Social classes, growth and primitive accumulation: some marxian views in a neoclassical framework

par

Jean-François Jacques & Antoine Rebeyrol
Abstract

We show, in contrast with Stiglitz, that an inegalitarian long run equilibrium can emerge in a Solow growth model framework, with linear consumption function. We then interpret this result in line with marxian economics. We extend the results by incorporating some features of the Pasinetti-Samuelson-Modigliani model, and provide an example of possible microfoundations.

JEL : B50, E25, O40

Le maître d’école : Dis-moi donc d’où la fortune de ton père lui est venue ?
L’enfant : Du grand-père.
Le maître d’école : Et à celui-ci ?
L’enfant : Du bisaïeul.
Le maître d’école : Et à ce dernier ?
L’enfant : Il l’a prise.

J.W. Goethe, quoted by Marx in the french edition of Capital.

Present-day neoclassical economic theory attempts at explaining the inequality among agents in the long run. Marxist analysis, whose number one concern is supposed to rest on social inequality, surprisingly has’nt come up yet with anything very significant about the long run, beyond Marx’ insights. The same can be said about classical and neo-ricardian analysis. The aim of this paper is to show that a crude neoclassical approach, such as Solow’s canonical simple...
growth model, with all its oversimplified hypothesis\(^1\), can be accommodated to grasp features relevant to these traditions, and so make the comparison richer between different economic approaches of the social inequality.

This aim is rather paradoxical, since Solow’s model cannot deal with asymptotic wealth inequality. The simplified intuition of this result is plain. In this model, the wages and the profit rate depend on the overall capital stock in the economy and so, with a given average (and marginal) propensity to save, the growth rate of an individual capital stock is inversely correlated with its level. Any inequality across agents vanishes asymptotically\(^2\). This result of convergence across agents, which parallels and slightly differs from the famous analogous one across countries, makes the relevance of this model questionable.

Stiglitz (1969) showed that, with a possible exception, the same result of an asymptotic even distribution of income and wealth is valid if we suppose a linear saving function, such that the marginal propensity to save is constant but the average propensity is increasing with the income and is endogenously determined in equilibrium. Then in contrast, Schlicht (1975) showed that an asymptotic inequality is possible with a strictly convex saving function. In Bourguignon (1981), the same idea repeats a very classical argument to the benefit of social inequality: with a strictly convex saving function the saving capacity and growth possibilities of an economy are boosted by the inequality of income distribution. Even if all agents are otherwise the same, in the long run everybody could benefit from a social inequality between them. As the title of his article formulated it, Bourguignon showed the ”Pareto superiority of unegalitarian equilibria”.

Stiglitz’ exception was indeed very different. What he had in mind was ”the possible exception, in the case of negative savings at zero income, of a group with zero wealth” (1969, p.382). In that case we’ll have no Pareto ranking possibility between egalitarian and unegalitarian equilibria, a group being better off and the other worse off in each situation compared to the other. We are interested in this case, because it gives a kind of indivisibility, a solution of continuity in the behaviour of agents with zero wealth. Nowadays inequality models do generally assume some kind of indivisibility too\(^3\), as a fixed cost for the investment for example. Besides, assuming asymmetric information on the credit market, poor agents who are rationed cannot overcome the indivisible cost; so we get an enduring inequality. But in contrast with this modern treatment of the question, Stiglitz’ exception may generate everlasting inequality without supposing any market imperfection: in a closed economy, without any asymmetric information at all, it is indeed rational to refuse to provide loans to agents with zero wealth who need that for consumption.

\(^1\)There is only one good which may be consumed or used in production, only one kind of labour, perfect technical substitution, factors payments at marginal productivity, etc.

\(^2\)See Jacques et Rebeyrol (2001), where it is also shown that the same result prevails if it is admitted that, as in Kaldor (1955-56), the propensity to save on profits is higher than the one on wages.

We first examine Stiglitz’ model to show that his exception has a much larger scope than he himself thought, by studying his model in a configuration of parameters he has not gone through. His article contains no flaw, but an interesting case is nevertheless neglected, which raises the questions of the genesis of social classes and of the necessary primitive accumulation in the growing process of the capitalist society. We comment this configuration in line with some Marx’ quotation, and then extend the result by considering, as in Pasinetti (1962), a model that distinguishes between two social classes, the workers and the capitalists, the latter being defined by the fact that they earn profits but no wages. In the last section, we show that this kind of results can easily be obtained from crude microfoundations.

1 Stiglitz’ model revisited

We first consider Stiglitz’ model. Let’s assume the following individual consumption function of agent \(h\), the same for all agents:

\[
C_h = \begin{cases} 
(1-s)Y_h + \beta & \text{if } K_h > 0 \text{ or } Y_h \geq \beta/s \\
Y_h & \text{if } K_h = 0 \text{ and } Y_h \leq \beta/s
\end{cases}
\]

\[ \beta > 0, \ s \in ]0,1[, \ K_h \geq 0. \]

\(1\)

The individual income \(Y_h\) is composed of the wage rate \(w\) (everybody works in this model) and of the profit earned on the individual capital \(K_h\) (if this capital is positive). This function is based on the idea that “there is a lower bound on the amount of capital that one can hold (an upper bound on indebtedness)” and that “the lower bound is zero” (Stiglitz, 1969, p.387, note 9). That is, nobody will lend to an agent devoid of any collateral, in order to allow his consumption.

Demographic growth occurs at rate \(n\). Agents are gathered in homogeneous groups with equal division of wealth among heirs. Group \(i\) is in the (constant) proportion \(a_i\) of the total population.\(^5\)

Let \(L\) be the total labour force, \(K_i\) the capital hold by group \(i\), \(k_i\) the ratio \(K_i/L\) and \(k\) the overall capital/labour ratio (of course, \(k = \sum_i k_i\)). The technology is described by a “well-behaved” aggregated production function which is homogeneous of degree one, twice differentiable and which satisfies Inada’s conditions. The average per capita income \(f(k)\) is divided between profits \(kf'(k)\) and wages \(w(k)\).

Group \(i\)'s savings is equal to \(a_i L [sw(k) - \beta] + sK_i f'(k)\). Thus, if the capital depreciate at rate \(\delta\), the dynamics of \(k_i\) is governed by the following equation:

\[
\dot{k}_i = \begin{cases} 
a_i [sw(k) - \beta] + [sf'(k) - (n + \delta)]k_i & \text{if } k_i > 0 \text{ or } w(k) \geq \beta/s \\
0 & \text{if } k_i = 0 \text{ and } w(k) \leq \beta/s
\end{cases}
\]

\[ 2\]

\(^4\)Stiglitz (1969, p.387, note 9) assumed that the savings was zero as soon as the individual capital \(K_h\) was nil. However, it is difficult to see why the savings should be zero if the wages earned are sufficient to ensure that the agent is planning positive savings, that is if \(w \geq \beta/s\).

\(^5\)Hereafter, the subscript \(h\) denotes an agent or a household, whereas the subscript \(i\) refers to an homogeneous group of agents.
while the total capital/labour ratio, at least as long as every $K_i$ is positive, moves according to:

$$\dot{k} = sw(k) - \beta + [sf'(k) - (n + \delta)]k = sf(k) - \beta - (n + \delta)k$$

(3)

Stiglitz assumed that the equation $\dot{k} = 0$ has two real roots. In that case, he showed that there are two long run positive egalitarian equilibria, corresponding to each of these roots. The lower equilibrium is unstable, whereas the higher one is stable. Thus the economy must disappear or converge to the high egalitarian equilibrium, depending on the initial capital/labour ratio. The "possible exception" he considers in a footnote (note 9 on p.387) is that unegalitarian equilibria appears near the lower root, in which there is a "poor group" with zero wealth. This possibility relies on the sign of the third derivative of $f(k)$; it doesn’t exist with a Cobb-Douglas production function.

Consider now, just the opposite of Stiglitz’ assumption, that parameter $\beta$ is great enough to prevent the existence of any positive egalitarian equilibrium, that is, the equation $\dot{k} = 0$ has no real roots. As long as any $K_i$ has not fallen to zero, the overall capital/labour ratio is continually decreasing. A possible phase diagram is given in Fig (1) in the case of two groups only.

The equation of the locus $\dot{k}_i = 0$ is $k_i = a_i \left( \frac{sw(k) - \beta}{n + \delta - sf'(k)} \right)$. The left branch of its graph, before its asymptote at $k_a = f^{-1}(\frac{n+\delta}{\delta})$, may lie entirely over the first bisector\(^6\), but we can always choose $a_i$ small enough to ensure that it crosses this bisector twice, as shown on Fig. (1). We define $k_r = w^{-1}(\frac{\beta}{\delta})$. Notice that

\(^6\)If this is the case for every group $i$, then the economy must definitely vanish.
Thus, on the left area of the diagram, the wage rate (which increases with $k$) is too small to assure positive savings from agents deprived of any capital (and profit).

This diagram can illustrate the problems of primitive economies whose productivity is small enough to threaten the existence. The economy can indeed disappear along trajectories that converge towards the origin. But it may happen that a group, if it initially holds enough of the total capital, sees its part of the total wealth growing to unity. If a (small !) group succeeds in holding enough capital, no matter how, it can find itself in a position to acquire all the capital. If this happens, as it does in the figure (1) at point B, at the close of a trajectory where the other group happens to be deprived of any capital, the dynamics of the overall capital/labour ratio will be changed. As $k_i = k$, the dynamics of $k$ is no more described by equation (3) but by equation (2)\(^8\).

The consumption of other agents, who earn a wage rate less than $\beta/s$ and are at this moment deprived of any capital, drop in a discontinuous manner. They have to tighten one's belt. This enhances the overall growth: the poverty felt by the other group diminishes the total consumption in a proportion sufficient to avoid the vanishing of the economy. Afterwards, the economy converges towards point A, which is an unegalitarian long run equilibrium. In this long run equilibrium, group $i$ holds all the existing capital whereas agents in the other group are "poor", not only because they don't get any capital at all, but also because they cannot consume as much as they would like to do at their level of wages income. They are constrained to consume no more than what they earn, despite the so-called "incompressible consumption level" $\beta$. As we said, this fact doesn't result from any market imperfection nor asymmetric information. In a closed economy, they couldn't consume more than their real wages without diminishing the overall capital/labour ratio, and this under duress level of consumption is the condition of the maintenance of the capital/labour ratio.

2 Marx and primitive accumulation

In the preceding model, inequality, even if it can be called violent and unfair (who belongs initially to the "rich" group? how and why?), is in no way arbitrary because the question of the extent of previous accumulation\(^9\), necessary to avoid the vanishing of the economy, cannot be separated from the question of the distribution of wealth.

At this point, it is tempting to consider that the agents are now gathered not only in two "groups", but that capitalist society social classes have emerged.

\(^7\)The hypothesis that the equation (3) admit no real roots implies that whatever $k$, $k < 0$.

Hence at $k_a$, as $sf'(k_a) - (n + \delta) = 0$ by definition, the equation (3) gives $k_a = sw(k_a) - \beta < 0$.

Because $sw(k_a) - \beta = 0$, we can conclude that $k_r > k_a$.

\(^8\)Refering to equations (2) and (3), remember that $0 < a_i < 1$ and that in the relevant area, $sw(k) - \beta < 0$ and $sf'(k) - (n + \delta) > 0$.

\(^9\)"The accumulation of stock must, in the nature of things, be previous to the division of labour." (Smith, 1876, p.292).
endogenously, through the appearance of a class of workers short of any means of production. In Karl Marx’ view, the “capital” is not primarily a stock of goods, but a social relationship which is historically determined:

“[The capital] can spring into life, only when the owner of the means of production and subsistence meets in the market with the free labourer selling his labour-power. And this one historical condition comprises a world’s history.” (1867, p.274)

As it is well known, for him the value of the labour-power must assure its physiological reproduction (which gives a minimum level), but also the reproduction of the social class of workers deprived of means of production (which gives a maximum socially determined level, beyond which workers would save and accumulate).

Marx thought that a capitalist society calls for a primitive accumulation, that is ”an accumulation not the result of the capitalistic mode of production, but its starting point” (chap.26), which has taken place through violence:

“In actual history it is notorious that conquest, enslavement, robbery, murder, briefly force, play the great part. (...) The methods of primitive accumulation are anything but idyllic.” (1867, p.874)

“Force is the midwife of every old society pregnant with a new one. It is itself an economic power.” (1867, p.916)

The historical process which results in a capitalist society is nevertheless not only an economic but also a political one:

“The immediate producer, the labourer, could only dispose of his own person after he had ceased to be attached to the soil and ceased to be the slaver, serf, or bondsman of another. (...) Hence, the historical movement which changes the producers into wage-workers, appears, on the one hand, as their emancipation from serfdom and from the fetters of the guilds, and this side alone exists for our bourgeois historians. But, on the other hand, these new freedmen became sellers of themselves only after they had been robbed of all their own means of production, and of all the guarantees of existence afforded by the old feudal arrangements. And the history of this, their expropriation, is written in the annals of mankind in letters of blood and fire.” (1867, p.875)

Of course, our modelisation cannot describe a precapitalist mode of production: in here, perfect labour and capital markets are presupposed from the beginning! We think nevertheless that it can partially grasp the idea that a violent process which results in the appearance of a class of workers, deprived of any means of subsistence apart from their labour-power, is a condition for the stability of a capitalist society, in which workers earn wages such that they live without being able to save and to accumulate any capital.

\[10\text{In terms of the consumption function (1), this maximal level is of course } \beta/s.\]
3 The model with non-worker-capitalists

We now turn to two technical limits of our representation of Marx’ insights. First of all, our description seems limited to poor and primitive economies (because we have rejected by assumption the Stiglitz’ hypothesis of the existence of a high egalitarian equilibrium). Second, in Marx’ analysis, capitalist agents do not work at all, they earn only profits, whereas, in the model of our first part, they earn wages too.

The idea that the capitalist class is defined by the ownership of capital and profits without working nor earning wages has been worked out by Pasinetti (1962) and Samuelson & Modigliani (1966). Pasinetti wanted to study not so much the ”functional distribution” between wages and profits but the ”social distribution” between workers and capitalists. In the ”Pasinetti case” prevails a long run inequalitarian equilibrium. But the mechanism at work is very different from the one contemplated by Stiglitz and in part I. In Pasinetti’s model, capitalists and workers have different (average and marginal) exogenous propensities to save, and the possibility of an unegalitarian long run equilibrium is generated by a high enough ratio of their respective savings propensities. Thus it illustrates the ideological and moralizing idylls that Marx gave as ”insipid childishness”. On the other side, in the Pasinetti’s long run equilibrium, if capitalists don’t work, workers nevertheless own a fraction of the capital and the profits. Marx’ idea that the worker ”is short of everything necessary for the realisation of his labour-power” is completely missed.

In order to avoid ”insipid childishness”, we now propose to introduce Pasinetti’s class definition (capitalists earn only profits whereas workers earn wages and profit if they get some capital) in the very Stiglitz’ model framework (each agent, whatever his class, will consume according to the very same equation (1)). Consider two groups of homogeneous agents, workers and capitalists. There are $L$ workers (denoted by the subscript $\ell$) and $M$ capitalists (denoted by the subscript $c$). Each of these numbers grow at rate $n$ through time and keeps the proportion $\theta = M/L$ constant. With $k_\ell = \frac{K_\ell}{L}$, $k_c = \frac{K_c}{L}$ and $k = \frac{K_\ell + K_c}{L}$, the dynamics of these ratios is given by the following equations:

$$
\dot{k}_\ell = \begin{cases} 
sw(k) - \beta + [sf'(k) - (n + \delta)] k_\ell & \text{if } k_\ell > 0 \text{ or } w(k) \geq \beta/s \\
0 & \text{if } k_\ell = 0 \text{ and } w(k) \leq \beta/s 
\end{cases}
$$

11 For a presentation of this model with a phase diagram, see Jacques & Rebeyrol (2001).

12 “This primitive accumulation plays in Political Economy about the same part as original sin in theology. Adam bit the apple, and thereupon sin fell on the human race. Its origin is supposed to be explained when it is told as an anecdote of the past. In times long gone-by there were two sorts of people; one, the diligent, intelligent, and, above all, frugal elite; the other, lazy rascals, spending their substance, and more, in riotous living. (...) Thus it came to pass that the former sort accumulated wealth, and the latter sort had at last nothing to sell except their own skins. And from this original sin dates the poverty of the great majority that, despite all its labor, has up to now nothing to sell but itself, and the wealth of the few that increases constantly although they have long ceased to work. Such insipid childishness is every day preached to us in the defence of property.” (chap.26)

13 Of course, Pasinetti’s aim was not to interpret Marx.
We assume that the equation \( sf(k) - (1 + \theta) \beta - (n + \delta)k = 0 \) has two distinct real roots, warranting the existence of a stable high equilibrium. A possible phase diagram, in the plane \((k, k_c)\), is drawn on figure (2) (a simple numerical example is given in footnote 18 below).

If the economy begins with an overall capital/labour ratio higher than \( k_3 \), it will converge towards a high equilibrium at \( k_4 \): workers acquire the capital and pure capitalist agents disappear asymptotically, along trajectories like the one denoted (1)\(^{14}\). On the other extreme of the spectrum, if the economy begins with \( k < k_1 \), it must disappear completely. The range between \( k_1 \) and \( k_3 \) raises the question of primitive accumulation. As in the model of part I some trajectories, like the one denoted (2) on figure (??), make the overall capital/labour ratio shrink to zero. But if pure capitalists agents own (by violent expropriation or any other means) a greater part of the total capital, as on the trajectory noted (3), workers will finally be completely deprived of any property of capital. At

\(^{14}\)This illustrates the walrasian dream of the extinction of social classes by every workers’ access to capital property.
this moment, workers earn less than \( \beta/s \) and must accordingly reduce discontinuously their consumption (indeed the point B, on the first bisector, shows a coordinate on the \( k \)-axis which is the level \( k_r \) such that \( w(k_r) = \beta/s \)\(^{15}\)). The dynamics of the overall capital/labor ratio is now given by \( \dot{k} = skf(k) - (n + \delta)k - \theta \beta \) (drawn as the dotted line on the left of figure (??)). Hence afterwards, the global accumulation is revived and the economy converges to a long run equilibrium of a "marxist type" at point A.

Of course, in this situation, every one is free to think that future technical progress might shift the \( k_c = 0 \) curve to the right, shifting the point A towards B in such a manner that workers begin to save and induce, after the vanishing of the "marxist type equilibrium", a virtuous dynamics towards a higher long run equilibrium. From a marxist point of view, it might nevertheless be recalled that the psychological parameter \( \beta \) may well increase with historical progress: the "value of the labor-power" is not physiologically but historically determined.

4 A numerical example with choice between leisure and work

Are there inequalities which are not the outcome of idiosyncratic features?\(^{16}\)

This is not a matter of fact. Idiosyncrasies do exist, but authors like Marx and Walras clearly think that individual differences are not meaningful to highlight the great social questions. If this very same idea has to be taken seriously, if the question of interest is the normative one of whether a society, by itself, can generate inequality among members, then it is important to exhibit schemes where all agents are completely alike, apart from the inheritance they received in a determined social framework. This is the reason why it seems more satisfactory to assign to everybody the same demographic and saving behaviour, as we do in this paper (it is not only a formal convenience). Nevertheless, in the model of part II, we don’t explain why capitalist agents do no work.

Now let’s turn to a neoclassical modelling where preferences, including the propensity to get leisure, are made explicit and completely alike across agents, so that the effects of a social inequal distribution of wealth can be analysed per se. We use crude "microeconomic foundations" and only give a numerical example, to show that "marxist type" equilibria can arise easily.

Suppose a model in which, in discrete time, agents live one period only. They maximize a static utility, which is function of their consumption \( C \), their labor \( L \) and the inheritance \( K_{t+1} \) they will bequeath to their children\(^{17}\). The utility

\(^{15}\)This can be seen because the two curves in dotted line meet at this level, which implies that \( skf'(k) = sf(k) - \beta \), or equivalently that \( sw(k) = \beta \).

\(^{16}\) "Allons au fait. Y a-t-il, dans notre société, d’autre misère que celle qui résulte logiquement de la paresse, de l’inintelligence ou des revers de la fortune?" Walras (1860, p.iii).

\(^{17}\) No dynamic optimization is involved because in this crude treatment, we admit that the inheritance of the children is directly useful to the parents, without working out the offspring’s
function is supposed to have the following form (where $a_1$, $a_2$, $a_3$, $\overline{C}$ and $\overline{K}$ are all positive parameters):

$$U(C, K_{t+1}, L) = a_1 \ln(C - \overline{C}) + a_2 \ln(K_{t+1} + \overline{K}) + a_3 \ln(2 - L) \tag{4}$$

First, consider that the individual income $Y_h$ is exogeneous. Then, maximizing (4) subject to $C_h + K_{h+1} \leq Y_h$, $C_h > \overline{C}$ and $K_{h+1} \geq 0$ yields to the following solution, if any, in $C_h$ and $K_{h+1}$:

$$\begin{aligned}
C_h &= (1 - s)Y_h + \beta \quad \text{(and } K_{h+1} = sY_h - \beta) \quad \text{if } Y_h \geq \beta/s \\
C_h &= Y_h \quad \text{(and } K_{h+1} = 0) \quad \text{if } \overline{C} < Y_h \leq \beta/s \tag{5}
\end{aligned}$$

where $s = \frac{a_2}{a_1 + a_2}$ lies between 0 and 1 and $\beta = \frac{a_1 \overline{K} + a_3 \overline{C}}{a_1 + a_2}$ is positive. Notice that $\beta/s = \overline{C} + \frac{a_1 \overline{K}}{a_2} > \overline{C}$. When $Y_h \geq \beta/s$, the agent wants a positive saving but when $\overline{C} < Y_h \leq \beta/s$, the constraint $K_{h+1} \geq 0$ is binding. In that case we have $K_{h+1} = 0$ and $C_h = Y_h$. The problem has no solution if $Y_h < \overline{C}$ (the parameter $\overline{C}$ can be viewed as a physiological subsistance level of consumption, so that it lies on the boundary of the consumption set). This solution (5) is clearly a discrete analogue of function (1).

Now the income of the agent is the sum of his wages if he works and the return from the capital $K_h$, inherited from his parents. This capital is used in production where it depreciates at rate $\delta = 1$. Its net return is then $r = f'(k) - 1$. We have:

$$Y_h = wL_h + (1 + r)K_h \tag{6}$$

We assume work hours to be indivisible: they can take only two values, 0 and 1. To solve the program the agent, using (5) and (6), compares his utility when working (with $L_h = 1$) and when enjoying leisure ($L_h = 0$).

This simple model can generate various dynamics. We are interested in situations where we have two homogeneous classes, the workers deprived of any capital and who do not save on the one hand, capitalists who earned only profits on the other hand. If there are, in such a situation, $L$ workers and $M$ capitalists, each of these numbers growing at rate $n$ (with $\theta = M/L$), the dynamics of the overall capital/labour ratio reflects only capitalists’ behaviour. It is given by:

$$(1 + n)k_{t+1} = skf'(k) - \theta \beta$$

Let’s exhibit a simple numerical example. Suppose the production function to be $y = \sqrt{K}$. Take the values $\overline{C} = 0.01$, $\overline{K} = 0.135$, $a_1 = 0.8$, $a_2 = 1.2$ (consequently $s = 0.6$ and $\beta = 0.06$), $a_3 = 3$, $n = 0$, $\delta = 1$. Now suppose there are $M$ agents who each owns a capital of 0.10177, and $L$ agents who get no capital at all, no other commodity for sale than their labour-power, with $\theta = 0.35$ 18. Then $K/L = 0.035619$, which is the highest, locally stable, of the two stationary solutions of the preceding difference equation (it will

 utilities.

18 This numerical example ($y = \sqrt{K}$, $s = 0.6$, $\beta = 0.06$, $n = 0$, $\delta = 1$) can be used to exemplify the configuration of parameters behind part II figure (??).
be denoted \( k^* \). We still have to check that in this situation, workers are indeed willing to work and willing to have zero savings, whereas capitalists choose leisure, the preferences of every one of them being given by the utility function (4).

With \( w(k^*) = 0.094365 \) (which is the amount of the wage rate if capitalists indeed do not work), workers deprived of any other source of income cannot save (because \( w(k^*) < \beta/s = 0.1 \)). If capitalists decided to work, the wage rate would be still lower (because the overall capital/labour would shrink), and the constraint still more binding. The \( L \) agents of course choose to work, because for if not their income will be nil and will not assure the survival level \( \overline{C} \).

As for the capitalists now, they each get an individual capital of \( \frac{K}{L} = 0.10177 \) and, with \( f'(k^*) = 2.6493 \), earn individual profits of the amount 0.26962. Without working each will consume 0.16785 and save 0.10177 (which will just assure the reproduction of capital, no more no less), reaching a level of utility of \(-1.1263\). What happens if he works ? Each of them might act as a competitive price taker, taking \( w \) and \( r \) as given. In that case, he will think that working gives him a global income of 0.36399, which will allow him to consume 0.2056 and save 0.15839, reaching a level of utility of \(-2.7768 \) only, because of the disutility of working\(^{19}\). So, he will indeed choose not to work. If, on the contrary, the agent acts strategically, he will consider that other capitalists could work too, which will consequently affect the prices. In that case, the incentive to work will be still less, because the wage rate will lower and the profit rate will rise\(^{20}\). In any case we can conclude that the situation is a long run inegalitarian equilibrium of a ” marxist type”, between absolutely identical agents, apart from their capital dotation.

In this economy, what happens if the existing capital is equally redistributed among the \( L + M \) agents? Every one now gets an individual capital of 0.026384. Consider only symmetrical solutions. If nobody works, there is no production and each one will consume his entire capital of 0.026384, which is between \( \overline{C} \) and \( \beta/s \), enjoying a level of utility equal to \(-3.6127 \). Afterwards the economy disappears (children all deprived of capital cannot secure the subsistence). On the other side, if every one works, the individual income is 0.16243. Every

\(^{19}\) In this example, the capitalist will choose leisure as soon as its weight in the utility function, \( a_3 \), is greater than 0.61877.

\(^{20}\) More generally, a capitalist \( h \) whose profits are greater than \( \beta/s \) will have an incentive to work if and only if

\[
 w > \left( \frac{2^{\frac{-a_3}{-(1-s)}}}{1 + s} - 1 \right) \left[ (1 + r)K_h + \overline{C} - \overline{C} \right].
\]

Higher \( r \) and smaller \( w \) both discourage working.

To prove it, denote by \( \Delta v \) the difference between the utility when working and when enjoying leisure. Such a capitalist will compute:

\[
 \Delta v = a_1 \ln \frac{w + (1 + r)K_h + \beta + \overline{C}}{(1 - s)(1 + r)K_h + \beta + \overline{C}} + a_2 \ln \frac{s(w + (1 + r)K_h) - \beta + \overline{C}}{s(1 + r)K_h - \beta + \overline{C}} - a_3 \ln 2
\]

Notice that \( \beta - \overline{C} = (1 - s)(\overline{K} - \overline{C}) \) and \( -\beta + \overline{K} = s(\overline{K} - \overline{C}) \), consequently:

\[
 \Delta v = (a_1 + a_2) \ln \frac{(1 + r)K_h + \overline{K} - \overline{C}}{(1 + r)K_h + \overline{K} - \overline{C}} - a_3 \ln 2
\]

The agent will work if \( \Delta v > 0 \), that is if

\[
 \frac{w + (1 + r)K_h + \overline{C} - \overline{C}}{s(1 + r)K_h + \overline{K} - \overline{C}} > 2^{\frac{-a_3}{-(1-s)}} - 1
\]

or equivalently if

\[
 w > \left( \frac{2^{\frac{-a_3}{-(1-s)}}}{1 + s} - 1 \right) \left[ (1 + r)K_h + \overline{C} - \overline{C} \right].
\]
one consumes 0.12497 and saves 0.037458, reaching a utility level of -3.8396 only, which is less than in the case of leisure. Hence the example illustrates the primitive accumulation problem: an egalitarian economy disappears while the concentration of capital among $M$ capitalists allows the stationary path analysed above. Now suppose the preference for the leisure to be less strong, say, with $a_3 = 1$ instead of 3. None of the preceding results will be affected, except that the agents of the egalitarian economy will now choose to work (reaching a utility level of -3.8396 instead of -4.9989 if they choose leisure), saving more (0.037458) than they have received (0.026384). The economy will converge towards a higher level of capital. This time the example illustrates the case of a growth path enhanced, beyond the "marxist type" equilibrium, by an egalitarian distribution of wealth.

In contrast with the recent tradition which puts emphasis on the interaction between indivisibilities and imperfections of the credit market to explain social inequality, we show, by elaborating on Stiglitz’ model (1969), that workers can be maintained in a poverty trap where they neither save nor consume more than their wage income, without market imperfections. This can be connected to the classical and marxian tradition of a wage rate that allows the workers to survive but not to accumulate capital. The modelisation has grasped the idea of a process of primitive accumulation, through which social classes and inequality emerge endogeneously, without supposing any difference at all in individual characteristics.

5 Bibliography


Marx, K. (1867), Capital, .


21 You can consider any value of $a_3$ between 0.61877 and 2.6726.
22 The dynamics of the capital/labour ratio will be given by $(1 + n)k_{t+1} = sf(k) - \beta$. 


Smith, A. (1776) : *The wealth of nations : An inquiry into nature and causes*.