demand is

$$X \equiv \int_1^{\theta^{\max}} x^*(\theta) f(\theta) d\theta. \tag{19}$$

Since each hedger has measure zero, they act as price-takers. When hit with a negative endowment shock, an individual hedger expects the security to be overpriced since a subset of hedgers submits positive demands, pushing prices higher as market-makers revise upward their assessment of the probability of the asset being of high value. Despite facing adverse selection, an individual hedger is willing to submit a buy order if his hedging demand parameter θ is sufficiently high.

In order to characterize hedging demand, it is useful to first compute the expected price of the asset conditional upon a negative endowment shock.

$$\begin{aligned} E[P|y_2 &= -\phi] \equiv \overline{P}^- \\ \overline{P}^- &= [q\sigma + (1-q)(1-\sigma)]P(2X) + [q(1-\sigma) + \sigma(1-q)]P(X) \\ &= qH + (1-q)L + q(1-q)(2\sigma - 1)(H-L). \end{aligned} \tag{20}$$

Equation (20) is consistent with the intuition that small investors face adverse selection when submitting buy orders, since the asset is overpriced relative to its unconditional expected value. Further, adverse selection is increasing in the precision of the speculator's signal. This latter finding is at the heart of the trade-off in the model. Securitization structures that encourage information production by the speculator simultaneously worsen adverse selection as perceived by small investors, discouraging them from insuring against negative endowment shocks.

Recalling the functional form for hedger utility in equation (1), consider the change in expected utility experienced by hedgers for various demand perturbations:

$$\begin{aligned} x &\in \left(0, \frac{\phi}{H}\right) \Longrightarrow \frac{\partial E(U|y_2 = -\phi)}{\partial x} = \theta[qH + (1-q)L] - \overline{P}^- \\ x &\in \left(\frac{\phi}{H}, \frac{\phi}{L}\right) \Longrightarrow \frac{\partial E(U|y_2 = -\phi)}{\partial x} = \theta(1-q)L - \overline{P}^- \\ x &> \frac{\phi}{L} \Longrightarrow \frac{\partial E(U|y_2 = -\phi)}{\partial x} = -\overline{P}^-. \end{aligned} \tag{21}$$

From the last line in (21) it is apparent that no individual hedger ever buys more than ϕ/L units of equity since this results in $c_2 > 0$ with probability one and zero marginal utility. In order to further characterize optimal hedging demand, we define two cutoff values for the preference parameter:

$$\begin{aligned} \theta_1 &\equiv \frac{\overline{P}^-}{qH + (1-q)L} \\ \theta_2 &\equiv \frac{\overline{P}^-}{(1-q)L}. \end{aligned} \tag{22}$$

From the demand perturbation equations (21) it follows that:

$$\begin{aligned} \theta &\in [1, \theta_1] \Rightarrow x^*(\theta) = 0 \\ \theta &\in (\theta_1, \theta_2) \Rightarrow x^*(\theta) = \frac{\phi}{H} \\ \theta &\in [\theta_2, \theta^{\max}] \Rightarrow x^*(\theta) = \frac{\phi}{L}. \end{aligned} \tag{23}$$

Hedgers with relatively low risk-aversion, as proxied by θ, go without any insurance, with perceived adverse selection swamping their inherent hedging motive. Such investors have $c_2 = -\phi$ if there is a negative endowment shock. Intermediate preferences lead to partial underinsurance, with $c_2 = 0$ if $\tau = H$ and $c_2 = -\phi(H - L)/H$ if $\tau = L$. Finally, investors with strong hedging preferences overinsure with $c_2 = 0$ if $\tau = L$ and $c_2 = \phi(H - L)/L$ if $\tau = H$.

Lemma 3 summarizes.

Lemma 3 *If both types adopt an all-equity financial structure, aggregate hedging demand in the event of a negative endowment shock is*

$$\begin{aligned} X &= \phi \cdot \left[\frac{F(\theta_2) - F(\theta_1)}{H} + \frac{1 - F(\theta_2)}{L} \right] \\ \theta_1 &= 1 + \frac{q(1-q)(H-L)(2\sigma - 1)}{qH + (1-q)L} \\ \theta_2 &= 1 + \frac{q(1-q)(H-L)(2\sigma - 1) + qH}{(1-q)L}. \end{aligned} \tag{24}$$

Finally, we may pull together the incentive compatible signal precision from Lemma 2 and the hedging demand from Lemma 3 to verify existence of equilibrium. We have the following proposition.

Proposition 1 *In the market-making game, for any $B_H > B_L$, there exists a unique equilibrium pair (σ^{eq}, X^{eq}) satisfying*

$$\begin{aligned} \sigma_{ic}(X^{eq}) &= \sigma^{eq} \in \left(\frac{1}{2}, 1\right) \\ X(\sigma^{eq}) &= X^{eq} \in \left(0, \frac{\phi}{B_L}\right). \end{aligned}$$

III. Baseline Model: Optimal Structuring under Full Securitization

This section determines optimal security design in a baseline model predicated upon the assumption that Owner must sell the entire asset. In this case, there is no possibility of a separating equilibrium. This is because the owner of a low type asset would find it optimal to mimic the structuring chosen by the owner of a high type asset, since any other structure would fully reveal the negative private information, leaving him to collect the minimum expected revenue L. It follows that in the baseline model one may confine attention to the securitization structure that gives the high type the highest pooling payoff.

This baseline model is useful for three reasons. First, it approximates a number of real-world settings. For example, one may think of a distressed firm or financial institution that has an immediate need for cash. Alternatively, one may think of a conglomerate that has decided to spin off a division in order to focus on its core business. Finally, one may think of the baseline model as approximating a structured finance transaction where a bank has created a bankruptcy remote special purpose vehicle which will issue asset backed securities. The second reason for performing this baseline analysis is that it facilitates comparison with the findings of Boot and Thakor (1993) since they consider a setting in which all cash flow rights are sold to outside investors. Finally, solving the baseline model is a necessary precursor to Section IV which allows the original owner to endogenously retain some cash flow rights.

An important simplifying result is that in the absence of market segmentation, as assumed throughout, attention can be confined to two publicly traded securities without loss of generality. This result is stated as Lemma 4.

Lemma 4 *When market-makers observe aggregate demands across all markets, any outcome attainable with three or more publicly traded securities is attainable with two publicly traded securities.*

Following Nachman and Noe (1994) and DeMarzo and Duffie (1999)), attention is confined to securities with nonnegative payoffs that are weakly increasing in the asset value in period 2. Monotonicity is assumed for two reasons. First, monotone securities are commonplace. Second, if one of the securities, say security A, was decreasing, the owner of security B could benefit at the expense of the owner of A by making a clandestine contribution of additional funds to the asset pool. As argued by DeMarzo and Duffie (1999), only securities with monotone payoffs will be observed if such hidden contributions are feasible, and demanding monotonicity is then without loss of generality.

The major difference between the analysis of one and two securities is that hedgers insure themselves in the most efficient way possible. In particular, they buy the security that exposes them to the lowest degree of adverse selection per unit payoff. Therefore, we begin the analysis of multiple securities with a focus on hedging demand.

A. Hedging Demand

Note first that hedgers have positive hedging demands only if they experience a negative endowment shock. By the same reasoning applied in Section II, hedgers are concerned about overpricing of securities in the event of a negative endowment shock since they correctly anticipate high aggregate demand.

Before proceeding, we first argue that in the baseline model where the original owner must sell off all cash flow rights, it is never optimal to issue a safe claim. To demonstrate the argument, suppose to the contrary Owner carves out a safe senior debt claim with $B_L = B_H = L$. Then all hedgers use security B, and only B, to fully insure in the event of a negative endowment shock, setting their demand to $x_B = \phi/L$. As argued by Gorton and Pennachi (1990) such a financial structure achieves perfect risk sharing. However, in the present setting, the goal of the original (high type) owner is to encourage information production since this increases his expected revenue (Lemma 1). With safe debt available there will be no information production since there is no market in which the speculator can make trading gains. We state this result as Lemma 5.

Lemma 5 *If the owner were to issue a safe claim, hedging demand would be confined to that claim, resulting in zero speculator effort ($\sigma^{eq} = 1/2$). Market-makers would then revert to their prior probability q of the asset being type H. If the owner must securitize the entire asset, he will never issue a safe claim.*

Lemma 5 highlights an inherent conflict between private and public incentives in the choice of securitization structures. In particular, the sale of a riskless claim would bring about perfect risk sharing. However, the original owner may be willing to sacrifice perfect risk sharing in order to encourage information acquisition. Further, in the baseline model costly acquisition of information is socially wasteful, serving no allocative role.

In light of Lemma 5, the remainder of this section focuses on securitizations in which both claims are risky. To compute the optimal demand of an individual hedger it is useful compute his conditional price expectation. To this end, let:

$$\overline{P}_A^- \equiv E[P_A | y_2 = -\phi]$$
$$\overline{P}_B^- \equiv E[P_B | y_2 = -\phi].$$

Under this section's working conjecture, to be verified, that all hedging demand is concentrated in a single security, say security B, there is zero aggregate demand for A since the speculator will also refrain from trading in that market given the lack of any cover provided by hedgers. Therefore, Table 1 continues to be the relevant table depicting aggregate demand (for security B). Since there is no market segmentation, the aggregate demand for security B is also used by the market-makers in setting prices for security A. That is, the market-makers will set prices as a function of aggregate demand for B as follows:

$$\begin{aligned} P_A(D) &= A_L + (A_H - A_L)\,\Pr[\tau = H|D] \quad \forall \quad D \in \{0, X, 2X\} \\ P_B(D) &= B_L + (B_H - B_L)\,\Pr[\tau = H|D] \quad \forall \quad D \in \{0, X, 2X\}. \end{aligned} \tag{25}$$

Recall, the only agent of positive measure is the speculator. Therefore, to support the perfect Bayesian equilibrium conjectured in Table 1, it is sufficient to verify that the speculator has no incentive to deviate. To that end, we assume the market-makers form adverse beliefs from the perspective of the informed trader, setting prices based upon $\Pr[s = s_H|D] = 1$ for any $D \notin \{0, X, 2X\}$ in the market for security B and/or for any nonzero demand for security A. It is readily verified that the speculator then has no incentive to deviate from the conjectured signal-contingent trading rule.

Using Table 1 we arrive at the following expressions for the expected prices computed by hedgers when they experience a negative endowment shock:

$$\begin{aligned} \overline{P_A^-} &= [q\sigma + (1-q)(1-\sigma)]P_A(2X) + [q(1-\sigma) + \sigma(1-q)]P_A(X) \\ &= qA_H + (1-q)A_L + q(1-q)(2\sigma - 1)(A_H - A_L) \end{aligned} \tag{26}$$

and

$$\begin{aligned} \overline{P_B^-} &= [q\sigma + (1-q)(1-\sigma)]P_B(2X) + [q(1-\sigma) + \sigma(1-q)]P_B(X) \\ &= qB_H + (1-q)B_L + q(1-q)(2\sigma - 1)(B_H - B_L). \end{aligned} \tag{27}$$

Equations (26) and (27) are consistent with the intuition that hedgers perceive overpricing for any security unless it is riskless. Further, the degree of perceived adverse selection is increasing in the precision of the speculator's signal.

Assume without loss of generality that security A is the convex security in the sense of taking a larger percentage claim in the event that $\tau = H$:

$$\begin{aligned} \frac{A_H}{H} &\geq \frac{A_L}{L} \\ \Rightarrow & \frac{\overline{P}_A^-}{qA_H + (1-q)A_L} \geq \frac{\overline{P}_B^-}{qB_H + (1-q)B_L} \\ \Rightarrow & \frac{\overline{P}_A^-}{(1-q)A_L} \geq \frac{\overline{P}_B^-}{(1-q)B_L}. \end{aligned} \tag{28}$$

The two inequalities that follow from the convexity of claim A imply that this security is viewed by hedgers as having higher adverse selection costs per unit of c_2 provided. The intuition behind optimal hedging demand is simple. For hedgers with θ sufficiently low, demand is zero for both securities, with adverse selection dominating insurance motives. For hedgers with intermediate values of θ the agent partially insures, buying enough units of period 2 cash flow such that his consumption is zero if the actual asset type is H, which implies that consumption is negative if the actual asset type is L. Finally, if θ is sufficiently high, the hedger overinsures in the sense of purchasing enough units of the security such that c_2 reaches zero even if the actual asset type is L, which implies $c_2 > 0$ if the asset type is H. Of course, for any given level of insurance, hedgers seek the least costly security combination.

In the event of a negative endowment shock, second period consumption can be expressed as a function of the actual asset type

$$c_2(x_A, x_B, \tau) = x_A A_\tau + x_B B_\tau - \phi \quad \forall \quad \tau \in \{L, H\}.$$

The optimal hedge portfolio is determined using perturbation arguments. Attention is confined to portfolios satisfying $c_2(x_A, x_B, L) \leq 0$. Otherwise, $c_2 > 0$ regardless of the actual asset type, despite the fact that marginal utility is then locally equal to zero. Letting χ be an indicator for $c_2(x_A, x_B, H) < 0$, the expected utility of a hedger on the relevant interval is:

$$E[U|y_2 = -\phi] = y_1 - x_A\overline{P}_A^- - x_B\overline{P}_B^- - \chi q\theta[\phi - x_A A_H - x_B B_H] - (1-q)\theta[\phi - x_A A_L - x_B B_L].$$

Grouping relevant terms, the associated maximand can be rewritten as follows:

$$m(x_A, x_B) \equiv x_A[\theta(\chi q A_H + (1-q)A_L) - \overline{P}_A^-] + x_B[\theta(\chi q B_H + (1-q)B_L) - \overline{P}_B^-].$$

Note that m is piece-wise linear, exhibiting a concave kink at points such that $c_2(x_A, x_B, H) = 0$. With this in mind, assume an initial portfolio consisting of zero units of either security, and then consider a local perturbation. We then have:

$$\begin{aligned} \frac{\partial m(0,0)}{\partial x_A} &= \theta\left[qA_H + (1-q)A_L\right] - \overline{P}_A^- \\ \frac{\partial m(0,0)}{\partial x_B} &= \theta\left[qB_H + (1-q)B_L\right] - \overline{P}_B^-. \end{aligned} \tag{29}$$

If θ is sufficiently low, both perturbations listed in (29) are negative and optimal hedging demand is zero. Specifically:

$$\theta \leq \frac{\overline{P}_B^-}{qB_H + (1-q)B_L} \equiv \theta_1^B \Leftrightarrow (x_A^*, x_B^*) = (0, 0). \tag{30}$$

Next, consider a portfolio (x_A, x_B) such that $c_2(x_A, x_B, H) = \varepsilon$ where ε is arbitrarily small. That is, we are considering a point just right of the kink in the maximand. Performing a perturbation at that point one finds:

$$\begin{aligned} \frac{\partial m(x_A, x_B)}{\partial x_A} &= \theta(1-q)A_L - \overline{P}_A^- \\ \frac{\partial m(x_A, x_B)}{\partial x_B} &= \theta(1-q)B_L - \overline{P}_B^-. \end{aligned} \tag{31}$$

If θ is sufficiently high, such a perturbation increases the maximand. Further, since the maximand is piece-wise linear, it would then be optimal to fully insure against negative consumption, achieving $c_2(x_A^*, x_B^*, L) = 0$. Finally, from the inequality in (28) the minimal cost means of achieving this full insurance is to purchase only security B. Formally, we have:

$$\theta \geq \frac{\overline{P}_B^-}{(1-q)B_L} \equiv \theta_2^B \Leftrightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_L}\right). \tag{32}$$

The final case to consider is $\theta \in (\theta_1^B, \theta_2^B)$. From the perturbation arguments given above, we know such hedgers partially insure, with $c_2(x_A^*, x_B^*, H) = 0$. Now, consider the marginal utility (MU) per unit of period 1 numeraire allocated to the purchase of each security (on the relevant region where the hedger is partially insuring). From the inequality in (28) we know:

$$MU_B = \frac{qB_H + (1-q)B_L}{\overline{P}_B^-} \geq \frac{qA_H + (1-q)A_L}{\overline{P}_A^-} = MU_A. \tag{33}$$

It follows that security B yields the highest marginal utility on the region of partial insurance, so that

$$\theta \in (\theta_1^B, \theta_2^B) \Rightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_H}\right). \tag{34}$$

This establishes Proposition 2.

Proposition 2 *(Baseline Model) There is zero hedging demand for the convex claim A. In the event of negative endowment shock, aggregate hedging demand for security B is*

$$\begin{aligned} X(\phi, B_L, B_H) &= \phi \cdot \left[\frac{F(\theta_2^B) - F(\theta_1^B)}{B_H} + \frac{1 - F(\theta_2^B)}{B_L}\right] \\ \theta_1^B &= 1 + \frac{q(1-q)(B_H - B_L)(2\sigma - 1)}{qB_H + (1-q)B_L} \\ \theta_2^B &= 1 + \frac{q(1-q)(B_H - B_L)(2\sigma - 1) + qB_H}{(1-q)B_L}. \end{aligned} \tag{35}$$

Proposition 2 can be contrasted with a result obtained by Boot and Thakor (1993). In their model, speculators make trading gains in the convex levered equity claim. This results from their particular reduced-form specification of noise trading. In our model, hedgers optimize their portfolios and insure themselves using the least informationally sensitive claim–the concave claim. Consequently, in our model the speculator is unable to make trading gains in the market for the convex claim. Rather, she hides behind the hedgers in the market for the concave claim.

B. Optimal Structuring

In the baseline setting in which all cash flow rights must be sold, the sole objective of the owner of the high value asset is to maximize the incentive compatible level of signal precision σ_{ic}. To see this, recall from Lemma 1 that the expected revenue of such an owner is increasing in σ. Next, note that the incentive compatible signal precision is defined implicitly by equation (16). From convexity of effort cost the function e it follows that the optimal security design solves:

$$\begin{aligned}
& && PROGRAM\ \ 1 && (36)\\
(B_L^*, B_H^*) &\in && \arg\max_{B_L, B_H} \ (B_H - B_L) X(\phi, B_L, B_H) && (37)\\
& && s.t. \\
(Concavity) \quad \frac{B_L}{L} &\geq && \frac{B_H}{H} \\
(Monotonicity) \quad B_H &\geq && B_L && (38)\\
(Limited\ Liability) \quad B_L &\leq && L.
\end{aligned}$$

Intuitively, the optimal financial structure under full securitization maximizes the product of the speculator's per-unit profit and endogenous hedge trading volume. This creates a natural trade-off given that uninformed demand decreases with informational sensitivity. The constraint labeled *Concavity* ensures that security B is the liquid security in which hedging demand is concentrated. The three listed constraints ensure all other limited liability and monotonicity constraints are respected since they imply:

$$\begin{aligned}
A_\tau(0, \tau) &\in [0, \tau] \quad \forall \quad \tau \in \{L, H\}\\
B_L &> 0\\
B_H &\in (0, H]\\
A_H &\geq \frac{H A_L}{L}.
\end{aligned}$$

Conveniently, Program 1 is independent of the choice of B_L provided $B_L \in (0, L]$. This is because hedging demand is homogeneous degree minus one in (B_L, B_H). For example, if Owner were to cut both state contingent payoffs in half, each hedger would simply double his hedging demand. Thus, examining the objective function in Program 1, the optimal policy is unique up to a scalar, since

$$(B_H - B_L) X(\phi, B_L, B_H) = (\zeta B_H - \zeta B_L) X(\phi, \zeta B_L, \zeta B_H) \quad \forall \quad \zeta \in (0, 1]. \tag{39}$$

Given this finding, let

$$B_H \equiv \kappa B_L.$$

In this case, the aggregate demand defined in Proposition 2 simplifies as follows (with slight abuse of notation):

$$\begin{aligned} X &= X(\phi, B_L, \kappa) = \frac{\phi}{B_L} \cdot \left[1 - \frac{F(\theta_1(\kappa))}{\kappa} - \frac{(\kappa-1)F(\theta_2(\kappa))}{\kappa}\right] \\ \theta_1(\kappa) &\equiv 1 + \frac{q(1-q)(2\sigma-1)(\kappa-1)}{1+q(\kappa-1)} \\ \theta_2(\kappa) &\equiv 1 + \frac{q(1-q)(2\sigma-1)(\kappa-1)+q\kappa}{(1-q)}. \end{aligned} \tag{40}$$

Increases in κ reduce hedging demand since both cutoffs are increasing in κ with:

$$\begin{aligned} \theta_1'(\kappa) &= \frac{q(1-q)(2\sigma-1)}{[1+q(\kappa-1)]^2} > 0 \\ \theta_2'(\kappa) &= q(2\sigma-1) + \frac{q}{(1-q)} > 0. \end{aligned} \tag{41}$$

Making the substitution $B_H = \kappa B_L$ throughout Program 1 also allows us to simplify the optimal structuring problem as follows.

Lemma 6 *Suppose total securitized cash flow is worth l if the asset is low quality and $h \geq l$ if the asset is high quality. Total expected revenue received for the securitized claims, conditional upon the asset being of high quality, is maximized with $B_L^* \in (0, l]$ and $B_H^* = \kappa^* B_L^*$ where κ^* solves*

$$\begin{aligned} & PROGRAM \;\; 2 \\ \kappa^* \;\in\; & \arg\max_{\kappa} \quad M(\kappa) \equiv \phi(\kappa-1)\left[1 - \frac{F(\theta_1(\kappa))}{\kappa} - \frac{(\kappa-1)F(\theta_2(\kappa))}{\kappa}\right] \\ & s.t. \\ \kappa \;\leq\; & \frac{h}{l}. \end{aligned}$$

Lemma 7 establishes a sufficient condition under which the objective function in Program 2 is strictly concave.

Lemma 7 *If the cumulative distribution function for the intensity of hedging preferences (F) is weakly convex, then the maximand in Program 2 (M) is strictly concave.*

The intuition behind Lemma 7 is as follows. If F is convex then marginal increases in κ result in ever larger reductions in aggregate hedging demand. Further, the benefit to the speculator of the increase in per-unit profits stemming from an increase in κ is spread over a progressively smaller trading base. Consequently, the maximand is strictly concave. For the remainder of the paper it is assumed that F is convex.

$$A2: \ F \text{ is weakly convex.}$$

The Lagrangian for Program 2 can be written as:

$$\mathcal{L}(\kappa) \equiv M(\kappa) + \lambda \left(\frac{h}{l} - \kappa \right). \tag{42}$$

The optimal policy is characterized by a unique pair (κ^*, λ^*) satisfying the following first-order condition

$$M'(\kappa^*) = \lambda^* \tag{43}$$

and the complementary slackness conditions:

$$\begin{aligned} \left(\frac{h}{l} - \kappa^* \right) \lambda^* &= 0 \\ \lambda^* &\geq 0. \end{aligned} \tag{44}$$

For the remainder of the analysis, we shall assume that H/L is sufficiently high such that the concavity constraint does not bind if the firm fully securitizes. To this end, let κ^{**} denote the unconstrained maximizer of M:

$$\kappa^{**} \equiv (M')^{-1}(0).$$

We adopt the technical assumption:

$$A3: \frac{H}{L} > \kappa^{**} \Rightarrow \kappa^*(L, H) = \kappa^{**}, \quad \lambda^*(L, H) = 0.$$

One can think of Owner as progressively raising κ, bringing the concave claim B closer and closer to a linear claim. Doing so raises the per-unit profit of the speculator, but also diminishes demand for B, with Assumption 2 implying the demand cost rises with κ. Assumption 3 is predicated on the notion that the demand cost dominates before B becomes linear. That is, Assumption 3 ensures that when Owner fully securitizes the underlying real asset, it is never optimal to package it as straight equity. To see this at a technical level, suppose instead that the asset is fully securitized and that the concavity constraint in Program 2 were actually binding at the optimum. Then B_H/B_L would be equal to H/L and the owner of B holds a claim with information sensitivity equal to that of ordinary equity.

Differentiating the maximand yields:

$$\begin{aligned} M'(\kappa) &= \phi\left[1 - \frac{F(\theta_1)}{\kappa} - \frac{(\kappa-1)F(\theta_2)}{\kappa}\right] \\ &\quad -\phi\left[\frac{\kappa-1}{\kappa}\right]\left[\frac{F(\theta_2)-F(\theta_1)}{\kappa} + f(\theta_1)\theta_1'(\kappa) + (\kappa-1)f(\theta_2)\theta_2'(\kappa)\right]. \end{aligned} \tag{45}$$

The first term in (45) captures the gain from increasing informational sensitivity (via κ), as it increases the speculator's per-unit trading gain. The negative term captures the cost of increasing informational sensitivity in terms of reducing equilibrium hedging demand, behind which the speculator hopes to hide her trading. Canceling terms one obtains:

$$M'(\kappa) = \phi\left[1 - F(\theta_2)\left(1-\frac{1}{\kappa^2}\right) - \frac{F(\theta_1)}{\kappa^2} - \left(\frac{\kappa-1}{\kappa}\right)[f(\theta_1)\theta_1'(\kappa) + (\kappa-1)f(\theta_2)\theta_2'(\kappa)]\right]. \tag{46}$$

We have then established the following proposition which characterizes optimal security design in the baseline model.

Proposition 3 *(Baseline Model) The optimal structuring consists of a liquid concave claim with $B_L^* \in (0, L]$ and $B_H^* = \kappa^{**}B_L^*$, where $\kappa^{**} < H/L$ is the unique solution to*

$$1 - \frac{F(\theta_1)}{\kappa^{**}} - \frac{(\kappa^{**}-1)F(\theta_2)}{\kappa^{**}} = \left[\frac{\kappa^{**}-1}{\kappa^{**}}\right]\left[\frac{F(\theta_2)-F(\theta_1)}{\kappa^{**}} + f(\theta_1)\theta_1'(\kappa^{**}) + (\kappa^{**}-1)f(\theta_2)\theta_2'(\kappa^{**})\right]. \tag{47}$$

The residual convex claim attracts zero aggregate hedging demand. All informed trading gains are derived in the market for the concave claim.

The first-order condition pinning down the optimal informational sensitivity of the liquid claim, as captured by κ^{**}, reflects the following fundamental trade-off. Under full securitization, the owner of a high quality asset maximizes the incentive of the speculator to generate more precise signals. This is accomplished by maximizing the product of per-unit profits and hedge trading volume. Marginal increases in informational sensitivity increase the speculator's trading gain on a per-unit basis, but also result in endogenous reductions in hedging demand as uninformed investors perceive greater exposure to adverse selection. At an interior optimum, the marginal benefits and costs are equated.

The following corollary shows that under full securitization optimal structuring can be achieved by combining standard securities.

Corollary *(Baseline Model) One optimal securitization structure consists of illiquid levered equity and liquid risky senior debt with face value $\kappa^{**}L$. Another optimal securitization structure consists of liquid equity and an illiquid call option on the whole asset with strike price $\kappa^{**}L$.*

At this point it is worth recalling the working assumption that the only party capable of issuing securities is Owner. That is, other economic agents cannot issue securities or short-sell. Now recall that the market-markets clear markets for all securities, buying one minus the combined aggregate demand of the hedgers and the speculator. But is it possible for aggregate demand to exceed supply? To address this question, notice that the maximum aggregate demand coming from the hedgers and speculator is $2X$. We can write hedging demand as:

$$X(\phi, B_L, \kappa) = \left(\frac{\phi}{B_L}\right) \left[\frac{F(\theta_2) - F(\theta_1)}{\kappa} + 1 - F(\theta_2)\right].$$

Therefore, to avoid the possibility of market-makers being called upon to short-sell, ϕ must be sufficiently small in relation to B_L. The no-shorting constraint is clearly easiest to satisfy if Owner chooses $B_L = L$, where the choice of B_L was otherwise arbitrary when we ignored the no-shorting constraint. From Assumption 1 it follows that the market-makers are never called upon to short-sell (even at suboptimal κ) since

$$\phi \leq L/2 \Rightarrow 2X(\phi, B_L^* = L, \kappa) < 1. \tag{48}$$

IV. General Model: Optimal Degree and Design of Securitization

The baseline model assumed Owner must sell the entire asset. Such a setting is relevant when there is a forced asset sale due to antitrust enforcement, bankruptcy liquidation, or unbounded liquidity needs. This section considers an alternative setting in which Owner chooses both the degree and design for securitization of the original asset.

The assumptions for the remainder of the paper are as follows. Owner is risk-neutral and values consumption equally in both periods, having utility of the form $c_1 + c_2$. Further, Owner has access to a linear production technology allowing him to convert each unit of numeraire received from investors in period 1 into $\beta > 1$ units of numeraire in that same period. In contrast to the original real asset, the value of this short-term production technology is not verifiable by courts, so this stream of cash flow cannot be securitized.

We consider this particular setup for two reasons. First, it approximates a number of real-world settings. For example, one may think of a distressed bank as placing high, yet bounded, value on the immediate receipt of cash coming from securitization of an underlying asset. Second, this setup allows us to retain our focus on the optimal securitization of a single real asset, here the original real asset with values in $\{L, H\}$. This allows us to address how the option to retain some cash flow rights affects the optimal securitization structure.

If there were no intrinsic benefit to receiving funds immediately ($\beta = 1$), the owner of the high quality asset would not sell any claims on the real asset given asymmetric information. He would then obtain his first-best payoff H by holding onto the entire asset. Conversely, if there were symmetric information and if β were greater than one, then an owner of either real asset type would sell all cash flow rights (full securitization). In the event of a negative endowment shock, the hedgers would then fully insure themselves by purchasing ϕ/τ units of the equity written on the real asset, ensuring they achieve $c_2 = 0$ with probability one, as is the case under perfect risk sharing.

A. The Security Design Game

Maskin and Tirole (1992) show the equilibrium set of signaling games can be narrowed and Pareto-improved (from the perspective of the privately informed party) by expanding the set of feasible initial actions. Tirole (2005) describes an application of the formulation of Maskin and Tirole (1992) to security issuance by a privately informed party. We adapt the game of Tirole (2005) to our setting.

We characterize PBE again requiring: all agents have a belief at each information set; strategies must be sequentially rational given beliefs; and beliefs are determined using Bayes' rule and the equilibrium strategies for all information sets on the equilibrium path.

The sequencing of events is as follows. The entire *security design game* actually consists of two connected games: an offer game and the market-making game. The latter game was already described in Section I. The *offer game* is a signaling game played between Owner and all outside investors. This game begins with Owner privately observing asset value. He then approaches the market-makers (e.g. investment banks) and publicly proposes a *menu* of two securitization structures, say $\Sigma \in \{\Sigma^1, \Sigma^2\}$, that he would like the option to choose from subsequently. This step resembles a shelf-registration in that Owner is locking in a pair of optional future financial configurations. Each structure stipulates all payoffs for claimants as a function of the verified asset value in period 2. The market-makers then agree to clear markets competitively for whatever structure Σ the owner subsequently chooses from his menu. All agents in the economy must have a belief regarding the asset type in response to any menu offer, including those off the equilibrium path. Beliefs at this stage are labeled *offer beliefs*. To support candidate PBE, initial menu offers off the equilibrium path are "punished" with outside investors inferring $\tau = L$ with probability one.

The sole difference between this formulation and the game described by Tirole (2005) is that in our model market-makers cannot agree to providing cross-subsidies. Rather, they simply agree to compete and clear markets for the securitization structure subsequently chosen by Owner. Using the terminology of Tirole (2005), competitive market-making implies that all investors in securities find them profitable type-by-type. This stands in contrast to a setting in which investors can pre-commit to subsequently buying some set of securities at a loss, a possibility allowed in the formulation of Tirole (2005).

In the next stage of the offer game, Owner selects a securitization structure Σ from the menu he initially proposed, with the choice being incentive compatible. After observing the selection of Owner, all other agents revise beliefs using Bayes' rule where possible. The beliefs formed at this stage are labeled *selection beliefs.* It is worth stressing that both types can offer the same menu, but they do not necessarily select the same securitization structure from the menu. In a *separating equilibrium* of the offer game, the initial securitization proposal is such that the Σ selected from the menu reveals the true asset type τ. In any separating equilibrium securities are correctly priced and all agents trade in full knowledge of the true type. There is no incentive for the speculator to put in effort in a separating equilibrium of the offer game.

In a *pooling equilibrium* of the offer game, both owner types propose the same trivial menu with $\Sigma^1 = \Sigma^2$. In such cases, no information is revealed about the asset type after the selection stage of the offer game. If and only if a pooling equilibrium is played in the offer game, play then passes to the market-making game. Recall, all relevant players enter the market-making game continuing to use their prior belief that $\Pr[\tau = H] = q$, as is appropriate when the offer game reveals no information regarding τ. Then a signaling game ensues between the speculator and market-makers, where market-makers use aggregate demand to form beliefs regarding the signal s and set prices accordingly.

B. The Least-Cost Separating Equilibrium

In the general model, the owner can credibly signal positive private information by retaining sufficient rights. To this end, assume Owner designs a third security C with value-contingent payoffs (C_L, C_H). Owner holds security C and sells the other two securities A and B to public investors in a competitive market. From Lemma 4 it follows that confining attention to no more than two publicly traded securities is without loss of generality.

We begin by evaluating the least-cost separating equilibrium (LCSE) from the perspective of the high type. Note that in the LCSE the speculator has no incentive to acquire information since the original owner's private information is fully revealed by his financing choice.

The LCSE minimizes the low type's incentive to mimic by giving him his first-best allocation in which he sells the entire asset in, say, equity form for L. The LCSE makes the high type as well off as possible subject to the constraint that the low type would not choose to mimic. It solves:

$$\begin{aligned} & \max \quad C_H + \beta B_H \\ & s.t. \\ No\ Mimic \ : \ & \beta L \geq C_L + \beta B_H \\ & Limited\ Liability \\ & Monotonicity. \end{aligned}$$

Clearly, the optimal policy is to relax the no-mimic constraint to the maximum extent by setting $C_L^* = 0$, implying $B_L^* = L$. Further, the no-mimic constraint must bind at the optimum , implying $B_H^* = L$ and $C_H^* = H - L$. This leads to the following proposition.

Proposition 4 *In the least-cost separating equilibrium, a low type asset is sold in its entirety in all-equity form. The owner of a high type asset sells only a safe senior debt claim with face value L, retaining a levered equity claim. Hedgers then perfectly insure against negative endowment shocks by purchasing correctly priced claims.*

The intuition behind Proposition 4 is simple. In the LCSE, the low type would always mimic if the high type were to sell any risky claim since he would then benefit from security overvaluation. Therefore, the best the high type can do is to get the maximum liquidity possible subject to zero informational-sensitivity. Debt with face value L achieves this objective.

In the LCSE, the high type experiences a deadweight loss relative to full information equal to $(\beta - 1)(H - L)$. This deadweight loss reflects that fact that first-best entails him selling off the entire asset instead of just the claim to L. As in the model of Myers and Majluf (1984), in the LCSE asymmetric information results in the high type cutting back the scale of his investment to below first-best.

The socially attractive feature of the LCSE is that it entails perfect risk sharing, regardless of the actual asset type. To see this, note that the equity of the low type and the debt of the high type are both correctly priced since the equilibrium is fully-revealing. Therefore, the hedgers can and will perfectly insure in the event of a negative endowment shock, having access to correctly priced securities. That is, the hedgers achieve $c_2 = 0$ with probability one giving up one unit of c_1 per unit of c_2 purchased.

Although not pursued here, Proposition 4 shows the results in Gorton and Pennachi (1990) are overly restrictive in that their model relies upon safe debt to achieve perfect risk sharing. However, perfect risk sharing is achieved in any *separating* equilibrium, even if the separation is predicated upon the issuance of risky claims.

C. The Equilibrium Set

This subsection maps some of the results of Maskin and Tirole (1992) and Tirole (2005) to our setting, relying on somewhat different proofs due to differences in the economic settings considered.

The next lemma places a lower bound on what each type must receive in any equilibrium.

Lemma 8 *In any equilibrium of the security design game, each owner type must receive a payoff weakly greater than his least-cost separating payoff.*

Proof. Suppose to the contrary that some type received less than his LCSE payoff. He could then profitably deviate by issuing safe debt with face value L and retaining residual cash flow rights. ■

The following lemma characterizes the equilibrium set.

Lemma 9 *The equilibrium set of the security design game always includes the least-cost separating equilibrium. It also includes a pooling equilibrium where there is a single contract on the offered menu provided that contract weakly Pareto dominates the least-cost separating equilibrium (from the perspective of both owner types).*

Proof. Consider first supporting the LCSE. If beliefs were set to $\Pr[\tau = H] = 0$ in response to any deviating menu, then no such deviation is profitable. Suppose next there is a pooling contract weakly Pareto dominating the LCSE. If beliefs were set to $\Pr[\tau = H] = 0$ in response to any deviating menu, the deviator would get weakly less than his LCSE payoff and the deviation is not profitable. ■

C. The Pooling Equilibrium

Consider next the nature of pooling equilibrium–an equilibrium in which both types offer a trivial menu such that $\Sigma^1 = \Sigma^2$. The objective of the owner of a high type asset is to maximize

$$C_H + E[P_A + P_B|\tau = H]. \tag{49}$$

Any pair (C_L, C_H) held by the original owner leaves a residual stream of payments that will be packaged and sold to outside investors:

$$(C_L, C_H) \Rightarrow (l, h) \equiv (L - C_L, H - C_H).$$

We characterize the optimal nature and scope of securitization using a two step procedure. First, Lemma 6 can be used to characterize the optimal structuring for the sale of residual cash flows after netting out Owner's claim. Then (C_L, C_H) are chosen in light of their effect on the value attainable in this residual structuring problem.

Before proceeding with the formal solution, it is useful to sketch the intuition. For the owner of a high quality asset, the benefit of increasing C_H is that he marginally reduces his exposure to underpricing. However, this retention of cash flow rights from the long-term tangible real asset reduces the amount he can invest in the profitable short-term project. Further, an increase in C_H reduces h/l. If $h/l \leq \kappa^{**}$, where κ^{**} is defined in Proposition 3, the concavity constraint in Program 2 is binding and the incentive compatible signal precision falls below that attainable under full securitization.

Let $M^*(l, h)$ denote the maximum value obtained in Program 2 given that the total value of publicly traded claims on the real asset is in $\{l, h\}$:

$$M^*(l,h) \equiv M[\kappa^*(l,h)]. \tag{50}$$

From (16) and the definition of M^* it follows that the maximized incentive compatible signal precision is:

$$\sigma^*(l,h) \equiv \psi[q(1-q)M^*(l,h)]. \tag{51}$$

From the Envelope Theorem we know:

$$\begin{aligned} M_1^*(l,h) &= \frac{\partial \mathcal{L}}{\partial l} = \frac{-h\lambda^*(l,h)}{l^2} \Rightarrow \sigma_1^*(l,h) = \frac{-q(1-q)h\lambda^*(l,h)}{l^2 e'(\sigma^*)} \leq 0 \\ M_2^*(l,h) &= \frac{\partial \mathcal{L}}{\partial h} = \frac{\lambda^*(l,h)}{l} \Rightarrow \sigma_2^*(l,h) = \frac{q(1-q)\lambda^*(l,h)}{l e'(\sigma^*)} \geq 0. \end{aligned} \tag{52}$$

The inequalities in (52) convey an important trade-off. Specifically, when the concavity constraint is binding, increases in l reduce the value obtained in Program 2 and with it the incentive compatible signal precision σ^*. Conversely, increases in h loosen the concavity constraint, potentially leading to higher σ^*. Thus, consistent with the intuition provided above, a high value of C_H imposes a cost in terms of the power of incentives that can be provided to the speculator. Lower incentives then lead to more severe mispricing of the public claims.

With this in mind, we turn to the solution of the following restated program.

PROGRAM 3

$$\max_{l,h} \quad \mu(l,h) \equiv H - h + \beta[l + (h-l)Z(\sigma^*(l,h))]$$

s.t.

Incentive Compatability : $\sigma^*(l,h) = \psi[q(1-q)M^*(l,h)]$

Monotonicity : $h \geq l$

Limited Liability : $h \in [0, H]$ and $l \in [0, L]$.

Program 3 is not necessarily concave. Therefore, instead of relying on first-order conditions, we pin down the optimal policy via perturbation and dominance arguments.

Casual intuition suggests the optimal pooling contract for the owner of a high quality asset entails $C_L^* = 0$ and $l^* = L$. After all, the owner of a high quality asset has no desire to retain any cash flow rights should the observed asset value be equal to L, since he knows this is a zero probability event. He also knows that the concomitant increase in l tends to boost his revenues, since investors are paying for the rights to l. However, there is a countervailing cost to such a policy, since increases in l tighten the concavity constraint, potentially reducing the incentive compatible level of signal precision. Formally, we have:

$$\mu_1(l,h) = \beta[1 - Z(\sigma^*(l,h)) + Z'(\sigma^*(l,h))\sigma_1^*(l,h)]. \tag{53}$$

The following lemma shows that the liquidity effect dominates in that the low payoff is always fully securitized in the pooling equilibrium.

Lemma 10 *The optimal pooling contract for the high type entails a zero payoff to the owner if asset value is low ($C_L^* = 0$ and $l^* = L$).*

Lemma 10 allows us to rewrite Program 3 as a one dimensional optimization:

$$\begin{aligned}
& PROGRAM \quad 3' \\
& \max_h \quad \mu(L,h) \equiv H - h + \beta[L + (h-L)Z(\sigma^*(L,h))] \\
& s.t. \\
Incentive\ Compatability \quad : \quad & \sigma^*(L,h) = \psi[q(1-q)M^*(l,h)] \\
LL\&Mono \quad : \quad & h \in [L,H].
\end{aligned}$$

It is readily verified that if the owner of the high quality asset opts to securitize only safe debt he gets

$$\mu(L,L) = H - L + \beta L. \tag{54}$$

Further,

$$\beta Z[\sigma^*(L,H)] \leq 1 \Rightarrow \mu(L,h) \leq \mu(L,L) \quad \forall \quad h \in (L,H]. \tag{55}$$

This leads directly to Proposition 5.

Proposition 5 *If $Z[\sigma^*(L,H)] \leq \beta^{-1}$ the payoffs and outcomes under the least-cost separating contract are the unique payoffs and outcomes.*

The intuition for Proposition 5 is straightforward. If the speculator cannot be incentivized to produce sufficiently precise signals, even when her incentives are maximized under full asset securitization, then the costs of underpriced securities exceed the value of immediate liquidity and the high owner of a high quality asset avoids issuing any risky security. Rather, he gets the maximal liquidity possible using safe debt.

Recall, the objective in Program 3 is to find the pooling contract preferred by the high type. Apparently, if $Z[\sigma^*(L,H)] \leq \beta^{-1}$ it is impossible to find a pooling contract that makes him better off. And since we ignored the welfare of the low type in that program, it follows that there is no Pareto-improving contract across the owner types. Also, the actual outcome for all agents under the pooling contract described in Proposition 5 is identical to that under the LCSE. Thus, the outcomes under the LCSE are the unique payoffs whenever $Z[\sigma^*(L,H)] \leq \beta^{-1}$. That is, when $Z[\sigma^*(L,H)] \leq \beta^{-1}$, private owners choose structures that achieve efficient risk sharing.

Consider next the outcome when the speculator can be incentivized to produce more precise signals. It is readily verified that

$$\beta Z[\sigma^*(L,H)] > 1 \Rightarrow \mu(L,H) > \mu(L,L). \tag{56}$$

It follows that when $\beta Z[\sigma^*(L,H)] > 1$, the high type can do strictly better than under the separating contract, which gives him $\mu(L,L)$. It is also readily verified that

$$\mu_2(L,h) = \beta Z[\sigma^*(L,H)] - 1 \quad \forall \quad h \in (\kappa^{**}L,H). \tag{57}$$

It follows that when $\beta Z[\sigma^*(L,H)] > 1$, setting $h = H$ strictly dominates any $h \in (\kappa^{**}L,H)$. Also, it is clear that the low type is better off under the pooling contract since he benefits from security mispricing, resulting in him receiving more than βL. The preceding results lead directly to Proposition 6.

Proposition 6 *If $Z[\sigma^*(L,H)] > \beta^{-1}$ the optimal pooling contract for the high type entails a risky securitization with $l^* = L$ and $h^* \in \{H \cup (L, \kappa^{**}L]\}$. Under this contract, information is produced by the speculator and hedgers are imperfectly insured. Both owner types are strictly better off under this pooling contract than under the least-cost separating equilibrium, so the latter is not a unique equilibrium of the security design game.*

Finally, we are interested in sufficient conditions such that the optimal pooling contract entails full securitization of the underlying tangible asset. To this end note that

$$\beta \geq q^{-1} \Rightarrow \mu_2(L,h) > 1 \quad \forall \quad h \in (L,H).$$

This leads directly to Proposition 7.

Proposition 7 *There exists a liquidity value $\beta^{full} \in [Z(\sigma^*(L,H))^{-1}, q^{-1})$ such that for all $\beta \geq \beta^{full}$ the optimal pooling contract for the high type entails full securitization with $l^* = L$ and $h^* = H$. Optimal structuring then follows Proposition 3. Under this structuring, information is produced by the speculator and the hedgers are imperfectly insured.*

D. Public versus Private Incentives in Securitization

For simplicity, assume the planner places equal weight on all agents including Owner, the hedgers, market-makers and the speculator. Under symmetric information, the owner sells the entire asset in the first period and converts the proceeds into β units of first-period consumption. The speculator and market-makers consume their endowments. Finally, the hedgers consume their first period endowment minus the cost of buying ϕ units of consumption in the second period with probability one half. That is, under symmetric information the hedgers attain $c_2 = 0$ with probability one. Thus, first best welfare is:

$$W^{FB} = \beta[qH + (1-q)L] + y_1^S + y_1^{MM} + y_1 - \frac{\phi}{2}. \tag{58}$$

Ex ante the planner computes the following welfare loss in the LCSE relative to first-best:

$$LOSS_{SEP} = q(\beta - 1)(H - L). \tag{59}$$

The only deadweight loss in the LCSE is the loss in NPV resulting from the high type operating the new short-term project below optimal scale. From a risk sharing perspective the LCSE is attractive, since full revelation of information allows hedgers to perfectly insure.

Consider next a pooling equilibrium in which the asset is fully securitized under the corresponding optimal structuring described in Proposition 3. In contrast to the separating equilibrium, a pooling equilibrium results in the constrained socially optimal investment in the short-term technology. However, it entails costly effort on the part of the speculator and inefficient risk sharing. Specifically, under full-securitization of the asset we have the following welfare loss in the pooling equilibrium:

$$\begin{aligned} LOSS_{POOL} \quad &= \quad e[\sigma^*(L,H)] + \frac{1}{2}\phi \int_{1}^{\theta_1(\kappa^{**})} (\theta - 1) f(\theta) d\theta \\ &+ \frac{1}{2}(1-q)\phi\left(1 - \frac{1}{\kappa^{**}}\right) \int_{\theta_1(\kappa^{**})}^{\theta_2(\kappa^{**})} (\theta - 1) f(\theta) d\theta + \frac{1}{2} q\phi(\kappa^{**} - 1)[1 - F(\theta_2(\kappa^{**}))] \end{aligned} \tag{60}$$

The first term in the welfare loss under pooling is the effort cost of the speculator. The second term represents the loss stemming from hedgers with low risk-aversion failing to insure, due to perceived adverse selection. The third term represents the loss stemming from a subset of hedgers imperfectly hedging. Finally, the last term represent the loss stemming from a subset of extremely risk-averse hedgers overinsuring, since they purchase insurance without knowing the actual asset type. We also know that:

$$\begin{aligned} \frac{\partial}{\partial \phi} LOSS_{POOL} \quad &= \quad e'(\sigma^*)\frac{\partial \sigma^*}{\partial \phi} + \frac{1}{2} \int_{1}^{\theta_1(\kappa^{**})} (\theta - 1) f(\theta) d\theta \\ &+ \frac{1}{2}(1-q)\left(1 - \frac{1}{\kappa^{**}}\right) \int_{\theta_1(\kappa^{**})}^{\theta_2(\kappa^{**})} (\theta - 1) f(\theta) d\theta + \frac{1}{2} q(\kappa^{**} - 1)[1 - F(\theta_2(\kappa^{**}))] > 0. \end{aligned} \tag{61}$$

It follows that there exists a unique value of ϕ, call it ϕ^{soc}, at which a neutral social planner would prefer the separating equilibrium over the pooling equilibrium for all $\phi > \phi^{soc}$. Now note that if $\beta \in (2/(1+q), 1/q)$ there exists a unique value of ϕ, call it ϕ^{own}, such that $\beta Z = 1$ and the owner of the high quality asset would prefer the pooling equilibrium over the separating equilibrium. It follows that there exists a unique value of ϕ, equal to the maximum of these two thresholds such that social and private objectives conflict, in the sense described in the following proposition.

Proposition 8 *If $\beta \in (2/(1+q), 1/q)$, there exists ϕ^{crit} such that for all $\phi \geq \phi^{crit}$ a neutral social planner prefers the separating equilibrium and the owner prefers the pooling equilibrium.*

The intuition behind Proposition 8 is as follows. The social welfare loss associated with the pooling equilibrium is increasing in the size of the endowment shock hitting hedgers, since the pooling equilibrium results in inefficient insurance against the shock. However, larger shocks hitting the hedgers increases the trading gains to informed speculation and the incentive compatible signal precision that will be attained in the pooling equilibrium. In turn, this reduces the degree of underpricing perceived by the owner of a high quality asset, which would tilt him towards the pooling equilibrium.

Similarly, an increase in risk-aversion via a first-order stochastic dominant shift in θ would also increase the welfare loss associated with the pooling equilibrium, while simultaneously making that equilibrium more attractive to the owner of a high quality asset. Taken together, these results indicate that the private sector will tend to prefer the pooling equilibrium, with inefficient risk sharing, precisely when this risk sharing has the highest social value.

Conclusions

This paper evaluates privately optimal securitization structures when the original asset owner has an intrinsic motive for raising funds immediately, but is concerned about mispricing given that he is privately informed regarding asset value. Securities markets are endogenously incomplete, with the securitization structure influencing risk sharing. Prices are set by competitive market-makers, with an endogenously informed speculator trading against uninformed hedgers placing rational orders.

If the speculator can be sufficiently incentivized, underpricing is low and all/most of the asset is securitized in a pooling equilibrium. Here, all speculator gains are derived in the market for the concave claim, with the optimal structuring maximizing the product of the speculator's per-unit gain and hedge trading volume. Hedgers imperfectly insure in pooling equilibria, as adverse selection distorts their trading decisions.

There also exists a separating equilibrium in which a low type sells the entire asset in equity form while a high type only sells safe debt, holding levered equity on his own books. In this separating equilibrium, the type is fully revealed so there is no motive for speculative activity and perfect risk sharing is achieved since investors do not fear adverse selection. However, there is a social cost to the separating equilibrium, since the high type operates below efficient scale.

The model highlights the following fundamental conflict between private and social incentives in choosing securitization structures: Private incentives to implement the pooling equilibrium are strongest precisely when the gains to efficient risk sharing are highest. Specifically, increases in the size of endowment shocks and/or risk-aversion encourage owners to rely upon speculative activity, rather than signaling, since higher hedge trading volume promotes information acquisition, which reduces the extent of mispricing. Thus, the private sector will engage in socially excessive securitization, and distort risk sharing, whenever society most highly values the risk sharing benefits that advocates commonly impute to such structures.

Appendix: Proofs

Lemma 1

Substituting beliefs from equation (6) into the expected revenue (10), we obtain:

$$\overline{R}_H(\sigma) = L + \left(\frac{q(H-L)}{2}\right)\left[\frac{\sigma^2}{1-q-\sigma+2\sigma q} + q + \frac{(1-\sigma)^2}{q+\sigma-2\sigma q}\right]. \tag{62}$$

We need only verify the square bracketed term is increasing. Let

$$\begin{aligned} a(\sigma) &\equiv q+\sigma-2\sigma q \\ \Omega(\sigma) &\equiv 1 + \frac{\sigma^2}{(1-a)} + \frac{(1-\sigma)^2}{a}. \end{aligned}$$

We need only verify Ω is increasing. Differentiating we obtain:

$$\begin{aligned} \Omega'(\sigma) &= \frac{2\,(1-a)\,\sigma + (1-2q)\sigma^2}{(1-a)^2} - \frac{2a(1-\sigma) + (1-2q)(1-\sigma)^2}{a^2} \\ &= \frac{[2\,(1-a) + (1-2q)\sigma]\,\sigma a^2 - (1-a)^2(1-\sigma)\,[2a + (1-2q)(1-\sigma)]}{(1-a)^2\,a^2} \end{aligned} \tag{63}$$

This is strictly positive if and only if.

$$\begin{aligned} [2\,(1-a) + \sigma(1-2q)]\,\sigma a^2 &> (1-a)^2(1-\sigma)\,[2a + (1-2q) - \sigma(1-2q)] \\ &\Updownarrow \\ [(1-a) + (1-q)]\,\sigma a^2 &> (1-a)^2(1-\sigma)\,[a + (1-q)] \\ &\Updownarrow \\ (1-q)\sigma a^2 &> (1-a)\left[(1-\sigma)(1-a)a + (1-\sigma)(1-q)(1-a) - \sigma a^2\right] \\ &\Updownarrow \\ (1-q)\sigma a^2 &> (1-a)\,[a(1-a) - a\sigma + (1-\sigma)(1-q)(1-a)] \\ &\Updownarrow \\ [(1-q)a + 1 - a]\,\sigma a &> (1-a)^2\,[a + (1-\sigma)(1-q)] \\ &\Updownarrow \end{aligned}$$

$$
\begin{aligned}
[1 - qa]\,\sigma a &> (1-a)^2(1-q\sigma) \\
&\Updownarrow \\
\sigma a - \sigma q a^2 &> (1-a)^2 - q\sigma(1-a)^2 \\
&\Updownarrow \\
q\sigma\left[(1-a)^2 - a^2\right] + \sigma a &> (1-a)^2 \\
&\Updownarrow \\
q\sigma + \sigma a(1-2q) &> (1-a)^2 \\
&\Updownarrow \\
a^2 + q(\sigma - a) &> (1-a)^2 \\
&\Updownarrow \\
q^2(2\sigma - 1) + 2\left[\sigma - q(2\sigma-1)\right] &> 1 \\
&\Updownarrow \\
(q-1)^2(2\sigma-1) - (2\sigma-1) + 2\sigma &> 1 \\
&\Updownarrow \\
(q-1)^2(2\sigma-1) &> 0.\blacksquare
\end{aligned}
$$

Proposition 1

Consider a graph with X on the vertical axis and σ on the horizontal axis. Plotting aggregate hedging demand, we know $X(1/2) \in (\phi/B_H, \phi B_L)$. Further, X is strictly decreasing in σ on $[1/2, 1]$. Plotting the incentive compatible signal precision, we know σ_{ic} is strictly increasing in X with $\sigma_{ic}^{-1}(1/2) = 0$ and the limit as σ_1 converges to one of $\sigma_{ic}^{-1}(\sigma_1) = \infty$. Thus, the two curves intersect once, and only once, implying a unique equilibrium exists.$\blacksquare$

Lemma 4

Suppose Owner sells $N \geq 3$ securities. Rank these securities in descending order in terms of the ratio of their payoff if value is low relative to their payoff if value is high. Section III establishes that hedge trading will be concentrated in security 1, and security 1 will be the only source of informed trading gains. Aggregate demand of the hedgers and informed trader will then be zero in securities 2 to N. Therefore, one may roll up these securities into a single (weakly convex) security having no effect on σ or expected revenues.■

Lemma 7

Differentiating the maximand one obtains

$$\frac{M'(\kappa)}{\phi} = 1 - F(\theta_2) + \kappa^{-2}[F(\theta_2) - F(\theta_1)] - (1 - \kappa^{-1})[f(\theta_1)\theta_1' + f(\theta_2)\theta_2'(\kappa - 1)].$$

And

$$\begin{aligned}\frac{M''(\kappa)}{\phi} &= -2\kappa^{-2}f(\theta_1)\theta_1' - 2\kappa^{-3}[F(\theta_2) - F(\theta_1)] \\ &\quad -(1 - \kappa^{-1})[f'(\theta_1)(\theta_1')^2 + f'(\theta_2)(\theta_2')^2] \\ &\quad -(1 - \kappa^{-1})[2(1 + \kappa^{-1})f(\theta_2)\theta_2' - f(\theta_1)|\theta_1''|].\end{aligned}$$

Since F is convex, a sufficient condition for $M'' < 0$ is $|\theta_1''| \leq \theta_2'$, which always holds.■

Lemma 10

This lemma is proved in a series of steps. First, we claim

$$\begin{aligned} h^* &= H \Rightarrow l^* = L. \\ & \textit{and} \\ l^* &< L \Rightarrow h^* < H. \end{aligned}$$

To demonstrate this, note

$$h^* = H \Rightarrow \forall l \in (0, L), \quad \lambda^*(l, h^*) = 0 \Rightarrow \mu_1(l, h^*) > 0.$$

Next we claim

$$\lambda^*(l^*, h^*) = 0 \Rightarrow l^* = L.$$

To demonstrate this suppose to the contrary that (l_0, h_0) are optimal with $\lambda^*(l_0, h_0) = 0$ but $l_0 < L$. Then consider increasing l by ε arbitrarily small, noting that such an increase meets all constraints including monotonicity since $\lambda^*(l_0, h_0) = 0$ implies $h_0 > l_0$. The gain is $\varepsilon\beta(1-Z) > 0$, contradicting the initial conjecture.

Next we claim

$$\lambda^*(l^*, h^*) > 0 \Rightarrow l^* = L.$$

To demonstrate this claim, suppose to the contrary that (l_0, h_0) are optimal with $\lambda^*(l_0, h_0) > 0$ but $l_0 < L$. Then let $\kappa_0 \equiv h_0/l_0$ and consider all pairs $(l, \kappa_0 l)$. By construction, all such pairs keep Z fixed at $Z[\sigma^*(l_0, h_0)] \equiv Z_0$. Then consider

$$\frac{d}{dl}\ \mu[l, \kappa_0 l] = \beta(1 - Z_0) + \kappa_0[\beta Z_0 - 1] \quad \forall \quad l \in (0, L).$$

Note that the value of this derivative is constant by construction. We next claim the derivative must be weakly positive. For if it is not, the optimal policy is to decrease both l and h to zero leaving the owner to collect $\mu = H$ which is strictly dominated by $l = h = L$. Finally, since the derivative is weakly positive $l^* = L$.■

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