Public deficit sustainability, public debt and monetization in an endogenous growth model: an application for Turkey

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Very Preliminary Version

Abstract

This paper analyzes the long term sustainability of budgetary policies in a general equilibrium framework. The analysis is based on an overlapping generations model where the government fixes a tax rate on factor incomes, conducts unproductive and productive public spendings and determines the weight of public bonds issuance and monetization in deficit financing. A budgetary rule setting the economy on a balanced growth path is considered sustainable. The model is used to evaluate the sustainability of Turkish budgetary policies since 1980. The results show that Turkish budgetary policies became sustainable in the period 1999-2007. Alternative policy simulations show that Turkey could have conducted more expansionist budget policies without risking sustainability during this period. The model shows also that the negative impact of inflation tax on private capital accumulation is greater than the impact of factor income tax.

1 INTRODUCTION

Several works in economics literature has studied public finance issues using economic growth models. Chalk (2000) and Rankin and Roffia (2003) affirm, using exogenous growth models, that public debt sustainability requires low initial public debt and public deficit ratios. Futagami and Shibata (2003) find the same result with an endogenous growth model. Bräuninger (2005) analyzes public debt sustainability using an endogenous growth model within an overlapping generations (OLG) setting. He studies the impact of different budgetary rules and concludes that above a public deficit/GDP ratio threshold, the budgetary policy becomes unsustainable, as there are no more balanced growth paths. Furthermore, there exists a threshold of initial public debt/private capital ratio above which the economy can not reach a balanced growth path. On the other hand, the study of Bräuninger assumes only non-productive public spendings. Arai (2008) and Yakita (2008) develop this approach by incorporating productive public spendings. Within an OLG setting, Arai assumes (2008) public spendings as a flow, as in Barro (1990). He

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demonstrates that for low initial values, increasing public spendings allows sustainability because its positive impact on production exceeds the negative impact on costs. In Yakita (2008), public spendings finance a public capital stock, as in Futagami, Morita and Shibata (1993). He concentrates the analysis on the "golden rule" of public finance which consists of issuing debt only for public investments. As in Bräuninger, he finds the existence of a public debt threshold, for each level of public capital, above which the budgetary policy is unsustainable in the long run.

The model developed in this paper is inspired by Yakita (2008). It contains two main differences: it includes money as an asset and it considers public deficit as an endogenous variable. In our model, the government intervenes in the economy in several ways. It realizes productive and unproductive public spendings, and collects taxes on factor income. These three exogenous parameters are expressed as a share in GDP. The public deficit generated by this policy is financed with public bonds and money creation. The government fixes the share of each financing method. Thus the government decides about four policy parameters. Fixed policy rules that enable the economy to reach a balanced growth path are considered sustainable.

In the model, money supply is represented by money creation aiming at financing the public deficit. Thus, money supply consists of seignorage. Money demand is represented by a Clower (1967) type liquidity constraint. Initially, this constraint assumes that agents have to finance the whole of their consumption with money, and thus have to hold money before the transaction. In an overlapping generations setting, such an assumption would block savings and consequently the private capital accumulation. This is why we will adopt the approach of Lucas and Stokey (1987) who distinguish goods bought with money and with credit. We will then assume, as in Hahn and Solow (1995), a partial liquidity constraint forcing old agents to realize a fixed portion of their spendings using money. In other terms, young agents are forced to hold cash balances in order to realize a fixed portion of their future consumption with monet. This way we obtain a liquidity constraint\(^1\) that creates distortions on capital accumulation without blocking it completely.

The introduction of money allows to take into account the transaction costs created by the need of holding money. The latter blocks a portion of the funds that could have been invested in other assets as public bonds or private capital. Money influences then the economic equilibrium and dynamics of accumulation (Crettez, Michel et Wigniolle 1999). Money supply being limited to monetization, the model enables to visualize inflation tax.

For the case of Turkey, public finance sustainability has been mostly analyzed in empirical works\(^2\). On the other hand, Yeldan (2002), analyzes the impact of Turkish budgetary policies conducted since 2000, with the support of the IMF, by using a theoretical endogenous growth model. The latter assumes an OLG setting where the government make public investments in the domain of education. He realizes a computable general equilibrium analysis which concludes that the budgetary policies implemented with the IMF are too restrictive and deprive the economy of funds needed to improve the quality

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\(^1\)For a detailed analysis of cash-in-advance models, see Villieu (1992).

of workforce through education policies.

The simulations of our model show that Turkish budgetary policies conducted in 1980-1988 and 1989-1998 periods were unsustainable. In other words they do not allow the economy to reach a balance growth path. However, the policy of the 1999-2007 period is sustainable. On the other hand, alternative policy simulations concludes that more expansionist and growth enhancing budgetary policies could have been implemented in Turkey in this period, without breaching sustainability in the long run. Our results are then similar to that of Yeldan (2002).

In order to better illustrate the impact of monetization, we simulate alternative policies also for the 1980-1988 where initially the budgetary policy was unsustainable. The objective is to compare the equilibria obtained with different stabilization measures. Results show that sustainability achieved by tax increases enables more growth than sustainability reached through monetization, in other terms, through inflation tax. The latter has a larger negative impact on private capital accumulation than direct taxes.

The paper is organized as follows: section II presents the model. Section III displays accumulation dynamics and defines public finance sustainability. Section IV shows the results of the simulations on Turkish budget policies. The last section concludes.

2 THE MODEL

2.1 Consumers

We consider an overlapping generations model (OLG). Two generations of agents coexist at each date $t$. Each agent lives only two periods. Agents are not altruist, thus we assume that there are no bequests. The anticipations of agents are perfect.

An agent born at period $t$ maximizes the intertemporal utility function given by:

$$U = \ln(c_{j,t}) + \beta \ln(c_{v,t+1})$$

(1)

In equation (1), $c_{j,t}$ stands for the consumption of the young agent at date $t$. $c_{v,t+1}$ represents the consumption of this same agent during retirement in period $t + 1$. The variable $\beta$ is the psychological discount factor reflecting the relative preference of the agent for present consumption. Agents are supposed to have a relative preference for present consumption, thus we assume $0 < \beta < 1$.

Agents work only during youth. Their labor supply is assumed to be exogenous and inelastic. We assume that there is no demographic growth and the active population’s size is normalized to 1. Thus, from now on, we pursue the analysis considering that at each period one young and one old agent co-exist.

The wage $w_t$ is the only source of revenue of an agent. We assume that the government determines a fixed tax rate $\theta$ on production factor incomes. Consequently, the wage rate is taxed and the disposable revenue of an agent is $(1 - \theta)w_t$. This revenue is split between
present consumption $c_{j,t}$, real non-monetary savings $s_t$, and real cash balances $\frac{M_{t+1}}{P_t}$. The budget constraint of a young agent is then:

$$c_{j,t} = (1 - \theta)w_t - s_t - \frac{M_{t+1}}{P_t}$$

(2)

The consumer agent born in period $t$ becomes old in $t+1$, stops working and consumes the real non-monetary savings and real cash balances. The interest rate being also taxed, the non-monetary savings realized during youth yields $(1 + (1 - \theta)r_{t+1})s_t$ at $t+1$. In addition to this non-monetary savings income, the old agent spends also his cash balances whose real value becomes $\frac{M_{t+1}}{P_{t+1}}$, following the modification of prices between period $t$ and $t+1$. The budget constraint of the old agent in $t+1$ is:

$$c_{v,t+1} = (1 + (1 - \theta)r_{t+1})s_t + \frac{M_{t+1}}{P_{t+1}}$$

(3)

For simplification purposes, we denote as $R_{t+1}$ the real detaxed return on non-monetary savings, and as $m_{t+1}$ the real cash balances hold in $t$ for consumption in $t+1$. These two variables are defined in the following way:

$$R_{t+1} = (1 + (1 - \theta)r_{t+1})$$

(4)

$$m_{t+1} = \frac{M_{t+1}}{P_t}$$

(5)

An agent born in $t$ saves money whose nominal value is $M_{t+1}$. We prefer to use the index $t+1$ instead of $t$ because this amount will be spent in period $t+1$ by this agent.

The agent faces a cash-in-advance constraint à la Hahn and Solow (1995). A fixed portion $\mu$ of future consumption is realized using money. The liquidity constraint is written in the following way:

$$\mu P_{t+1} c_{v,t+1} \leq M_{t+1}$$

(6)

The return of money is supposed to be lower than that of non-monetary savings\(^3\) ($\frac{P_t}{P_{t+1}} < R_{t+1}$). Money assures less return than non-monetary savings. That’s why agents do not hold money more than the portion $\mu$ imposed by the liquidity constraint. The liquidity constraint is then binding:

$$\mu P_{t+1} c_{v,t+1} = M_{t+1}$$

(7)

We develop the retired agents budget constraint given by (3) by including the cash-in-advance constraint (7):

$$c_{v,t+1} = \frac{R_{t+1}s_t}{1 - \mu}$$

(8)

\(^3\)This assumption will be verified at the steady state
The liquidity constraint (7) can be written, by dividing both sides of the equation by $P_t$:

$$m_{t+1} = \mu \pi_{t+1} c_{v,t+1}$$

(9)

where $\pi_{t+1}$ stands for the inflation factor $\left(\frac{P_{t+1}}{P_t}\right)$ between $t$ and $t+1$.

We include the liquidity constraint in the young agent’s budget constraint by replacing the expression of real balances $m_{t+1}$ figuring in (2) by the one given in equation (9):

$$c_{j,t} = (1 - \theta)w_t - s_t - \mu \pi_{t+1} c_{v,t+1}$$

(10)

By eliminating the non-monetary savings variable $s_t$ in the modified budget constraints of retirement period (8) and active period (10), we obtain the intertemporal budget constraint of an agent born in $t$:

$$c_{j,t} = (1 - \theta)w_t - c_{v,t+1} \frac{1 - \mu}{R_{t+1}} - c_{v,t+1} \mu \pi_{t+1}$$

(11)

This expression can be written in the following way:

$$c_{j,t} = (1 - \theta)w_t - c_{v,t+1} \left( \frac{1 - \mu}{R_{t+1}} + \mu \pi_{t+1} \right)$$

(12)

In the intertemporal budget constraint (12), the expression $\left( \frac{1 - \mu}{R_{t+1}} + \mu \pi_{t+1} \right)$ represents the discount rate which takes into account the return on total savings $(s_t + \frac{M_{t+1}}{P_t})$. A portion $(1 - \mu)$ of future consumption is discounted using the real return $R_{t+1}$, while the portion $\mu$ left is discounted using the return on money $\frac{1}{\pi_{t+1}}$.

The consumer decides about consumption levels during youth $c_{j,t}$, and retirement $c_{v,t+1}$, and about non-monetary savings $s_t$, by maximizing the intertemporal utility function (1) with respect to the intertemporal budget constraint (12):

$$\left\{ \begin{array}{l} \text{Max } U(c_{j,t}, c_{v,t+1}) \\ \text{s.c. } c_{j,t} = (1 - \theta)w_t - c_{v,t+1} \left( \frac{1 - \mu}{R_{t+1}} + \mu \pi_{t+1} \right) \end{array} \right.$$

The resolution of this program yields the following results:

$$s_t = \frac{\beta(1 - \mu)(1 - \theta)w_t}{(1 + \beta)(1 - \mu + \mu \pi_{t+1} R_{t+1})}$$

(13)

$$c_{j,t} = \frac{(1 - \theta)w_t}{1 + \beta}$$

(14)

$$c_{v,t+1} = \frac{\beta(1 - \theta)w_t R_{t+1}}{(1 + \beta)(1 - \mu + \mu \pi_{t+1} R_{t+1})}$$

(15)
2.2 Production

We assume that firms produce one good serving both for consumption and investment, using private capital $K_t$, labor $L_t$. In our model, public capital $G_t$ enters the production function, thus affects the level of production. In the same way as Yakita (2008), we assume that the production technology of a firm $i$ is given by the following Cobb-Douglas production function:

$$Y_{i,t} = A K_{i,t}^\alpha (G_t L_{i,t})^{1-\alpha}$$  \hspace{1cm} (16)

with $0 < \alpha < 1$ and $A > 0$

The variable $G_t$ represents the public capital stock and acts as a positive externality on individual production of firms. In the individual level, $G_t$ is an exogenous variable, identical for all firms. In the aggregate level, public capital becomes an endogenous variable, whose level is determined by budget policies. Public capital can be thought as the infrastructure of an economy which permits firms to produce.

Labor supply is exogenous, inelastic and normalized to 1 ($L_t = 1$). Under the hypothesis of perfect competition, marginal productivities of production factors are equal to their net utilisation costs. Denoting $\delta$ as the depreciation rate of both private and public capital, we write the interest rate $r_t$ and the wage rate $w_t$ in the following way:

$$r_t + \delta = \frac{\partial Y_{i,t}}{\partial K_{i,t}} \implies r_t = \alpha \frac{Y_{i,t}}{K_{i,t}} - \delta$$  \hspace{1cm} (17)

$$w_t = \frac{\partial Y_{i,t}}{\partial L_{i,t}} \implies w_t = (1-\alpha)Y_{i,t}$$  \hspace{1cm} (18)

2.3 Government

Government intervenes in the economy using the following instruments: unproductive public spendings, public investment spendings, taxes, financing of the public deficit by monetization and/or public bonds.

The public investment level of a period $t$ is fixed proportionally to the GDP of the economy. We denote as $i$ the weight of public investments in GDP. Thus, the amount of public investments in a period $t$ is denoted as $iY_t$.

The amount of unproductive public spendings is also expressed proportionally to the GDP. The weight of these spendings in GDP is denoted $g$. Thus, the amount of unproductive public spending in a period $t$ is $gY_t$.

The factor incomes are subject to the tax rate $\theta$. Denoting as $B_t$ the public bonds stock reimbursed at the end of period $t$, the total fiscal revenue of the government in period $t$ is:

$$T_t = \theta w_t L_t + \theta r_t K_t + \theta r_t B_t$$  \hspace{1cm} (19)

We replace the variables $w_t$ and $r_t$ by their expressions shown in (18) et (17). Given that we have $L_t = 1$, the fiscal revenue becomes:
\[ T_t = \theta(1 - \alpha)Y_t + \theta \left( \frac{\alpha Y_t}{K_t} - \delta \right) K_t + \theta r_t B_t \]  

(20)

In period \( t \), total taxes collected by the government is:

\[ T_t = \theta (Y_t - \delta K_t) + \theta r_t B_t \]  

(21)

In our model, the tax rate is not endogenous as it is the case in Bräuninger (2005) and Yakita (2008). In those models, which analyzes public debt sustainability, the government fixes its public deficit objective and chooses the tax rate which verifies its budget constraint. In our model, the tax rate, the public spending plan and the structure of public deficit financing are exogenous, while the public deficit is endogenous. This way, we can analyze budget realizations instead of budgetary objectives.

The government fixes three budgetary instruments \((g, i, \theta)\), each of them being expressed proportionally to the GDP. Furthermore, the government decides how the public deficit generated will be financed.

The public deficit, at the end of period \( t \) is written in the following way:

\[ Def_t = (g + i)Y_t + r_t B_t - T_t \quad \Rightarrow \quad Def_t = (g + i - \theta)Y_t + (1 - \theta)r_t B_t + \theta \delta K_t \]  

(22)

Public deficit can be financed by bond issuances or by creating money. A fixed portion \( \lambda \) of this deficit is financed by public bonds while the remaining \((1 - \lambda)\) is financed by monetization.

The public bond stock \( B_{t+1} \), issued at the end of \( t \), verifies the following equality:

\[ B_{t+1} - B_t = \lambda Def_t \quad \Rightarrow \quad B_{t+1} - B_t = \lambda ((g + i - \theta)Y_t + (1 - \theta)r_t B_t + \theta \delta K_t) \]  

(23)

In the same way, money creation, or money supply evolves in the following way:

\[ \frac{M_{t+1} - M_t}{P_t} = (1 - \lambda) Def_t \quad \Rightarrow \quad \frac{M_{t+1} - M_t}{P_t} = (1 - \lambda) ((g + i - \theta)Y_t + (1 - \theta)r_t B_t + \theta \delta K_t) \]  

(24)

Money supply consists only of deficit monetization. It depends on the level of public debt generated by the policy rule adopted by the government and by the parameter \( \lambda \), reflecting the weight of bond issuance and money creation in public deficit financing.

We define the "total public debt", denoted as \( D_t \), as the sum of public bonds stock and real cash balances in a given date:
The evolution of public debt is obtained by taking the difference between the two expressions above. By simple manipulations, we write the dynamics of public debt:

\[ D_{t+1} - D_t = B_{t+1} - B_t + \frac{M_{t+1} - M_t}{P_t} \rho_t \]  

(27)

with \( \rho_t \) representing the inflation rate \( \frac{P_t - P_{t-1}}{P_{t-1}} \). In the dynamic given in (27), we can see the two public deficit financing instruments: public bonds issues \( (B_{t+1} - B_t) \) and monetization \( \left( \frac{M_{t+1} - M_t}{P_t} \right) \). In addition to these two mechanisms, the public debt dynamic includes the expression \( \frac{M_t}{P_t} \rho_t \). By developing \( \rho_t \) in the latter, we obtain \( \frac{M_t}{P_{t+1}} - \frac{M_t}{P_t} \). This expression corresponds to the difference between the values of real balances following an inflation between \( t - 1 \) and \( t \). This erosion generated by the inflation is defined in the literature as the inflation tax.

Taking into account the dynamics (23), (24) and (27), we finalize the development of the public debt accumulation:

\[ D_{t+1} - D_t = (g + i - \theta)Y_t + (1 - \theta)r_tB_t + \theta \delta K_t - m_t \left( 1 - \frac{1}{\pi_t} \right) \]  

(28)

Public investments, denoted as \( iY_t \), adds up to the existing (depreciated) public capital stock and assures its accumulation:

\[ G_{t+1} = (1 - \delta)G_t + i Y_t \]  

(29)

### 2.4 Private Capital Accumulation

Labor is normalized to 1. Thus individual non-monetary savings given by (13) corresponds to the total non-monetary savings of the economy.

Non-monetary saving \( s_t \) is split between private capital and public bonds:

\[ s_t = K_{t+1} + B_{t+1} \]  

(30)

The non-monetary saving of the young agent is:
Yet, the value of consumption during youth \((c_{j,t})\) is given in equation (14). Putting the latter in (31) we obtain:

\[
s_t = (1 - \theta)w_t - c_{j,t} - m_{t+1}
\]

Replacing non-monetary saving in (30) by its value given by (32), the accumulation dynamic of private capital is found:

\[
K_{t+1} = \frac{\beta(1 - \theta)w_t}{1 + \beta} - (m_{t+1} + B_{t+1})
\]
3 RESOLUTION OF THE MODEL AND STABILITY

Equations and dynamics constituting our model are as follows:

\[
\begin{align*}
Y_t &= AK_t^\alpha G_t^{1-\alpha} \\
r_t &= \alpha \frac{Y_t}{K_t} - \delta \\
w_t &= (1-\alpha)Y_t \\
B_{t+1} - B_t &= \lambda ((g + i - \theta)Y_t + (1 - \theta)r_tB_t + \theta \delta K_t) \\
\frac{M_{t+1} - M_t}{P_t} &= (1-\lambda) ((g + i - \theta)Y_t + (1 - \theta)r_tB_t + \theta \delta K_t) \\
D_{t+1} - D_t &= (g + i - \theta)Y_t + (1 - \theta)r_tB_t + \theta \delta K_t - m_t \left(1 - \frac{1}{\pi_t}\right) \\
K_{t+1} &= \frac{\beta(1-\theta)w_t}{1+\beta} - (m_{t+1} + B_{t+1}) \\
G_{t+1} &= (1-\delta)G_t + iY_t \\
\mu P_{t+1} c_{t+1} &= M_{t+1}
\end{align*}
\]

We will use growth factors of aggregates $K_t$, $G_t$, $B_t$, $D_t$ and $m_{t+1}$ to study budgetary policy sustainability. This study aims thus to see whether a budgetary policy can place the economy on a balanced growth path or not. The work will then be concentrated on the steady states of the model.

We define the following variables which will be used further to define the steady state:

\[
\begin{align*}
\frac{D_t}{K_t} &= x_t \quad (35) \\
\frac{G_t}{K_t} &= z_t \quad (36) \\
\frac{B_t}{D_t} &= \phi_t \quad (37) \\
\frac{m_t}{D_t} &= 1 - \phi_t \quad (38)
\end{align*}
\]

$x_t$ represents total public debt/private capital ratio at date $t$. $z_t$ is the public capital/private capital ratio at $t$. Finally, $\phi_t$ corresponds to the weight of public bonds in total public debt, while, inversely, $(1 - \phi_t)$ gives the portion of cash balances in the total government debt.

From now on, we write the growth factors of each aggregate in terms of $x_t$, $z_t$ and $\phi_t$ defined above. We then demonstrate that the stabilization of these three variables gives the steady state of the economy by equalizing all growth factors. A budgetary policy $(g, i, \theta, \lambda)$ allowing to calculate steady state values $x^*$, $z^*$ and $\phi^*$ enables a long run equilibrium and is thus considered sustainable.
3.1 Evolution dynamics of public bonds stock

To determine the growth dynamic of public bonds stock, we divide equation (23), depicting the accumulation of public bonds, by $B_t$:

$$\frac{B_{t+1}}{B_t} = \lambda(g + i - \theta) Y_t B_t + \lambda(1 - \theta) r_t + \lambda \theta \delta K_t + 1$$  \hspace{1cm} (39)

Equation (39) is the growth factor of public bonds stock. The objective is to write this growth factor in terms of $x_t$, $z_t$ and $\phi_t$ defined in equations (35)-(38). We re-write $\frac{Y_t}{B_t}$, $\frac{K_t}{B_t}$ and $r_t$ appearing in (39) in terms of the preceding ratios.

The first expression is developed the following way:

$$\frac{Y_t}{B_t} = \frac{Y_t}{K_t} \cdot \frac{K_t}{D_t} \cdot \frac{D_t}{B_t}$$

Yet we have $\frac{D_t}{K_t} = x_t$ and $\frac{B_t}{D_t} = \phi_t$. The expression above then becomes:

$$\frac{Y_t}{B_t} = \frac{Y_t}{K_t} \frac{1}{x_t \phi_t}$$  \hspace{1cm} (40)

In addition, according to the production function (16) we have:

$$\frac{Y_t}{K_t} = A \cdot \left( \frac{G_t}{K_t} \right)^{1-\alpha} = A z^{1-\alpha}$$  \hspace{1cm} (41)

Putting (41) in (40), we obtain:

$$\frac{Y_t}{B_t} = A z^{1-\alpha} \frac{1}{x_t \phi_t}$$  \hspace{1cm} (42)

As for the ratio $\frac{K_t}{B_t}$, it can be developed as follows:

$$\frac{K_t}{B_t} = \frac{K_t D_t}{D_t B_t} = \frac{1}{\phi_t x_t}$$  \hspace{1cm} (43)

The variable $r_t$ is transformed by a similar procedure. According to equation (17) which states the equality between the interest rate and the marginal productivity of capital, we have:

$$r_t - \delta = \alpha \frac{Y_t}{K_t}$$
Putting (41) in the latter equation, the interest rate becomes:

\[ r_t = \alpha A_t^{1-\alpha} - \delta \]  

(44)

Taking into account (42), (43) and (44) the growth factor of public bonds stock is written as follows:

\[ \frac{B_{t+1}}{B_t} = \lambda (g + i - \theta) \frac{A_t^{1-\alpha}}{x_t \phi_t} + \lambda (1 - \theta) (\alpha A_t^{1-\alpha} - \delta) + \frac{\lambda \theta \delta}{x_t (1 - \phi_t)} + 1 \]  

(45)

3.2 The accumulation of cash balances

The starting point of the cash balances growth factor calculation is the equation (24) depicting the money creation mechanism:

\[ \frac{M_{t+1} - M_t}{P_t} = (1 - \lambda) ((g + i - \theta) Y_t + (1 - \theta) r_t B_t + \theta \delta K_t) \]

The left side of this equation can be written as follows:

\[ \frac{M_{t+1} - M_t}{P_t} = m_{t+1} - m_t \frac{1}{\pi_t} \]

We divide the latter by \( m_t \) to obtain, by using the money creation equation above:

\[ \frac{m_{t+1}}{m_t} - \frac{1}{\pi_t} = (1 - \lambda) (g + i - \theta) \frac{Y_t}{m_t} + (1 - \lambda) (1 - \theta) r_t \frac{B_t}{m_t} + (1 - \lambda) \theta \delta \frac{K_t}{m_t} \]

Our objective is to express this growth factor in terms of \( x_t, z_t \) and \( \phi_t \). According to the calculations, whose details are given in appendix, the growth factor of monetary mass is:

\[ \frac{m_{t+1}}{m_t} = (1 - \lambda) (g + i - \theta) \frac{A_t^{1-\alpha}}{x_t (1 - \phi_t)} + (1 - \lambda) (1 - \theta) \left( \frac{\phi_t}{1 - \phi_t} \right) \left( \alpha A_t^{1-\alpha} - \delta \right) + \frac{1 - \lambda \theta \delta}{x_t (1 - \phi_t)} + \frac{1}{\pi_t} \]  

(46)

3.3 Total public debt dynamic

The starting point is the dynamic of total public debt (28):

\[ D_{t+1} - D_t = (g + i - \theta) Y_t + (1 - \theta) B_t + \theta \delta K_t - m_t \left( 1 - \frac{1}{\pi_t} \right) \]

According to the calculations presented in appendix, the growth factor of total public debt expressed in terms of \( x_t, z_t \) and \( \phi_t \) is:

\[ \frac{D_{t+1}}{D_t} = (g + i - \theta) \frac{A_t^{1-\alpha}}{x_t} + (1 - \theta) \phi_t \left( \alpha A_t^{1-\alpha} - \delta \right) + \frac{\theta \delta}{x_t} - (1 - \phi_t) \left( 1 - \frac{1}{\pi_t} \right) + 1 \]  

(47)
3.4 Public capital accumulation

We first divide the accumulation dynamic of public capital stock (29) by $G_t$:

$$\frac{G_{t+1}}{G_t} = 1 - \delta + i \frac{Y_t}{G_t}$$

The growth factor of public capital stock (for details see appendix) is:

$$\frac{G_{t+1}}{G_t} = 1 - \delta + Aiz_t^{-\alpha} \quad (48)$$

3.5 Dynamics of private capital

According to equation (33), the evolution dynamics of private capital is:

$$K_{t+1} = \frac{\beta(1 - \theta)w_t}{1 + \beta} - (m_{t+1} + B_{t+1})$$

The growth factor of private capital is (for calculation details see appendix):

$$\frac{K_{t+1}}{K_t} = \frac{\beta(1 - \theta)(1 - \alpha)}{1 + \beta} A z_t^{1-\alpha} - (g + i - \theta)A z_t^{1-\alpha} - \theta \delta - (1 + (1 - \theta)(\alpha A z_t^{1-\alpha} - \delta)) \phi_t x_t - \frac{(1 - \phi_t)x_t}{\pi_t} \quad (49)$$

3.6 Inflation

The consumer must respect the liquidity constraint (cash-in-advance constraint) given in equation (6). The modification of monetary mass, reflecting the budgetary rule of the government, influences the price level if it is non-proportional to retired agents’ consumption increase. This liquidity constraint let us calculate the evolution of inflation.

The cash-in-advance constraint states that a fixed portion $\mu$ of retired period consumption must be done with money: $\mu P_{t+1} c_{v,t+1} = M_{t+1}$. By dividing both sides by $P_t$ and introducing variables $m_{t+1} = \frac{M_{t+1}}{P_t}$ and $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ we obtain:

$$\pi_{t+1} = \frac{m_{t+1}}{\mu c_{v,t+1}}$$

In the latter, we replace $c_{v,t+1}$ by its value given by the intertemporal maximisation program of the agent in (13). Developing the equation this way (for details see appendix) we obtain the evolution dynamics of the price level:

$$\pi_{t+1} = \frac{(1 + \beta)(1 - \mu)}{\mu (1 + (1 - \theta)(\alpha A z_t^{1-\alpha} - \delta)) \left( \frac{\beta(1 - \theta)(1 - \alpha)A z_t^{1-\alpha}}{\pi_t(1 - \phi_t)} \gamma_m - 1 - \beta \right)}$$

with $\gamma_m = \frac{m_{t+1}}{m_t} \quad (50)$
3.7 Steady states and sustainability of budgetary policies

In the preceding section, we defined growth factors of five aggregate:

- \( \frac{D_{t+1}}{D_t} = \gamma_d(x_t, z_t, \phi_t) \): growth factor of total public debt given by equation (47)
- \( \frac{B_{t+1}}{B_t} = \gamma_b(x_t, z_t, \phi_t) \): growth factor of public bonds given by (45)
- \( \frac{m_{t+1}}{m_t} = \gamma_m(x_t, z_t, \phi_t) \): growth factor of cash balances given by (46)
- \( \frac{K_{t+1}}{K_t} = \gamma_k(x_t, z_t, \phi_t) \): growth factor of private capital given by (49)
- \( \frac{G_{t+1}}{G_t} = \gamma_g(z_t) \): growth factor of public capital shown in (48)

The five growth factors shown above describes the functioning of the economy. A budgetary policy would be sustainable if it allows a steady state where all growth factors are equal. Yet, these factors are functions of \( z_t = \frac{G_t}{K_t} \), \( x_t = \frac{D_t}{K_t} \) and \( \phi_t = \frac{B_t}{D_t} \). Thus, the definition of a sustainable budgetary policy is:

**Définition**: A budgetary policy defined by a vector \((g, i, \theta, \lambda)\) of budgetary parameters is said to be sustainable if, for given structural economic parameters \((A, \alpha, \beta, \delta, \mu)\), it enables to resolve the system of equations given by \( \gamma_b = \gamma_m = \gamma_d = \gamma_g = \gamma_k \), to obtain a solution \((x^*, z^*, \phi^*)\).

The system of equations allowing to calculate the steady state of the economy can be written, for instance, the following way:

\[
\begin{cases}
\frac{B_{t+1}}{B_t} = \frac{G_{t+1}}{G_t} \\
\frac{D_{t+1}}{D_t} = \frac{G_{t+1}}{G_t} \\
\frac{K_{t+1}}{K_t} = \frac{G_{t+1}}{G_t}
\end{cases}
\]

We have then a system of non-linear equations with three unknowns \((x_t, z_t, \phi_t)\). By using the substitution method, it is possible to obtain a polynomial equation of the variable \( z_t \), in the form \( H(z) = 0 \). The roots of this polynomial gives the steady states of the economy. Furthermore, the system above gives the relation, at the steady state, between \( z^* \) and \( x^* \) and between \( z^* \) and \( \phi^* \). Thus, the roots of the polynomial \( H(z) \) allows us to obtain the solutions \((x^*, z^*, \phi^*)\).

On the other hand, the theoretical solution of this system is very complicated and makes opaque the analysis on the sustainability frontier of budgetary policies. This is why we will not solve the system theoretically. The study on the sustainability frontier is realised by using simulations illustrating Turkish budgetary policies conducted during the last decades.
For most of the simulations that will be presented, the polynomial $H(z)$ has two roots. Thus we have multiple equilibria. This is explained by increasing returns incorporated as an externality through public capital. Only the saddle point, attributed to a public debt model is analysed\(^4\). In fact, the stable equilibrium gives negative deficits and debts.

4 SIMULATIONS - SUSTAINABILITY OF BUDGETARY POLICIES IN TURKEY

4.1 Methodology

As we defined it earlier, a budgetary policy is considered sustainable if it sets the economy on a balanced growth path. The objective of the paper being to evaluate budgetary policies, the simulations will be focalised on steady states of the model. We simulate\(^5\) Turkish budgetary policies and verify if the system $\gamma_b = \gamma_d = \gamma_m = \gamma_k = \gamma_g$ allows a solution\(^6\) $(x^*, z^*, \phi^*)$. The existence of a solution implies that the simulated policy is sustainable in the long run.

Once the solution is found\(^7\), and thus the sustainability confirmed, various long run economic indicators can be calculated by using the stationary values $x^*$, $z^*$ and $\phi^*$. The long term growth rate is calculated by substituting these values in any growth factor equation, say $\gamma_g$. We assume that one period corresponds to thirty years\(^8\).

Inflation is calculated the same way, by substituting $(x^*, z^*, \phi^*)$ in $\pi^\ast$ given by equation (50). Besides inflation and growth, we also simulate the long run values of public deficit/GDP and public debt/GDP ratios, the interest rate, the weight of cash balances, public bonds and private capital in total savings and finally the return on money holdings.

First of all, we evaluate the sustainability of Turkish budgetary policies conducted in periods 1980-1988, 1989-1998 and 1999-2008. Then we pursue simulations to see the impact of budgetary parameters on sustainability and growth. The objective is to analyse the sustainability frontier by modifying the budget parameters. This analysis is done within two steps.

First the 1999-2007 period is studied, during which, according to the model and simulations, the budget policy conducted was sustainable. Taking off from this initial configuration, we simulate alternative and more expansionist policies. This

\(^4\)See Azariadis (1993)
\(^5\)We use the mathematics software Maple 9.5 for simulations.
\(^6\)At the steady state, we assume $\pi_{t+1} = \pi_t$ and $r_{t+1} = r_t$.
\(^7\)Solutions must verify the superiority of interest rate income over money holdings return.
\(^8\)Barro et Sala-i Martin (1995).
analysis enables to study the impact of each budget parameter on sustainability and various indicators of the economy. The results show that Turkey could have implemented more expansionist and growth enhancing policies, without risking long run sustainability. In fact, the policy on course in this period is relatively restrictive and not close to the sustainability frontier.

Secondly, the policy of period 1980-1988, which is unsustainable, is studied. We simulate alternative policies to see how sustainability could have been achieved. More precisely, the following stabilisation policies are studied: a reduction in non productive public spendings, a decrease in public investments, an increase in taxes, and an increase in monetization. The results show that monetization acts as a tax: it makes the budgetary policy more sustainable and creates a distortion in portfolio choices of the agent. It triggers a fall in non-monetary savings, thus slows down public and private capital accumulation. Its negative impact on private capital accumulation appears to be more important than that of the increase of factor income tax.

4.2 Sustainability of Turkish budgetary policies

We analyse budgetary policies conducted in Turkey between 1980 and 2008. We split this interval into three phases:

- **1980-1988**: It is the first liberalization period where internal public debt started to rise as the dominant deficit financing method. Peaks of monetization are observed at the beginning and at the end of the period.

- **1989-1998**: Following the liberalization of the capital account, the economy becomes more unstable. Public debt accumulation goes on. Monetization is used at the beginning of the period while it disappears completely in 1998.

- **1999-2007**: This decade witnessed more important efforts in macroeconomic stabilisation. Particularly in the aftermath of the crisis in 2001, successive governments conducted very restrictive budgetary policies recommended by the IMF.

Structural parameters of the economy, namely \((A, \alpha, \beta, \delta)\) are considered identical for the three distinct periods. They are calibrated in the following way:

- Private capital elasticity\(^9\) : \(\alpha = 0.27\)
- Total exogenous productivity\(^10\) : \(A = 46\)

Table 1: Sustainability of Turkish budgetary policy since 1980

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Public spendings (g)</td>
<td>7.8%</td>
<td>10.4%</td>
<td>9.5%</td>
</tr>
<tr>
<td>Public investments (i)</td>
<td>9.2%</td>
<td>6.3%</td>
<td>4.2%</td>
</tr>
<tr>
<td>Taxes (θ)</td>
<td>13.5%</td>
<td>17.1%</td>
<td>18.3%</td>
</tr>
<tr>
<td>Share of monetization (1 − λ)</td>
<td>24%</td>
<td>12%</td>
<td>0%</td>
</tr>
<tr>
<td>Liquidity constraint (µ)</td>
<td>15%</td>
<td>8.6%</td>
<td>7.4%</td>
</tr>
<tr>
<td>Sustainability</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

- Intertemporal discount factor\(^{11}\) : \(β = 0.55\)
- Depreciation rate of capital\(^{12}\) : \(δ = 0.78\)

The parameters \((g, i, θ, λ)\) concerning budgetary policies and the liquidity constraint \(µ\), differ according to the studied period:

- The shares of current public expenditures, public investments and fiscal income in the GDP \((g, i, θ)\) are calculated using data from the overall public sector accounts. For the parameter \(θ\), only fiscal revenues have been taken into account. All data is taken from the online databases of the State Planning Organization (DPT).

- The share of monetization in public deficit financing \((1 − λ)\), is estimated by dividing the stock of credits granted by the Central Bank to the Treasury by the public sector borrowing requirement of the corresponding year.

- The liquidity constraint \(µ\) is estimated by dividing the M1 monetary aggregate by the annual total consumption. Data on M1 aggregate are taken from the online database of the Central Bank of Turkey, while consumption data are taken from the Statistical Indicators review of the Turkish Statistical Institute (TurkStat, 2007).

For each period mentioned, we estimate an "average" budgetary policy by simply calculating the annual average of the budgetary parameters. Table (1) shows the calibration of each period and the result of the simulation concerning sustainability.

According to the results, budgetary policies of periods 1980-1988 and 1989-1998 do not set the economy on a balanced growth path, thus are not sustainable. On

\(^{11}\)Barro et Sala-i Martin (1995)

\(^{12}\)Barro et Sala-i Martin (1995)
Table 2: Economic indicators simulated for period 1999-2007

<table>
<thead>
<tr>
<th></th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public/Private capital ratio:</td>
<td>$z^* = 0.34$</td>
</tr>
<tr>
<td>Public debt/Private capital ratio:</td>
<td>$x^* = 0.57$</td>
</tr>
<tr>
<td>Public bonds/Public debt ratio:</td>
<td>$\phi^* = 0.66$</td>
</tr>
<tr>
<td>Annual growth</td>
<td>3.5%</td>
</tr>
<tr>
<td>Inflation</td>
<td>-3.4%</td>
</tr>
<tr>
<td>Public deficit (% of GDP)</td>
<td>3.3%</td>
</tr>
<tr>
<td>Primary deficit (% of GDP)</td>
<td>-5.5%</td>
</tr>
<tr>
<td>Public bonds (% of GDP)</td>
<td>5.1%</td>
</tr>
<tr>
<td>Share of cash balances in total savings</td>
<td>12.3%</td>
</tr>
<tr>
<td>Share of public bonds in total savings</td>
<td>24%</td>
</tr>
<tr>
<td>Share of private capital in total savings</td>
<td>63.7%</td>
</tr>
<tr>
<td>Public bonds (private capital) return $R^*$</td>
<td>5</td>
</tr>
<tr>
<td>Money return $\frac{1}{\pi^*}$</td>
<td>2.8</td>
</tr>
</tbody>
</table>

on the other hand, the policy conducted in 1999-2007 is sustainable.

Table (2) shows the long run equilibrium calculated for the given budgetary parameters. Let us remind that the equilibrium analyzed is a saddle point\textsuperscript{13}. According to table (2), at the steady state, the economy grows at an annual rate of $-3.4\%$. Annual inflation is negative and equal to $-3.4\%$. This is due to the fact that the only inflation source in the model is monetization and that there is no money creation in the simulated policy ($\lambda = 1$). In this context, the initial money stock $M_0$ is transferred from one generation to the other. At the steady state, prices adjust in such a way that cash balances grow at the same rate than other aggregates. The price variation is then equal to the inverse of the steady state growth factor.

On the balanced growth path, the public deficit is 3.3\% of GDP and there is a primary surplus of 5.5\%. The share of public bond stock in GDP $\frac{B_t}{Y_t}$ is 5.1\%. This means that in the long run, the recent policy would bring the economy to a public bonds/GDP ratio of 5.1\%.

\textsuperscript{13}Simulation for this period reveals multiple equilibria. The second equilibrium, which is stable locally, is characterized by $z^* = 0.14$, $x^* = -0.35$ et $\phi^* = 1.1$. It is a situation where the weight of public capital stock is lower ($z^*$) compared to the equilibrium presented in table (2). Furthermore the steady state public debt is negative, in other terms, for the same budgetary parameters, we have a second equilibrium where government subsidizes the young agent. At this equilibrium, the growth rate is higher. High growth, combined with no money creation, ($\lambda = 1$), generates high deflation. According to the results, at this second equilibrium, return on money is 3.47, while return on bonds (both public and private) is 2.59. This situation breaches the binding liquidity constraint. Money is no more a dominated asset. In this context agents would only hold money and there would be no capital accumulation.
According to the results, at the steady state, real cash balances \((m_{t+1})\) would constitute 12.3\% of total savings of an agent. Public bonds \((B_{t+1})\) and private capital \((K_{t+1})\) would be equal to 24\% and 63.6\% of total savings, respectively.

### 4.3 The 1999-2007 period and alternative policies

Was it possible to conduct more expansionist policies without violating the sustainability frontier? If the response is positive, would the new equilibria obtained be better or worse than the initial one?

In order to give a response to these questions, we conduct three series of simulations depicting more expansionist budgetary policies than the initial one. First, we analyze the impact of an increase in public investments on the stability. Then, we study the effects of an increase in non-productive public spendings. The last serie of simulations tests policies imposing less tax than the initial one. A final simulation is realized in order to test the impact of more monetization.

The equilibrium of period 1999-2007 is a saddle point. Thus, the analysis presented here is not a comparative statics. It rather aims at determining if other more expansionist policies could have been adopted without passing the sustainability frontier. It does not analyze the impact of a change in policy on the initial equilibrium.

#### 4.3.1 Impact of public investments

Table (3) presents equilibria obtained with public investment rates \((i)\) higher than that of the policy conducted in period 1999-2007. The first column reminds the results of the initial policy. The others expose alternative policies, with higher levels\(^\text{14}\) of \((i)\). We see that, above \(i = 0.085\) approximately, the budgetary policy becomes unsustainable.

According to table (3), policies with more public investments would allow higher growth rates. The initial growth rate of 3.5\%, with \(i = 0.042\), goes up to 5.4\% for a policy with \(i = 0.082\). Public capital is productive in the model and intervenes as an externality in private production. The marginal productivity of private capital \(rt = \alpha Az_i^{1-\alpha} - \delta\) is increasing in \(G\) the public capital stock. Moreover, the increase in \(R^*\) shown in table (3) confirms this impact. Increasing \(i\) moves the economy closer to its sustainability frontier but provides more growth enhancing equilibria.

As the growth rate becomes higher, we observe an increase in deflation. Given that there is no money creation in this policy \((\lambda=1)\), at the steady state return on money (inverse of inflation) is equal to the long run growth rate. Deflation goes

\(^{14}\text{The initial levels of other budgetary parameters are not modified}\)
In this table, we also see that as the rate of public investments goes up, the long run public deficit/GDP ratio decreases. We can associate this fall to the important growth rate of production. It should be noted that, paradoxically, as public investments rise, public deficit/GDP and primary deficit/GDP get closer to 0.

One should also point the modification in the composition of savings. As public investments increase, the structure of savings is modified in favour of private capital. In fact, when \( i \) is higher, public capital stock goes up and increases the marginal productivity of private capital. Consequently, the share of private assets in total savings rises.

### 4.3.2 Impact of non-productive public spendings

We now analyze the impact of an increase in non productive public spendings on sustainability. Table (4) contains the results of simulations of more expansionist budgetary policies with higher non-productive public spendings. The first column gives the initial results of the 1999-2007 period budgetary policy. Above 13.4% of public expenses, the other parameters being constant, the budget policy becomes unsustainable.

The influence of current budget expenses on sustainability and growth is similar to that of public investments. A budgetary policy containing more non-productive public spendings grows faster on its balanced growth path. However the effect on growth is less than that of public investments. This is due to the fact that \( g \) has
Table 4: Impact of an increase in non-productive public spendings

<table>
<thead>
<tr>
<th></th>
<th>$g=0.095$</th>
<th>$g=0.105$</th>
<th>$g=0.115$</th>
<th>$g=0.125$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z^*$</td>
<td>0.34</td>
<td>0.32</td>
<td>0.29</td>
<td>0.26</td>
</tr>
<tr>
<td>$x^*$</td>
<td>0.57</td>
<td>0.5</td>
<td>0.37</td>
<td>0.24</td>
</tr>
<tr>
<td>$\phi^*$</td>
<td>0.66</td>
<td>0.64</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Annual growth</td>
<td>3.5%</td>
<td>3.57%</td>
<td>3.62%</td>
<td>3.7%</td>
</tr>
<tr>
<td>Inflation</td>
<td>-3.38%</td>
<td>-3.4%</td>
<td>-3.5%</td>
<td>-3.6%</td>
</tr>
<tr>
<td>Public deficit (% du PIB)</td>
<td>3.3%</td>
<td>2.8%</td>
<td>2.2%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Primary deficit (% du PIB)</td>
<td>-5.5%</td>
<td>-4.2%</td>
<td>-2.7%</td>
<td>-1.2%</td>
</tr>
<tr>
<td>Public bonds (% du PIB)</td>
<td>5.1%</td>
<td>4.3%</td>
<td>3.3%</td>
<td>1.9%</td>
</tr>
<tr>
<td>Cash balances/Savings</td>
<td>12.3%</td>
<td>11.7%</td>
<td>11%</td>
<td>10%</td>
</tr>
<tr>
<td>Public bonds/Savings</td>
<td>24%</td>
<td>20.5%</td>
<td>16%</td>
<td>9%</td>
</tr>
<tr>
<td>Private capital/Savings</td>
<td>63.7%</td>
<td>67.8%</td>
<td>73%</td>
<td>81%</td>
</tr>
<tr>
<td>Interest return $R^*$</td>
<td>5</td>
<td>4.8</td>
<td>4.5</td>
<td>4.2</td>
</tr>
<tr>
<td>Return on money $\frac{1}{\pi^z}$</td>
<td>2.8</td>
<td>2.9</td>
<td>2.9</td>
<td>3</td>
</tr>
</tbody>
</table>

not a direct impact on the production function. In the absence money creation, inflation gets lower as the growth rate increases.

We also see that the more current public spendings increase the less is the public deficit/GDP ratio. This surprising result is also mentioned by Azariadis (1993, p.326) in an analysis of an exogenous growth model with fixed budget rules: "(...)

Another process takes place at the saddle $G_1$ in which a higher deficit produces the counterintuitive outcome of more capital and less public debt. The message from this economy is that there is no simple rule that relates the size of government deficits to the steady state values of capital and debt or, for that matter, the statistical correlation of physical capital and national debt. This relationship rather straightforward for stable steady states, is quite subtle for the saddles that are quite often the most economically interesting stationary equilibria."

In fact, as the budgetary policy becomes more expansionist, public deficit/GDP and public debt/GDP ratios decrease on the saddle-point equilibrium. On the sustainability frontier, there is a unique equilibrium with the highest growth rate. From this point on, more expansionist budgetary policies push the economy in an unsustainable situation (no balanced growth path).

4.3.3 Impact of taxes

Simulation results for policies including less tax ($\theta$) are presented in table (5).

As the tax rate diminishes, balanced growth rate increases, because available income of agents $(1-\theta)w_t$ rises. Agents save more. The decrease in interest return
### Table 5: Effect of a decrease in tax rate

<table>
<thead>
<tr>
<th></th>
<th>$\theta = 0.183$</th>
<th>$\theta = 0.173$</th>
<th>$\theta = 0.163$</th>
<th>$\theta = 0.153$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z^*$</td>
<td>0.34</td>
<td>0.31</td>
<td>0.29</td>
<td>0.26</td>
</tr>
<tr>
<td>$x^*$</td>
<td>0.57</td>
<td>0.48</td>
<td>0.37</td>
<td>0.25</td>
</tr>
<tr>
<td>$\phi^*$</td>
<td>0.66</td>
<td>0.64</td>
<td>0.59</td>
<td>0.5</td>
</tr>
<tr>
<td>Annual growth</td>
<td>3.5%</td>
<td>3.57%</td>
<td>3.6%</td>
<td>3.73%</td>
</tr>
<tr>
<td>Inflation</td>
<td>-3.4%</td>
<td>-3.44%</td>
<td>-3.5%</td>
<td>-3.6%</td>
</tr>
<tr>
<td>Budget deficit (% du PIB)</td>
<td>3.3%</td>
<td>2.9%</td>
<td>2.3%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Primary deficit (% du PIB)</td>
<td>-5.5%</td>
<td>-4.1%</td>
<td>-2.7%</td>
<td>-1.33%</td>
</tr>
<tr>
<td>Public bonds (% du PIB)</td>
<td>5.1%</td>
<td>4.4%</td>
<td>3.5%</td>
<td>2.2%</td>
</tr>
<tr>
<td>Cash balances/savings</td>
<td>12.3%</td>
<td>11.7%</td>
<td>11%</td>
<td>10%</td>
</tr>
<tr>
<td>Public bonds/savings</td>
<td>24%</td>
<td>20.5%</td>
<td>16%</td>
<td>10%</td>
</tr>
<tr>
<td>Private capital/savings</td>
<td>63.7%</td>
<td>67.7%</td>
<td>73%</td>
<td>80%</td>
</tr>
<tr>
<td>Interest return $R^*$</td>
<td>5</td>
<td>4.8</td>
<td>4.5</td>
<td>4.3</td>
</tr>
<tr>
<td>Money return $\frac{1}{\pi^*}$</td>
<td>2.8</td>
<td>2.9</td>
<td>2.9</td>
<td>3</td>
</tr>
</tbody>
</table>

($R^*$) also reveals a rise in non-monetary savings. Private capital is accumulated faster. The steady states $z^* = \frac{G}{K}$ and $x^* = \frac{D}{K}$ gets lower, implying an increase in private capital $K$ relatively to public capital $G$ and total public debt $D$. However, under $\theta = 0.153$ the budgetary policy becomes unsustainable as there is no more steady state solution.

### 4.3.4 Effect of monetization

This paragraph analyzes the effect of monetization on growth and sustainability. As we did for the preceding studies, we simulated different policies by decreasing progressively the level of $\lambda$ (thus increasing monetization), while we keep constant the other budgetary parameters. The results, presented in table (6) show that monetization acts like a tax. As we increase monetization, we increase the inflation tax, thus the new saddle-point equilibria become "more sustainable".

More monetization implies a higher money supply. Inflation is then higher as the return on money $\frac{R_{t+1}}{\pi_{t+1}}$ decreases. Due to the liquidity constraint, agents are forced to hold more money. Thus non-monetary savings are lower. The latter being the source of private capital accumulation, its decrease implies a slow down in growth.

The results in table (6) confirm this explanation. When monetization rises, steady state values of $z^* = \frac{G}{K}$ and $x^* = \frac{D}{K}$ rise too. In other words, a larger money supply causes a fall in the share of private capital $K$ in the economy. The marginal productivity of capital being decreasing, the rise in $R^*$ also implies the fall of private capital's relative weight.
Monetization forces the agents to hold more money in order to maintain the value of their real cash balances and thus slows down capital accumulation. This policy also enables the financing of the public deficit. Then, we observe a resource transfer from agents toward the government. Monetization acts as a tax. In other words, the rise of monetization increases the inflation tax level and creates a source of income for the government. Moetization is considered then as a policy enabling sustainability.

### 4.4 Alternative policies for the 1980-1988 period

This last subsection studies the 1980-1988 period policy, which is unsustainable as shown in table (1). The economy in this period is situated over the sustainability frontier and can not reach a balanced growth path.

We analyze possible modifications on the budgetary policy which would allow the economy to reach a steady state. More precisely, we study the impacts of an increase in monetization (decrease in $\lambda$), a decrease in non-productive public spendings ($g$), a reduction in public investments ($i$) and finally a rise in the tax rate ($\theta$). For each simulation serie, we modify the level of the analyzed instrument while keeping the others constant. We modify the parameters until an equilibrium is reachd, in other words until the policy becomes sustainable.
4.4.1 Monetization as a stabilization instrument

Let us first remind the Turkish budgetary parameters of period 1980-1988 which is unable to reach a balanced growth path:

- Share of public deficit financed by bond issuance : $\lambda = 0,76$
- Share of public investments in GDP : $i = 0,092$
- Share of current public expenditures in GDP : $g = 0,078$
- Tax rate : $\theta = 0,135$

We simulate alternative policies by reducing $\lambda$, in other words by increasing monetization. The policy becomes sustainable from $\lambda = 0,18$ on. This means that financing 82% of public deficit by creating money would have allowed the economy to reach a balanced growth path. Table (7), presented in appendix, shows the details of the saddle point equilibrium obtained with this new policy.

The growth rate recorded at this new equilibrium (5,1%) is much higher than the rate obtained with the policy of period 1999-2007 (3,46%). This difference is due to the high level of public investments in period 1980-1988.

4.4.2 Other methods of stabilization

In addition to the increase of monetization, three other stabilization policies are simulated. According to the results:

- Sustainability is obtained if non-productive public spendings/GDP falls from $g = 0,078$ to $g = 0,039$. Details of the equilibrium obtained are presented in table (8) in appendix.

- A reduction of public investment/GDP ratio from $i = 0,092$ to $i = 0,052$ stabilizes the economy. Table (9) in appendix presents the details of the new equilibrium.

- A rise in taxes from $\theta = 0,135$ to $\theta = 0,175$ makes the budgetary policy sustainable. The details of the equilibrium are shown in table (10) in appendix.

Comparing tables (7), (8), (9) and (10), we see that the stabilization method allowing maximum growth rate is the decrease in non-productive public spendings (5,7%). The rise in taxes allows a stable growth rate of 5,6%. On the other hand, the most unfavourable method in growth terms appears to be the decrease of public investments 4,3%), due to the important influence of this variable in productivity.
It is interesting to compare the impact of tax increase and monetization in the stabilization of the economy. The balanced growth path obtained by raising taxes on factor income ($\theta$) allows a higher growth rate (5.6%) than the one obtained by raising inflation tax (5.1%).

It is then possible to correct an unsustainable budgetary policy by increasing monetization and thus by collecting an inflation tax. On the other hand, with given structural and budgetary parameters, reaching sustainability through a rise in factor income taxes is more advantageous. The most advantageous stabilization method, according to our results, is a reduction of non-productive public spendings. Finally, the most unfavourable method is a fall in public investments.

5 CONCLUDING REMARKS

This paper presented an endogenous growth model with OLG settings where productive public capital is incorporated as an externality and money is included through a liquidity constraint. The government, in the model, realizes productive and unproductive spendings whose amounts are expressed as a share of GDP. It also fixes a tax rate on factor income and determines the share of bond issuance and monetization in public deficit financing. The latter is exogenous in our model, while the other budgetary parameters are constant and allows to fix budget rules. A fixed budget policy allowing the economy to reach a balanced growth path is considered as sustainable.

This theoretical framework enables to analyze the sustainability of Turkish budgetary policies between 1980 and 2007. Our simulation results show that policies conducted in 1980-1988 and 1989-1998 periods are unsustainable, while the "average" policy applied in period 1999-2007 is sustainable. Further simulations on alternative policies showed that more expansionist policies could have been conducted in 1999-2007 period without breaching sustainability.

The model also illustrates the tax inflation mechanism. Simulations show how monetization acts as a tax. It enables sustainability but creates inflation and slows down private capital accumulation through a fall in non-monetary savings. The latter is due to the liquidity constraint which forces agents to hold more money when money supply and inflation rise. Although monetization can be considered as a tax, its negative effect on private capital accumulation is greater than that of factor income taxes. Thus, according to the model, increasing taxes is a better policy than increasing monetization in order to reach a balanced growth path. However, increasing taxes in a less developed or developed country is not as easier as it is in a developed country.
6 APPENDIX

6.1 Dynamic of cash balances

We have the equality:

\[
\frac{m_{t+1}}{m_t} - \frac{1}{\pi_t} = (1 - \lambda)(g + i - \theta)\frac{Y_t}{m_t} + (1 - \lambda)(1 - \theta)r_t \frac{B_t}{m_t} + (1 - \lambda)\theta \delta \frac{K_t}{m_t}
\]

The ratio \(\frac{Y_t}{m_t}\) can be re-written the following way:

\[
\frac{Y_t}{m_t} = \frac{Y_t}{K_t} \frac{K_t}{D_t} \frac{D_t}{m_t}
\]

We know \(\frac{Y_t}{K_t}\) from the relation (41), and \(r_t\) from (44). Introducing these terms we express the growth of cash balances as:

\[
\frac{m_{t+1}}{m_t} = (1 - \lambda)(g + i - \theta)\frac{A z_t^{1-\alpha}}{x_t(1 - \phi_t)} + (1 - \lambda)(1 - \theta)\left(\frac{\phi_t}{1 - \phi_t}\right) \left(\alpha A z_t^{1-\alpha} - \delta\right) + (1 - \lambda)\theta \delta \frac{K_t}{x_t(1 - \phi_t)} + 1
\]

6.2 Growth of total public debt

Our starting point is the accumulation dynamic given by (28):

\[
D_{t+1} - D_t = (g + i - \theta)Y_t + (1 - \theta)B_t + \theta \delta K_t - m_t \left(1 - \frac{1}{\pi_t}\right)
\]

We divide this equation by \(D_t\):

\[
\frac{D_{t+1}}{D_t} = (g + i - \theta)\frac{Y_t}{D_t} + (1 - \theta)r_t \frac{B_t}{D_t} + \frac{\theta \delta K_t}{D_t} + \left(1 - \frac{1}{\pi_t}\right) \frac{m_t}{D_t} + 1
\]

Yet we already know the following:

\[
\frac{Y_t}{D_t} = \frac{Y_t}{K_t} \frac{K_t}{D_t} = \frac{A z_t^{1-\alpha}}{x_t}
\]

\[
\frac{B_t}{D_t} = \phi_t
\]

\[
\frac{K_t}{D_t} = \frac{1}{x_t}
\]

\[
r_t = \alpha A z_t^{1-\alpha} - \delta
\]

\[
\frac{m_t}{D_t} = 1 - \phi_t
\]
Introducing these terms in the growth factor of total public debt, we obtain:

\[
\frac{D_{t+1}}{D_t} = (g + i - \theta) \frac{A z_t^{1-\alpha}}{x_t} + (1 - \theta) \phi_t (\alpha A z_t^{1-\alpha} - \delta) + \theta \delta \frac{x_t}{x_t} - (1 - \phi_t) \left(1 - \frac{1}{\pi_t}\right) + 1
\]

### 6.3 Public capital dynamic

The calculation of the public capital stock growth starts by dividing the public capital dynamic (29) by \( G_t \):

\[
\frac{G_{t+1}}{G_t} = 1 - \delta + i \frac{Y_t}{G_t}
\]

According to the production function, the production/public capital ratio is written as:

\[
\frac{Y_t}{G_t} = A \frac{K_t^\alpha G_t^{1-\alpha}}{G_t}
\]

\[
\frac{Y_t}{G_t} = A \left(\frac{K_t}{G_t}\right)^\alpha
\]

According to (36) we know that \( z_t = \frac{G_t}{K_t} \). The preceding expression then becomes:

\[
\frac{Y_t}{G_t} = A \left(\frac{1}{z_t}\right)^\alpha
\]

Thus the public capital stock growth:

\[
\frac{G_{t+1}}{G_t} = 1 - \delta + A i z_t^{-\alpha}
\]

### 6.4 Private capital growth

We study the private capital accumulation dynamic (33) in order to calculate its growth factor:

\[
K_{t+1} = \frac{\beta (1 - \theta) w_t}{1 + \beta} - (m_{t+1} + B_{t+1})
\]

We replace \( B_{t+1} \) and \( m_{t+1} \) by their respective values given by the accumulation dynamic of public bonds (23) and cash balances (24). We also replace the interest rate\(^{15} r_t \) and the labor cost per unit \( w_t \) by their values given in (44) and (18).

---

\(^{15}\)When studying the steady state, we consider \( r_{t+1} = r_t \)
\[ K_{t+1} = \frac{\beta(1 - \theta)(1 - \alpha)}{1 + \beta} Y_t - (g + i - \theta)Y_t - \theta\delta K_t - (1 - \theta)Br_t - B_t - \frac{m_t}{\pi_t} \]

Then we divide this expression by \( K_t \). During this operation, we realize the following transformations: \( \frac{Y_t}{K_t} = Az_t^{1-\alpha} \), \( \frac{B_t}{K_t} = \frac{B_t}{D_t} \frac{D_t}{K_t} = \phi_t x_t \) and \( r_t = (\alpha Az_t^{1-\alpha} - \delta) \).

We obtain:

\[ \frac{K_{t+1}}{K_t} = \frac{\beta(1 - \theta)(1 - \alpha)}{1 + \beta} Az_t^{1-\alpha} - (g + i - \theta)Az_t^{1-\alpha} - \theta\delta - (1 + (1 - \theta)(\alpha Az_t^{1-\alpha} - \delta)) \phi_t x_t - \frac{(1 - \phi_t)x_t}{\pi_t} \]

### 6.5 Inflation

The cash-in-advance constraint requires:

\[ \mu P_{t+1}c_{v,t+1} = M_{t+1} \]

Dividing both sides by \( P_t \) and introducing variables \( m_{t+1} = \frac{M_{t+1}}{P_t} \) and \( \pi_{t+1} = \frac{P_{t+1}}{P_t} \) we get:

\[ \pi_{t+1} = \frac{m_{t+1}}{\mu c_{v,t+1}} \]

In this expression, we replace \( c_{v,t+1} \) by its value given by the intertemporal maximizing program of the agent shown in (13):

\[ \pi_{t+1} = \frac{m_{t+1}(1 + \beta)(1 - \mu + \mu \pi_{t+1} R_{t+1})}{\mu \beta(1 - \theta)w_t R_{t+1}} \]

Replace labor cost \( w_t \) by its marginal productivity with (18):

\[ \pi_{t+1} = \frac{m_{t+1}(1 + \beta)(1 - \mu + \mu \pi_{t+1} R_{t+1})}{\mu \beta(1 - \theta)(1 - \alpha)Y_t R_{t+1}} \]  

(51)

Divide both the numerator and the denominator of the fraction situated on the right side by \( m_{t+1} \). We then have to express \( \frac{Y_t}{m_{t+1}} \) in terms of \( x_t, z_t \) et \( \phi_t \):

\[ \frac{Y_t}{m_{t+1}} = \frac{Y_t m_t}{m_t m_{t+1}} \]  

(52)

The part \( \frac{Y_t}{m_t} \) can easily be written in terms of \( x_t, z_t \) and \( \phi_t \):
\[
\frac{Y_t}{m_t} = \frac{Y_t}{D_t} \frac{D_t}{m_t} = \frac{Y_t}{K_t} \frac{1}{D_t} \frac{1}{1 - \phi_t} = \frac{A_t^{1-\alpha}}{x_t(1 - \phi_t)}
\]

We denote the growth factor of cash balances shown in (52) as \( \gamma_m = \frac{m_{t+1}}{m_t} \). Incorporating (52) in (51) and developing \( R_{t+1} \) according to (4) we determine the price dynamic:

\[
\pi_{t+1} = \frac{(1 + \beta)(1 - \mu)}{\mu(1 + (1 - \theta)(\alpha A_t^{1-\alpha} - \delta)) \left( \frac{\beta(1-\theta)(1-\alpha) A_t^{1-\alpha}}{x_t(1 - \phi_t)^\gamma} - 1 - \beta \right)}
\]
6.6 Results of Simulations of Stabilization Policies

Table 7: Sustainability obtained by monetization

<table>
<thead>
<tr>
<th></th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>public capital/private capital : ( z^* )</td>
<td>1</td>
</tr>
<tr>
<td>public debt/private capital : ( x^* )</td>
<td>1,4</td>
</tr>
<tr>
<td>Public bonds/total public debt: ( \phi^* )</td>
<td>0,1</td>
</tr>
<tr>
<td>Annual growth</td>
<td>5,1%</td>
</tr>
<tr>
<td>Inflation</td>
<td>-2,2%</td>
</tr>
<tr>
<td>Public deficit (% of GDP)</td>
<td>7,6%</td>
</tr>
<tr>
<td>Primary deficit (% of GDP)</td>
<td>3,1%</td>
</tr>
<tr>
<td>Public bonds (% du PIB)</td>
<td>1,7%</td>
</tr>
<tr>
<td>cash balances/savings</td>
<td>50,2%</td>
</tr>
<tr>
<td>public bonds/savings</td>
<td>7,4%</td>
</tr>
<tr>
<td>private capital/savings</td>
<td>42,4%</td>
</tr>
<tr>
<td>return on public bonds ( R^* )</td>
<td>11,2</td>
</tr>
<tr>
<td>return on money ( \frac{1}{\pi} )</td>
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</tr>
</tbody>
</table>
Table 8: Sustainability reached by reducing non-productive public spendings

<table>
<thead>
<tr>
<th></th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>public capital/private capital : $z^*$</td>
<td>0,54</td>
</tr>
<tr>
<td>public debt/private capital : $x^*$</td>
<td>0,27</td>
</tr>
<tr>
<td>public bonds/public debt: $\phi^*$</td>
<td>0,07</td>
</tr>
<tr>
<td>Annual growth</td>
<td>5,7%</td>
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<tr>
<td>Inflation</td>
<td>-5,3%</td>
</tr>
<tr>
<td>public deficit (% of GDP)</td>
<td>0,35%</td>
</tr>
<tr>
<td>primary deficit (% of GDP)</td>
<td>-0,1%</td>
</tr>
<tr>
<td>public bonds (% of GDP)</td>
<td>0,3%</td>
</tr>
<tr>
<td>cash balances/savings</td>
<td>20%</td>
</tr>
<tr>
<td>public bonds/savings</td>
<td>1,5%</td>
</tr>
<tr>
<td>private capital/savings</td>
<td>78,5%</td>
</tr>
<tr>
<td>return on bonds $R^*$</td>
<td>7,2</td>
</tr>
<tr>
<td>return on money $\frac{1}{\pi}$</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 9: Sustainability attained through public investments reduction

<table>
<thead>
<tr>
<th></th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>public capital/private capital: $z^*$</td>
<td>0,3</td>
</tr>
<tr>
<td>public debt/private capital: $x^*$</td>
<td>0,2</td>
</tr>
<tr>
<td>public bonds/public debt: $\phi^*$</td>
<td>-0,1</td>
</tr>
<tr>
<td>Annual growth</td>
<td>4,3%</td>
</tr>
<tr>
<td>Inflation</td>
<td>-4,2%</td>
</tr>
<tr>
<td>public deficit (% of GDP)</td>
<td>-0,5%</td>
</tr>
<tr>
<td>primary deficit (% of GDP)</td>
<td>0,1%</td>
</tr>
<tr>
<td>public bonds (% of GDP)</td>
<td>-0,1%</td>
</tr>
<tr>
<td>cash balance/savings</td>
<td>18,7%</td>
</tr>
<tr>
<td>public bonds/savings</td>
<td>-2,2%</td>
</tr>
<tr>
<td>private capital/savings</td>
<td>83,5%</td>
</tr>
<tr>
<td>bonds return $R^*$</td>
<td>4,7</td>
</tr>
<tr>
<td>money return $\frac{1}{\pi}$</td>
<td>3,6</td>
</tr>
</tbody>
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Table 10: Sustainability obtained through tax increase

<table>
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<th></th>
<th>steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>public capital/private capital : $z^*$</td>
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</tr>
<tr>
<td>public debt/private capital : $x^*$</td>
<td>0,28</td>
</tr>
<tr>
<td>public bonds/public debt : $\phi^*$</td>
<td>0,08</td>
</tr>
<tr>
<td>annual growth</td>
<td>5,6%</td>
</tr>
<tr>
<td>Inflation</td>
<td>-5,2%</td>
</tr>
<tr>
<td>budgetary deficit (% of GDP)</td>
<td>0,4%</td>
</tr>
<tr>
<td>primary deficit (% of GDP)</td>
<td>-0,2%</td>
</tr>
<tr>
<td>public bonds (% of GDP)</td>
<td>0,4%</td>
</tr>
<tr>
<td>cash balances/savings</td>
<td>20,2%</td>
</tr>
<tr>
<td>public bonds/savings</td>
<td>1,7%</td>
</tr>
<tr>
<td>private capital/savings</td>
<td>78,1%</td>
</tr>
<tr>
<td>return on bonds $R^*$</td>
<td>7,2</td>
</tr>
<tr>
<td>return on money $\pi^*$</td>
<td>5</td>
</tr>
</tbody>
</table>

References


