Partial Yardstick Regulation and Collusion*

Cécile Aubert† and Jérôme Pouyet‡

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Abstract

Entrants may provide information to a regulator, even when they cannot be regulated. With correlated costs, yardstick-like regulatory contracts based on the output of unregulated firms nullify information rents. But they give strong incentives to the regulated incumbent to bribe competitors and have them hide information.

We consider a regulated Stackelberg leader and an unregulated follower, that have correlated costs and may collude. We show that, although regulation is partial, collusion-proofness can still be obtained, but is more costly. When offering collusion-proof contracts, the regulator cannot benefit from the asymmetric information between firms, contrary to the case of complete regulation. Moreover, the regulator can no longer use the information provided by the competitor’s behavior to obtain efficiency.

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†EURIsCO, Université Paris IX Dauphine. Address: Place du Maréchal de Lattre de Tassigny, 75 016 Paris, France. E-mail: Cecile.Aubert@dauphine.fr
‡Department of Economics, Ecole Polytechnique, 91128 Palaiseau Cedex, France. E-mail: Jerome.Pouyet@polytechnique.edu
1 Introduction

One of the major benefits of competition is that it helps solve information problems. In particular, yardstick competition can be quite effective: a regulator uses the information provided by all operating firms in his contract with a particular firm\(^1\). This type of competition has been increasingly used since entry has been allowed in regulated industries; for instance, OFWAT, the UK water regulator, uses yardstick mechanisms for water and sewage services across the country. We stress here that this type of regulation gives rise to strong stakes of collusion. Collusion will strongly reduce the ability of the regulator to use information from competitors when regulation is “incomplete” or “partial”, i.e. when the regulator cannot control all the relevant market participants.

Regulation of public utilities has long been studied in the context of monopolies. Yet since the major wave of deregulation began in the 1980s in the US and Europe, and was followed by Developing Countries, new market structures have emerged. The introduction of competition in regulated industries requires a modification of the standard paradigm of regulation. The asymmetry in regulatory practices appears crucial, although it has received little attention: entrants are typically much less regulated than incumbents in practice. In addition, regulators are often constrained by institutional dispositions to regulate only specific sectors, or even only specific industries. If competing firms produce goods that are sufficiently weak substitutes, one may be subject to regulation, and not the other.

Although theoretical analyses are lacking, this situation is well documented. The example of telecommunications is particularly striking.\(^2\) The incumbent is usually heavily regulated, especially regarding interconnection to its copper wire network and to final prices, contrary to entrants, particularly on the mobile segment. More striking, Internet telephony is not subject to regulation. Yet, market interactions between the regulated incumbent and entrants, and between providers of mobile, Internet, and fixed telephony, are important. The incumbent in

\(^1\)See Shleifer, 1985, for a seminal article on yardstick with moral hazard.

\(^2\)Sharkey (1994), among others, underlines the discrepancies in regulation that emerge, due to changing technologies and the deregulation of certain portions of the global communications network (p. 34). He also stresses the role of markets internationalization in leading to asymmetric regulation.
the Postal Service also tends to be more regulated than competitors on parcel delivery or express mail, and its ability to choose prices is restricted — geographical price discrimination is generally forbidden for letter delivery.

Collusion in regulated industries is also a relevant concern. Illustrations are given by recent decisions of the French competition authority, the Conseil de la Concurrence, with regard to cartels in mobile telephony (November 2005), in public transportation (July 2005) or in public road construction (December 2005).

We consider competition between a regulated firm and an unregulated competitor, with correlated marginal costs and imperfectly substitutable goods. We assume that the regulator is a Stackelberg leader of the game (which corresponds to the case of a regulated incumbent) and must choose the quantity produced by the regulated firm before the unregulated competitor undertakes its production decision. However, the regulator can condition the transfers paid to the regulated firm, on the quantity produced by the unregulated competitor. The ex post observation of the unregulated firm’s quantity enables to obtain a signal on the cost parameter of this firm. If costs are correlated, this signal can be used ex ante to improve the efficiency of the regulation through the use of yardstick competition contracts, an ‘information transmission’ effect. In this context, stakes of collusion arise.

The stakes of collusion in the context we study here stem, not only from the ‘information transmission’ effect, but also from a ‘market competition’ effect. This second effect is isolated in Aubert and Pouyet (2004), in which the type of the unregulated firm is known by all economic agents (the ‘information transmission’ effect therefore does not arise). A regulated inefficient firm always produces a lower quantity than an efficient one, thereby enabling the unregulated firm to make higher profits. The latter is therefore willing to bribe the regulated one, in order to have it always pretend being inefficient. The regulator cannot directly affect the profit of the unregulated firm (and therefore the maximum possible bribe), and can only use, as indirect tools, the production levels of the regulated firm. Fighting collusion therefore forces the regulator to distort the production of both an efficient and an inefficient regulated firm. But non collusion-proof contracts can, in this context and if the goods are sufficiently substitutable, improve on collusion-proof ones. They indeed allow the regulator to tax unregulated profits, by making the
unregulated firm pay for the revelation of the firm’s inefficiency.

In this paper, on the other hand, both firms have private information on their type. The ‘information transmission’ effect then arises. The quantity produced by the unregulated competitor constitutes an informative signal on the type of the regulated firm, and the regulator can use transfers contingent on the ex post observation of this quantity. A yardstick-like mechanism allows to totally extract the information rent of the regulated firm. As a consequence, the regulated firm would like to collude with the unregulated competitor to counter the effects of yardstick competition.

In the last part of the paper, we look at the way in which both stakes of collusion interact. We model collusion along the lines of Laffont and Martimort (2000), that is, by explicitly taking into account the asymmetries of information at the level of the coalition. In order to better understand the consequences of incompleteness in the regulation, we compare our results to the work of Laffont and Martimort (2000), who study collusion in a centralized organization. In their setting, a Weak Collusion-Proofness Principle applies, according to which there is no loss of generality in restricting the regulator to use collusion-proof contracts. Moreover, informational asymmetries at the level of the coalition may prevent the implementation of an efficient collusive agreement, and they show that the regulator can affect directly this inefficiency in the collusive agreement.

Due to regulation incompleteness, we are unable to prove the Collusion-Proofness Principle. In addition, focusing on collusion-proof regulatory contracts, this regulation incompleteness makes the regulator unable to benefit from the asymmetries of information between colluding firms. This model of incomplete regulation gives an example in which ill-designed institutions do not succeed in fighting efficiently collusion. The cost of ensuring collusion-proofness with asymmetric regulation stresses the necessity of carefully adapting the regulatory structure before opening markets to competition, as recommended by Laffont (1998).

**Relevant literature** This paper uses the literature on mechanism design in a correlated environment. Following Myerson (1981), Crémer and McLean (1988) have shown (in an auction setting) that when firms’ private information parameters are correlated, the first-best allocation becomes optimally implementable. Our approach is closer to Riordan and Sappington
(1988), who show that the ex post observation of a correlated signal enables a regulator to offer contingent contracts to a firm, and find conditions under which the first-best allocation is implementable despite the informational advantage of the firm. A difference with this paper is that the ex post signal we consider is not exogenous since it depends on the regulatory contract offered in the first place.

Finally, we extensively use the work on coalition formation by Laffont and Martimort (1997, 2000), adapting it to imperfectly differentiated products, to study collusion with consistent assumptions on information asymmetries. Tangerás (2002) also studies collusion-proofness with yardstick mechanisms, in a slightly different context as colluding firms have to commit to a collusive strategy ex ante, before they know their types. Conversely, we assume that the firms are informed at the time they enter a collusive agreement, so that our analysis is closer to Laffont and Martimort (1997, 2000). These papers all consider complete regulation, contrary to ours.

The paper is organized as follows. The model is presented in Section 2. Section 3 derives the benchmark of regulation under perfect information. Section 4 shows how the regulator can infer information from the quantity produced by the unregulated firm. We then consider collusion under asymmetric information between firms in Section 5 and derive in Section 6 the equilibrium outcome with collusion-proof contracts. Concluding remarks are gathered in Section 7. All the proofs are in the Appendix.

2 The model

The consumers. Consumers can buy two differentiated goods, denoted by $a$ and $b$, produced respectively by firm $F^a$ and firm $F^b$. Gross consumers surplus for respective quantities $q^a$ and $q^b$ is quadratic and given by

$$GS(q^a, q^b) = d^aq^a + d^bq^b - \frac{1}{2}(q^a)^2 - \frac{1}{2}(q^b)^2 - sq^aq^b.$$  

Parameter $s \in (0, 1)$ measures the degree of substitutability between the two goods whereas $d^i$ is the size of market for good $i$. The inverse demand functions for goods $a$ and $b$ are therefore
given respectively by
\[
\begin{align*}
P_a(q^a, q^b) &= d^a - q^a - sq^b, \\
P_b(q^a, q^b) &= d^b - q^b - sq^a.
\end{align*}
\]

The firms  Competition takes place in quantities\(^3\). The quantities produced by the firms are common knowledge, but not their costs. Firm \(F^i\) \((i = a, b)\) has a constant marginal cost \(\theta^i\) that is private information. It is common knowledge that \((\theta^a, \theta^b)\) takes its value in \(\Theta^2 = \{\theta, \theta\}^2\) according to the symmetric joint probability distribution \(p(.)\). We use the following notations:

\(p_{11} = p(\theta^a = \theta, \theta^b = \theta), p_{12} = p(\theta^a = \theta, \theta^b = \bar{\theta}) = p_{21} = p(\theta^a = \bar{\theta}, \theta^b = \theta)\) and \(p_{22} = p(\theta^a = \bar{\theta}, \theta^b = \bar{\theta})\). The degree of correlation is then defined by \(\rho = p_{11}p_{22} - p_{12}p_{21}\). We assume \(\rho > 0\).

The regulator  Firm \(F^a\) —the ‘incumbent’— is regulated by regulator \(R^a\), while firm \(F^b\) is left unregulated. The regulatory contract specifies the production of some quantity \(q^a\) together with a transfer \(t^a\) to the regulator. For a given contract \(\{q^a, t^a\}\), total ex post profits for \(F^a\) and \(F^b\) respectively are equal to

\[
\begin{align*}
\pi^a &= [P^a(q^a, q^b) - \theta^a]q^a - t^a, \\
\pi^b &= [P^b(q^a, q^b) - \theta^b]q^b.
\end{align*}
\]

The regulator cannot contract with the entrant \(F^b\). The analysis could be extended in a straightforward way to fixed lump-sum taxes\(^5\), or to taxes proportional to either profits or output\(^6\), but non linear taxation is ruled out.

\(^3\)Our framework could be extended to the case of Bertrand competition. Under Bertrand competition, only the most efficient firm produces, and sells at a price just below the marginal cost of its competitor. When there is no collusion, observing a strictly positive production by the unregulated firm, and its price, allows to infer the marginal cost of the regulated firm. As in our framework, the behavior of the unregulated firm constitutes an informative signal on the type of the regulated one. Yet, the process of Bertrand competition, which is usually interpreted as a tâtonnement process, becomes rather unrealistic when one of the firms is subject to an \textit{ex ante} regulation which restricts its capacity to react to market conditions.

\(^4\)We use similar notations for ex post profits and transfers: subscript ‘11’ means that \((\theta^a = \bar{\theta}, \theta^b = \bar{\theta}),\) subscript ‘12’ means that \((\theta^a = \theta, \theta^b = \bar{\theta})\), subscript ‘21’ means that \((\theta^a = \bar{\theta}, \theta^b = \theta)\) and subscript ‘22’ means that \((\theta^a = \theta, \theta^b = \theta)\).

\(^5\)Shut-down of \(F^b\) would occur more often.

\(^6\)A tax proportional to quantity \(q^b\) could be assimilated to a decrease in \(d^b\) or an increase in \(\theta^b\).
For simplicity, we assume that the firms’ profits do not enter the social welfare function of the regulator\(^7\), so that this objective is reduced to net consumers surplus plus the transfer paid by the regulated firm. Hence, the rents left to the regulated firm are socially costly for the regulator.

Social welfare can therefore be written as

\[
SW^a = GS(q^a, q^b) - \theta^aq^a - P^b(q^a, q^b)q^b - \pi^a.
\]

**The timing**  The timing without collusion is the following.

1. Nature draws the cost parameter of each firm, \(\theta^i\) (\(i = a, b\)), which is then privately learned by this firm only.

2. Regulator \(R^a\) proposes a contract \(\{q^a(\cdot), t^a(\cdot)\}\) to the regulated firm \(F^a\).

3. The regulated firm decides whether to accept or reject this contract. In case of refusal, it gets a reservation gain, exogenously normalized to 0. If it accepts, it sends a message to the regulator and produces the corresponding quantity.

4. The unregulated firm chooses its production after having observed the regulatory contract offered to \(F^a\), and the quantity the latter produces.

5. The regulated firm pays the specified transfer.

The possibility that firms collude will be added to this timing in Section 5.

The regulator can condition the tax paid by the regulated firm to the quantity produced by the unregulated firm, \(q^b\) (\(R^a\) can indeed wait until \(q^b\) is produced to tax the regulated firm). But given the timing, it cannot condition the quantity produced, \(q^a\), on \(q^b\): Firm \(F^a\) has to produce its quantity before the unregulated firm undertakes its production decision.\(^8\)

\(^7\)The results can be extended to positive weights on the sum of firms’ profits. For rent extraction to be desirable, the weight on the regulated firm’s profit must be strictly smaller than 1.

\(^8\)If the regulator had been able to also condition \(q^a\) on \(q^b\), he could easily have implemented the socially optimal quantity, under complete information. Consider indeed the following quantity scheme: "Produce the socially optimal quantity if firm \(F^b\) adopts a perfectly competitive behavior on its market; or, ‘flood’ the market..."
The unregulated competitor chooses its quantity after the incumbent $F^a$. Simultaneous competition would indeed be less compatible with the regulatory process — and with the incumbent-entrant scenario often prevailing when regulation is incomplete.  

3 Complete information

Let us assume in this section that the firms’ efficiency parameters $\theta^a$ and $\theta^b$ are known to all economic actors.

- **The unregulated firm’s output decision**

The best response of the unregulated firm $F^b$ to the observation of the quantity $q^a(\theta^a)$ selected by $F^a$ is given by:

$$q^b(\theta^b, q^a(\theta^a)) = \frac{1}{2}[d^b - \theta^b - sq^a(\theta^a)].$$

Due to product substitutability, the unregulated quantity decreases in the regulated one.

- **The optimal quantity with complete information and incomplete regulation**

The socially optimal quantities are the ones the regulator would ideally choose, but they are not achievable unless there is complete information and complete regulation, i.e. regulation of ($q^a \to +\infty$) if firm $F^b$ tries to exploit its market power. If regulator $R^a$ can commit to such a scheme, firm $F^b$ always prefers to behave competitively and, in equilibrium, the socially optimal allocation is implemented. Such an equilibrium is however not subgame perfect. The timing we adopt better reflects, in our opinion, the type of sequential competition that may occur between incumbents and entrants in regulated industries.

\(^9\)We would nevertheless obtain similar qualitative results with the following timing: The unregulated firm chooses its output $q^b(\theta^b)$, while simultaneously, the regulator offers a contract and the firm responds to it, which yields a quantity $q^a(\theta^a)$. Regulatory transfers would occur in the following stage, once quantities are known to all agents in the economy. The best response of $F^b$ would then be computed taking the expectation of the firm’s profits on $\theta^a$, conditionally to $\theta^b$. The informative characteristics of $q^b$ would remain unchanged. The regulator would be able to detect any type of collusion allowing $F^b$ to choose its quantity under full information on $\theta^a$, and to punish it by a very negative transfer for $F^a$. A coalition would then prefer to restrict $q^b$ to be one of the two values it can take without collusion, $q^b(\overline{\theta})$ and $q^b(\underline{\theta})$.
both firms. When the regulator cannot contract with firm $F^b$, quantity $q^a$ will be optimal given $q^b$, but $q^b$ will result from the maximization problem of the unregulated firm. The rent of $F^a$ is socially costly, so that the regulator nullifies it in equilibrium. Maximization of his objective function yields the following schedule of output:

$$q^a_s(\theta^a, \theta^b) = \frac{1}{4 + s^2} [4(d^a - \theta^a) - s(d^b - \theta^b)].$$

The quantity produced by the unregulated firm $F^b$ can then be obtained using (1):

$$q^b_s(\theta^a, \theta^b) = \frac{1}{4 + s^2} [2(d^b - \theta^b) - 2s(d^a - \theta^a)].$$

A larger quantity $q^a$ than the socially optimal one is needed to compensate for the tendency of the unregulated firm to contract output.\footnote{11}

### 4 Information transmission in the absence of collusion

This section focuses on information effects when the cost parameter of each firm is its private information, and when there is no collusion. Collusion is considered in Section 5 and Section 6.

#### 4.1 The role of observability

Ex post observation of the quantity of the unregulated firm allows to infer information on cost $\theta^b$, and therefore, due to correlation, on $\theta^a$. This quantity can therefore be used as a correlated signal by the regulator. But one should note that this quantity depends on the regulated quantity $q^a$. The signal constituted by $q^b$ is therefore not completely exogenous from the point of view of the regulator.

After the regulated firm has revealed its private information $\theta^a$ and produced quantity $q^a(\theta^a)$, the unregulated firm has to choose its quantity. If $q^b(\bar{\theta}, q^a(\theta^a)) \neq q^b(\bar{\theta}, q^a(\bar{\theta}))$, then regulator

\footnote{10}The socially optimal quantities, indexed by ‘so’, are such that price equals marginal cost in each market:

$$q^a_{so}(\theta^a, \theta^b) = \frac{1}{1 + s^2} [(d^a - \theta^a) - s(d^b - \theta^b)],$$

$$q^b_{so}(\theta^a, \theta^b) = \frac{1}{1 + s^2} [(d^b - \theta^b) - s(d^a - \theta^a)].$$

\footnote{11}We assume that firm $F^B$ prefers indeed to produce rather than exit the market $(d^b - \theta^b - s(d^a - \theta^a) \geq 0)$.}
$R^a$ can condition the regulatory transfer to the true type of the unregulated firm because the ex post observation of $q^b$ completely reveals $\theta^b$.

To simplify the exposition, we will focus here on values of the parameters for which firm $F^b$ always produces.\textsuperscript{12} The well-known result of Crèmer and McLean (1988) applies, as we show below: The possibility to construct contingent transfers for the risk-neutral regulated firm enables the regulator to extract all the informational rents that $F^a$ could command due to its private information, as long as types are strictly correlated.

### 4.2 The optimal regulatory contract

The Revelation Principle applies, so that the regulator maximizes its objective under participation and incentive constraints.

The output level chosen by $F^b$ necessarily reveals its type (when it is positive), and provides a correlated signal on $\theta^a$. The regulator should use this quantity to condition the transfer paid by the regulated firm, but also to update beliefs on the type of this firm.

At the time at which the regulated firm decides whether to participate in the contract, and which announcement to make to the regulator, it is under imperfect information on $\theta^b$ (and $q^b$). Participation and incentive constraints are therefore Bayesian ones. The beliefs of $F^a$ as to the type of the unregulated firm depend on its own type, $\theta^a$, due to the correlation of the two costs.

\textsuperscript{12}The intuition for the other cases is as follows: (i) If the entrant $F^b$ preferred to produce only when its competitor has a high cost, quantity $q^b$ would be informative only in some states of nature. But it would still constitute a —sometimes non informative— correlated signal on $\theta^a$, which would be enough for the results stated in the text to hold. (ii) The decision by $F^b$ to enter the market is endogenous to the contract offered by the regulator. If the quantities preferred by the regulator were such that $F^b$ was not willing to enter, using yardstick mechanisms would be impossible with these quantities. But regulator $R^a$ could offer $F^b$ a stochastic mechanism entailing production of the optimal quantity for a regulated monopoly with a given probability, and production of a quantity small enough to induce entry by $F^b$ with the complementary probability. Since the regulated firm can be made to suffer large penalties, a small probability of producing a quantity inducing entry from $F^b$ would be enough to induce truthful revelation by $F^a$ along the lines of the mechanism described in the text.
The individual participation constraints for firm $F\alpha$ are thus:

\[ p_{11}^{a} + p_{12}^{a} \geq 0 \quad BIR^{a}(\bar{\theta}), \]
\[ p_{21}^{a} + p_{22}^{a} \geq 0 \quad BIR^{a}(\theta). \]

The Bayesian incentive constraints are given by:

\[ p_{11}^{a} \pi_{11}^{a} + p_{12}^{a} \pi_{12}^{a} \geq p_{11}^{a} \pi_{21}^{a} + p_{12}^{a} \pi_{22}^{a} + \Delta \theta(p_{11}^{a} q_{21}^{a} + p_{12}^{a} q_{22}^{a}) \quad BIC^{a}(\bar{\theta}), \]
\[ p_{21}^{a} \pi_{21}^{a} + p_{22}^{a} \pi_{22}^{a} \geq p_{21}^{a} \pi_{11}^{a} + p_{22}^{a} \pi_{12}^{a} - \Delta \theta(p_{21}^{a} q_{11}^{a} + p_{22}^{a} q_{12}^{a}) \quad BIC^{a}(\theta). \]

Let us introduce the following condition:

\[ d^{b} - \theta - s(d^{a} - \bar{\theta}) \geq \Delta \theta(1 - s^{2} \frac{p_{11}}{4 p_{11} + p_{12}}) \quad A_{1}. \]

This condition ensures that firm $F^{b}$ prefers to produce rather than exit the market for the optimal regulated quantities when the regulator uses contingent transfers. Then, we obtain the following result:

**Lemma 1 (A Crémer and McLean -type result)** Under Condition $A_{1}$, regulator $R^{a}$ can nullify the (expected) rent of the regulated firm. The optimal quantity profile is defined as follows

\[ q^{a}(\bar{\theta}) = q^{a*}(\bar{\theta}) \equiv \frac{1}{4 + s^{2}}[4(d^{a} - \bar{\theta}) - s(d^{b} - E(\theta^{b}/\bar{\theta}))], \]
\[ q^{a}(\theta) = q^{a*}(\theta) \equiv \frac{1}{4 + s^{2}}[4(d^{a} - \theta) - s(d^{b} - E(\theta^{b}/\theta))]. \]

The quantity profile corresponds to quantities implemented when the regulator knows the cost parameter of the regulated firm but remains uninformed on that of the unregulated firm. Indeed, because of the timing, the regulator bases its production decisions on the expected cost of the unregulated firm (these expectations depend on $F^{a}$’s cost parameter due to the correlation).

Condition $A_{1}$ simply ensures that, faced to this quantity profile, the unregulated firm produces a positive quantity in every state of nature.

The possibility to observe ex post a signal correlated with $F^{a}$’s private information enables $R^{a}$ to leave no expected rent to the regulated firm in equilibrium. In the Appendix, we show that as soon as the degree of correlation is non null, regulator $R^{a}$ can satisfy all the individual constraint (that is, the constraints $BIR^{a}(\bar{\theta})$, $BIR^{a}(\theta)$, $BIC^{a}(\bar{\theta})$ and $BIC^{a}(\theta)$) as equalities.
This in turn implies that the expected rents vanish in the expected social welfare function and that no distortion on the quantity profile are needed for incentive reasons: This is the ‘information transmission’ effect.

This ‘information transmission’ effect gives rise to new stakes of collusion: Since the regulated firm $F^a$ obtains large profits in some states of nature —and suffers large losses in others— it has strong incentives to bribe the unregulated firm to have it produce the quantity for which its own profits are the largest \(^{13}\). It also has incentives to pay the unregulated firm so as to have it produce a non informative quantity, i.e. a quantity that does not vary according to its cost parameter. This indeed prevents the use of yardstick mechanisms by the regulator, and allows an efficient regulated firm to obtain an information rent. Collusion is studied in the following sections.

5 Coalition formation

Collusive negotiations take place between agents who try to take advantage of their private information, and can in some situations commit to a behavior even though collusive agreements are not enforceable by a Court. The modeling used here attempts to capture the outcome of such a bargaining process with asymmetric information.

5.1 Modeling collusion

Following Laffont and Martimort (1997, 2000), we model bargaining within the coalition as follows: An uninformed third party, $T$ chooses the report made by $F^a$ to the regulator, the quantity produced by $F^b$, and side-transfers between the two firms, so as to maximize the expectation of the sum of firms’ profits, $\pi^a + \pi^b$. The side-transfers must be budget-balanced in all states of nature.

\(^{13}\)We showed in a previous version of this work that although the unregulated firm suffers an ex post loss from the fact of not fully adapting its quantity to its type, it may gain ex ante, even if the regulated firm does not compensate it with a bribe, for some parameter values. This comes from the downward distortion that the regulator imposes on regulated quantity when it does not have access to a correlated signal.
We assume that if both firms choose to participate in a collusive agreement, the decisions taken according to this process are enforceable in some way.\footnote{Commitment issues are obviously important in practice. Even if $F^b$ is ex ante willing to accept a collusive agreement, it will always prefer ex post, once $q^a$ has been determined, to choose the quantity that maximizes its profits, i.e. the one corresponding to (1). A similar commitment problem arises for $F^a$ as well: It has no longer an incentive to pay the bribe once $q^b$ has been produced according to its wishes. A practical way out of such commitment issues may consist of delegating the choice of action to a third party. For instance, firm $F^b$ may, at the time of the collusive agreement, sign a binding contract with a retailer, advertise a specific price or commit to some production capacity.}

The third party is formally a principal offering a (side) contract under incomplete information. The Revelation Principle therefore applies: There is no loss of generality in restricting the side contract to be direct and inducing truthful revelation. A side contract can be written as $SC = \{ \phi(.), q^b(.), \{ y^i(.) \}_{i=a,b} \}$ where $\phi(.)$ is the ‘manipulation function’ that determines the report made by $F^a$ to $R^a$ as to its type, $q^b(.)$ is the quantity produced by the unregulated competitor, and $\{ y^i(.) \}_{i=a,b}$ is a couple of side transfers paid by each firm to the third party.

### 5.2 Timing of the game

The timing of the game is as follows:

1. Nature determines the type $\theta^i$ of firm $F^i$ ($i = a, b$). Each firm only learns its own cost parameter.

2. Regulator $R^a$ proposes a contract to his firm. $F^a$ decides to accept or to refuse this contract. If it rejects the contract, it does not produce and the unregulated firm sets its quantity as a monopoly. The following steps are not played. The game otherwise is as follows.

3. The third party $T$ proposes a side contract $SC$ to both firms. If both firms accept the side contract, they report their cost parameters to the third party.

4. Firm $F^a$ reports to $R^a$,

   - according to the side contract if both firms have accepted it in the previous stage,
• non cooperatively otherwise.

The quantity $q^a$ corresponding to this report is produced.

5. Quantity $q^b$ is produced$^{15}$,

• according to the side contract if both firms have accepted it in stage 3,
• according to the non cooperative choice of $F^b$ otherwise.

6. The transfers specified by the regulatory contract take place as well as the side transfers promised by $T$.

According to this timing, the third party must ensure that each firm earns more, in expectation, when it accepts the side contract than when it decides to refuse it and to play non-cooperatively the subsequent game. We assume that firms can commit to behave cooperatively: If they accept, under incomplete information as to each other’s type$^{16}$, to participate to the side contract, they are bound to follow the decisions of the third party, regarding the report and the quantity $q^b$, respectively. Allowing for commitment within the coalition means that quantities $q^b$ that differ from the non cooperative best response (1) have to be considered. Participation constraints are Bayesian constraints.

If the side contract has been refused by at least one firm, then the regulated firm plays non-cooperatively the regulatory contract and the unregulated firm chooses non-cooperatively its production. But after observing that a firm refuses to participate in the collusive contract, the other firm updates its beliefs as to the cost of the first firm. The non cooperative choices may therefore be taken under different beliefs than the initial ones. The status quo gains of each firm in case of refusal of the side contract thus depend on the way in which beliefs are modified by an out-of-equilibrium action. To simplify the analysis, but also for lack of a better assumption, we assume that the refusal to play the side contract by a firm does not change the beliefs of the other firm (that derive initially from the common knowledge prior probability distribution).$^{17}$

$^{15}$For simplicity, we restrict the analysis to situations in which the unregulated competitor always produces.

$^{16}$The participation and incentive constraints faced by the third party are therefore Bayesian ones.

$^{17}$This assumption of ‘passive beliefs’ allows in addition to compare our results with those of Laffont and Martimort (1997, 2000) for complete regulation.
5.3 The program of the third party

All the constraints relating to the program of the third party will be indexed by \( T \).

Let \( \Pi^a(\theta^a, \tilde{\theta}^a) \) be the interim gain of the regulated firm with type \( \theta^a \) when it announces \( \tilde{\theta}^a \) to the third party. Similarly, the interim gain of an unregulated firm with type \( \theta^b \) that announces \( \tilde{\theta}^b \) to \( T \) is \( \Pi^b(\theta^b, \tilde{\theta}^b) \) (their expression is given in Appendix A.3.). Firm \( F^i \) \((i = a, b)\) will reveal truthfully to the third party if

\[
\Pi^i(\theta^i, \theta^i) \geq \Pi^i(\tilde{\theta}^i, \theta^i) \quad \forall (\theta^i, \tilde{\theta}^i) \in \Theta^2 \quad BIC^i_T(\theta^i).
\]

Firm \( F^i \) \((i = a, b)\) will accept the collusive side contract if the following constraint is met:

\[
\Pi^i(\theta^i, \theta^i) \geq \hat{\Pi}^i(\theta^i) \quad BIR^i_T(\theta^i),
\]

where \( \hat{\Pi}^i(\theta^i) \) is \( F^i \)'s gain when the side contract has been refused by at least one firm and firms play non-cooperatively the subsequent game with the prior beliefs.

Finally, the ex post budget-balance condition simply writes as follows:

\[
\sum_{i=a,b} y^i(\theta^a, \theta^b) = 0 \quad \forall (\theta^a, \theta^b) \in \Theta^2 \quad BB_T(\theta^a, \theta^b).
\]

In the game of coalition formation, an equilibrium consists in a direct and truthful side contract which i) is accepted by both firms, given their reservation gains \{\( \hat{\Pi}^a(\theta^a), \hat{\Pi}^b(\theta^b) \)\}, and ii) maximizes the third party’s objective function.\(^\text{18}\)

Let us now introduce a definition that will be useful for the rest of the analysis.

**Definition 1** A regulatory contract is said to be collusion-proof if and only if it is a direct contract inducing truthful revelation and if the null side contract \( SC^0 = \{\phi^0 = Id_{\Theta}, q^b \in \arg\max q^b \pi^b, y^i = 0, i = a, b\} \) is a continuation equilibrium of the game of coalition formation.

Whatever the contract offered by \( R^a \) to \( F^a \), the third party can always implement the null side contract. In equilibrium there will therefore always be formation of the coalition. The problem

\(^{18}\)Every attempt by the regulator to elicit from \( F^a \) information on the collusion equilibrium can be countered by the third party, who can design collusive contracts so as to make the firm indifferent between hiding and revealing its information to the regulator. We assume that when the firms are indifferent between two actions, they choose the one preferred by the third party.
is to determine whether that coalition will be active (i.e., $\phi \neq I_dQ$ and/or $q^b \notin \arg \max_q \pi^b$ for instance) or not in equilibrium.

If both firms were under the jurisdiction of the regulator, then, following Laffont and Martimort (1997, 2000) we could apply the Weak Collusion-Proofness Principle to characterize the implementable allocations. This principle states that there is no loss of generality in restricting attention to contract such that the choice of the null side contract is an equilibrium (sustained with out-of-equilibrium passive beliefs) of the game of coalition formation.

However, because of the incompleteness of regulation, such a principle does not apply our context: It is possible that some outcomes cannot be obtained in a game for which the null side contract is an equilibrium. This point is discussed at the end of the following section.

6 Collusion-proof equilibrium allocations

Writing the conditions such that it is optimal for the third party not to manipulate the report of the regulated firm enables to obtain the ‘collusion-proofness’ constraints. We choose to use the term collusion-proofness although, in the present context, the regulator cannot influence directly the choice of the unregulated quantity and consequently cannot ensure that the firms will not be colluding by manipulating the quantity of the unregulated firm. Nonetheless, if $R^a$ ensures the revelation of $\theta^a$, then this very strongly constrains the collusive choice of the unregulated quantity, as we will show below.

6.1 Conditions for truthful revelation by the regulated firm

Under incomplete information between the colluding members the constraints for truthful revelation by the regulated firm —the collusion-proof constraints— are as follows (see the Appendix):

- For an efficient regulated firm $F^a$

$$
\begin{align*}
\pi^a_{11} + \pi^b_{11} & \geq \pi^a_{21} + \pi^b_{21} + \Delta \theta q^a(\bar{\theta}) & CPC_1, \\
\pi^a_{12} + \pi^b_{12} & \geq \pi^a_{22} + \pi^b_{22} + \Delta \theta q^a(\bar{\theta}) & CPC_2.
\end{align*}
$$
And for an inefficient regulated firm $F^a$

$$\begin{align*}
\pi^a_{21} + \pi^b_{21} &\geq \pi^a_{11} - \Delta \theta q^a(\bar{q}) + \pi^b_{11} - \frac{p_{11}}{p_{12}} \epsilon^a \Delta \theta q^a(\bar{q}) - q^a(\bar{q})] \quad CPC_3, \\
\pi^a_{22} + \pi^b_{22} &\geq \pi^a_{12} - \Delta \theta q^a(\bar{q}) + \pi^b_{12} - f(\epsilon^a) \Delta \theta q^a(\bar{q}) - q^a(\bar{q})] \quad CPC_4,
\end{align*}$$

where $\epsilon^a$ and $f(\epsilon^a)$ are functions of Lagrangean multipliers (their expression is given in the Appendix). What matters from our point of view is that $\epsilon^a = 0$ if the incentive compatibility constraint for an efficient firm is slack in the program of the third party.

$\epsilon^a$ reflects the asymmetry of information within the coalition. In order to obtain truthful revelation by an agent, a principal may have to distort outcomes so as to satisfy incentive compatibility in a less costly way. Similarly here, to obtain revelation by the firms, the third party has to make some distortions on its decisions with respect to the situation of collusion under complete information: Informational problems can be the source of inefficiency from the viewpoint of the coalition. This is no longer the case if the incentive compatibility constraint is not binding. Hence, when $\epsilon^a = 0$, no distortion arises even though there is asymmetric information within the coalition to begin with. The constraints for a high-cost firm then have the same shape as the constraints for a low-cost firm, as with collusion under complete information.

### 6.2 The reaction function of the unregulated firm

The collusive reaction function for $q^b$ should be distorted by the third party, first, in order to maximize the sum of the gains of the colluding members, and second, because of informational problems from the perspective of the third party. The program faced by the coalition is given in subsection A.3.1. of the Appendix: As for firm $F^a$, a term $\epsilon^b$, defined in a similar way as $\epsilon^a$, appears in the optimality conditions for the quantity produced by the unregulated firm.

However, to be willing to delegate the choice of its quantity the unregulated firm must be compensated with some monetary side transfers paid by $F^a$ to the third party. Indeed, let us consider the behavior of $F^a$, and the quantity produced $q^a$, as fixed. If $F^b$ accepts the agreement, then $q^b$ will be chosen so as to maximize the sum of the gains of the coalition. On the other hand, if it refuses the side contract, then it earns the gain obtained by playing non-cooperatively and setting $q^b$ so as to maximize its own profit.
Let us assume that the regulator proposes a contract such that the optimal manipulation function is the identity function: The regulated firm reveals truthfully. In this situation, the regulated firm will obviously not be willing to pay a bribe to the unregulated firm to obtain a modification of its quantity, since a modification in $q^b$ does not increase its profits: It obtains the same rent with and without collusion, since this rent is taken as given at the time at which collusion takes place.\footnote{Given our timing, the regulator knows that the coalition will react in a differentiated way if it offers a regulated contract that differentiate according to the four states of nature. It might be the case that the coalition would have been better off committing ex ante, before the regulated contract is offered, to produce the same quantity whatever the type of the unregulated firm, for instance. But such an ex ante commitment is not available here.}

Therefore, if the regulator proposes a contract that induces truthful revelation by the regulated firm, then the third party cannot make this firm pay side transfers. But this in turn implies that the unregulated firm cannot be compensated for the change in its reaction function. This yields the following lemma.

**Proposition 1** Assume that regulator $R^a$ proposes to the regulated firm a contract such that the third party finds it optimal not to manipulate the announcement of $F^a$. Then, this contract is collusion-proof (the only implementable side contract is the null side contract $SC^*_0$).

The important point here is that by destroying the incentives of the third party to manipulate the report on the type of $F^a$, the regulator also destroys the possibility for the third party to modify the quantity produced by the unregulated firm.

It should be noticed that in the setting of Laffont and Martimort (1997, 2000), contracts that induce truthful revelation are equivalent by definition to collusion-proof contracts. Here, collusion-proofness should not be a priori restricted to truthful revelation of the type of the regulated firm $F^a$, since it also bears on the unregulated quantity $q^b$ and on possible side-transfers. Yet if a contract induces truthful revelation of $\theta^a$, it necessarily is collusion-proof.\footnote{If the coalition were formed with at least two unregulated firms, the standard motive for coordinating decision}
Finally, this proposition implies that along the equilibrium path in which the regulator proposes a contract such that the optimal manipulation function is the identity, the participation constraints of both firms will be binding in the program of the third party.\(^\text{22}\)

### 6.3 Can the regulator benefit from asymmetric information within the coalition?

When regulator \(R^a\) offers a contract such that the third party finds it optimal not to manipulate the report of \(F^a\), the side transfers become incompatible with the participation constraints in the program of the third party, and the null side contract is proposed. But for this contract, since the quantity produced by the unregulated firm is such that it maximizes its profits, firm \(F^b\) has no incentive to misreport on its cost. Therefore the incentive constraint of the efficient firm \(F^b\) is not binding in the program of the third party and the associated Lagrange multiplier is zero (this result does not depend on the value of the parameters).

**Lemma 2** Assume that the third party offers the null side contract to the firms. Then, the Bayesian incentive compatibility constraints of the unregulated firm \(F^b\) are strictly satisfied in the program of the third party. This implies that \(\epsilon^b = 0\).

When the third party chooses to implement the null side contract, the Bayesian incentive compatibility constraints faced by the regulator are identical to the ones faced by the third party.

**Lemma 3** Assume that regulator \(R^a\) proposes to the regulated firm a contract such that the third party finds it optimal not to manipulate the announcement of \(F^a\). If the incentive constraints of the regulated firm \(F^a\) are strictly satisfied in the program of the regulator, then they are also strictly satisfied in the program of the third party. In this case \(\epsilon^a = 0\).

\(^\text{22}\)As a result the complementarity slackness conditions do not bring more information on the multipliers associated to these constraints in the third party’s program. In Laffont and Martimort (1997, 2000) the same reasoning holds for the incentive compatibility constraints in the program of the third party implying that \(\epsilon^a\) and \(\epsilon^b\) are a priori choice variables for the regulator. We show nevertheless in the following that in our setting, and contrary to the results obtained by Laffont and Martimort (1997, 2000), those variables are constrained in a very strong way.
We show in the next subsection that this is indeed the case for a collusion-proof contract. In Laffont and Martimort (1997, 2000) the regulator can always choose the values of the \( \epsilon \)s because the Bayesian incentive constraint of an efficient firm is binding in equilibrium; these values are then chosen to maximize the welfare of the regulator, and lessen the cost of preventing collusion.

The institutional incompleteness destroys this possibility. Anticipating on the result of the next subsection, we obtain:

**Proposition 2** The fact that regulation is incomplete prevents the regulator from benefiting from information asymmetries within the coalition, when fighting collusion.

### 6.4 The allocation with collusion-proof regulation

We have seen that if the regulator induces truthful revelation, the quantity produced by the unregulated firm will be given by the non collusive best response given in equation (1). The best collusion-proof contract is thus the solution to the following program:

\[
\max_{\{q_{a_{ij}}, \pi_{a_{ij}}\}_{i,j=1,2}} \begin{align*}
& p_{11}[(GS(q_{a_{11}}, q_{b_{11}})) - \theta q_{11}^a - P^b(q_{a_{11}}, q_{b_{11}})q^b(q_{a_{11}}) - \pi_{11}^a] \\
& + p_{12}[(GS(q_{a_{12}}, q_{b_{12}})) - \theta q_{12}^a - P^b(q_{a_{12}}, q_{b_{12}})q^b(q_{a_{12}}) - \pi_{12}^a] \\
& + p_{12}[(GS(q_{a_{21}}, q_{b_{21}})) - \theta q_{21}^a - P^b(q_{a_{21}}, q_{b_{21}})q^b(q_{a_{21}}) - \pi_{21}^a] \\
& + p_{22}[(GS(q_{a_{22}}, q_{b_{22}})) - \theta q_{22}^a - P^b(q_{a_{22}}, q_{b_{22}})q^b(q_{a_{22}}) - \pi_{22}^a]
\end{align*}
\]
subject to

\[ \begin{align*}
\pi_{11}^a + \pi_{11}^b & \geq \pi_{21}^a + \pi_{21}^b + \Delta \theta q^a(\bar{\theta}) & \text{CPC}_1 \\
\pi_{12}^a + \pi_{12}^b & \geq \pi_{22}^a + \pi_{22}^b + \Delta \theta q^a(\bar{\theta}) & \text{CPC}_2 \\
\pi_{21}^a + \pi_{21}^b & \geq \pi_{11}^a - \Delta \theta q^a(\bar{\theta}) + \pi_{11}^a - \frac{P_{12}}{P_{21}} \epsilon^a \Delta \theta [q^a(\bar{\theta}) - q^a(\bar{\theta})] & \text{CPC}_3 \\
\pi_{22}^a + \pi_{22}^b & \geq \pi_{12}^a - \Delta \theta q^a(\bar{\theta}) + \pi_{12}^a - f(\epsilon^a) \Delta \theta [q^a(\bar{\theta}) - q^a(\bar{\theta})] & \text{CPC}_4 \\
p_{11} \pi_{11}^a + p_{12} \pi_{12}^a & \geq p_{11} \pi_{21}^a + p_{12} \pi_{22}^a + \Delta \theta (p_{11} q_{21}^a + p_{12} q_{22}^a) & \text{BIC}^a(\bar{\theta}) \\
p_{21} \pi_{21}^a + p_{22} \pi_{22}^a & \geq p_{21} \pi_{11}^a + p_{22} \pi_{12}^a - \Delta \theta (p_{21} q_{11}^a + p_{22} q_{12}^a) & \text{BIC}^a(\bar{\theta}) \\
p_{11} \pi_{11}^a + p_{12} \pi_{12}^a & \geq 0 & \text{BIR}^a(\bar{\theta}) \\
p_{21} \pi_{21}^a + p_{22} \pi_{22}^a & \geq 0 & \text{BIR}^a(\bar{\theta}).
\end{align*} \]

The optimal collusion-proof contract is such that:

- The collusion-proofness constraints CPC\(_1\) and CPC\(_2\) as well as the ex post participation constraint of an inefficient regulated firm are binding.

- The regulator cannot benefit from the asymmetry of information within the coalition: \(\epsilon^a = \epsilon^b = 0\).

- The optimal quantity profile is

\[ \begin{align*}
q_{cp}^a(\bar{\theta}) &= \frac{1}{4 - 3c^2} \left[ 4(d^a - \bar{\theta}) - 3s(d^b - E(\bar{\theta}^b/\bar{\theta})) \right], \\
q_{cp}^b(\bar{\theta}) &= \frac{1}{4 + s^2(3 - \frac{2}{p(\bar{\theta})})} \left[ 4(d^a - \bar{\theta}) - \frac{p(\bar{\theta})}{p(\bar{\theta})} \Delta \theta - s(3 - \frac{2}{p(\bar{\theta})})(d^b - E(\bar{\theta}^b/\bar{\theta})) \right].
\end{align*} \]

- The remaining collusion-proofness constraints are satisfied in equilibrium if \(q_{cp}^a(\bar{\theta}) \geq q_{cp}^b(\bar{\theta})\). Bunching otherwise occurs.

The threat of collusion prevents the regulator from fully using the information conveyed by the unregulated firm’s choice of quantity: The collusion-proof quantities are not differentiated according to the unregulated firm’s type, contrary to the contract without collusion. The regulator has to satisfy collusion-proofness constraints (CPC\(_1\) and CPC\(_2\)) that are more stringent than dominant strategy incentive constraints for a low-cost regulated firm, as well as ex post participation constraints for a high-cost one.
The pattern of distortions on the quantity profile is extremely similar to the quantities that would have emerged in a collusion-proof contract if the two firms had been under complete information with respect to each other’s type, with ex post participation constraints. Here, $R^a$ is uninformed on $\theta^b$ at the time of implementing the regulated quantity and therefore bases its production decisions on the expected cost of the unregulated competitor. This explains the sole difference in the two quantity profiles. This similarity underlines again the fact that the regulator has to react to collusion as if there was complete information within the coalition: He is unable to take advantage of asymmetric information between the two firms.

When the degree of substitutability tends to zero, the regulated quantities tend to the ones corresponding to asymmetric information and a single firm: $q^a(\bar{\theta}) \to q^e_a(\theta) = d^a - \theta$ and $q^a(\bar{\theta}) \to q^e(\theta) = d^a - \bar{\theta} - \frac{d^a}{p(\theta)} \Delta \theta$. Since the goods become less and less substitutable, $F^a$ is closer to a monopolistic situation.

When, on the other hand, substitutability is very strong, the pattern of quantities depend on the likelihood that the regulated firm be inefficient. If $p(\theta)$ tends to zero, the regulated quantity for an inefficient firm also tends to zero.

6.5 Non collusion-proof contracts: A heuristic discussion

In a centralized organization, the question of collusion-proofness is not relevant: When a unique entity regulates both firms, the regulator and the third party have the same information and, loosely speaking, the same contracting abilities. Moreover any transfers that could be implemented by the third party could be exactly replicated by the regulator at the same cost. This is the essence of the Weak Collusion-Proofness Principle: There is no loss of generality in restricting the regulator to offer a regulatory contract such that there is no manipulation of announcements and no side transfers implemented by the third party in equilibrium. Notice that it does not mean that coalition formation cannot occur in equilibrium but rather that an equilibrium in which the firms actively collude cannot perform strictly better, from a welfare point of view, than the optimal collusion-proof contract. This principle may no longer be valid in the structure of asymmetric regulation we have considered. Indeed the third party can coordinate the rents of both firms and, in this sense, has more leeway than a regulator that contracts only with one
One may therefore wonder whether letting firms actively collude might not be a better policy. We show in Aubert and Pouyet (2004) that this may enable, in a particular game, the regulator to tax away the unregulated firm’s profits, something that is not possible with collusion-proof regulation. The main issue associated with the characterization of non collusion-proof contracts consists in defining the set of possible such contracts. Indeed, there is no longer any reason for the Revelation Principle to apply between the regulator and the regulated firm. The ‘true agent’ is now the coalition itself. In Aubert and Pouyet (2004), this problem is solved relatively easily since there is only one parameter of asymmetric information. With asymmetric information on the type of both firms, on the other hand, it becomes more difficult for the regulator to induce a particular outcome: $R^a$ has only one instrument whereas the private information of the agent (the coalition) is bi-dimensional.

Some characteristics of non collusion-proof contracts with incomplete regulation can nevertheless be informally described.

- With a non collusion-proof contract, the quantity of the unregulated firm could be set so as to maximize the sum of the gains of the colluding members. With substitute products, this tends to decrease the quantity produced by the unregulated firm.

- The quantity of the unregulated firm is ex post observable by regulator $R^a$. However, with a collusion-proof contract, he cannot condition the transfer given to $F^a$ on the fact that firms are colluding or not (i.e., $q^b$ not set non-cooperatively) as $F^a$ is indifferent between colluding or not in equilibrium. On the other hand, in a non collusion-proof contract, if the firms are strictly willing to collude, then the regulator can impose a ‘tax’ on the coalition, through the transfer given to the regulated firm. Yet, the regulator may find it difficult, due to the collusive participation constraints of both firms within the coalition, and the individual participation constraint of the regulated firm.

- We have seen that if the contract is such that the regulated firm $F^a$ truthfully reveals its information, then it is necessarily collusion-proof. Hence, a non collusion-proof contract must be such that the firm mis-report on its type when there is no collusion (as in\textsuperscript{23} Aubert

\textsuperscript{23}In their paper, the optimal collusion-proof contract is such that, if there was no collusion, the regulated firm
and Pouyet, 2004).

- With a collusion-proof contract we have shown that regulator $R^a$ was not able to benefit from the informational asymmetries at the level of the coalition. This might not be the case with a non collusion-proof contract.

- Last, letting collusion happen could enable to implement a quantity profile for the regulated firm that accounts for all the relevant information.

A more precise characterization is left for future research.

7 Conclusion

Aside from other numerous reasons (additional variety, increased cost efficiency . . . ), a reason to open markets to competition is to obtain information.

In our model, authorizing private firms to compete with a public firm on a given market may bring new information correlated with the information privately held by the regulated public firm. This should help to regulate the firm in a more efficient way. However, the use of yardstick mechanisms can also trigger the formation of collusive coalitions in response to a regulation that extracts all the rent of the regulated firm. If the regulator cannot regulate the whole industry, the threat of collusion may strongly limit the efficiency of yardstick mechanisms. We have shown how incomplete regulation in the form of restricted mandates for regulatory agencies can give rise to important inefficiencies in the fight against collusion. A regulator is still able to deter active collusion despite regulatory incompleteness, but may no longer make use of the information provided by the competitor’s behavior. Collusion therefore destroys the possibility to use actual yardstick mechanisms in this setting. And the incompleteness of regulation further destroys the possibility for the regulator to benefit from asymmetric information between the colluding firms, when the regulator deters actual collusion.

One can interpret our results as indicators that antitrust control is needed even for industries that are subject to regulation. It is important to note that collusion can be a strong concern even would always pretend being efficient. The regulated firm bribes it in order to have it reveal its efficiency, when its type is indeed $\theta$. 

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when the degree of substitutability between the products is low. Antitrust authorities tend to focus on a definition of the relevant market based on the degree of substitutability between products. As we see here, the validity of this approach may not be warranted for all types of collusive behavior: Firms may be able to collude efficiently when their products are weak substitutes. The stake of collusion that we have highlighted indeed exists even for independent products, for instance for local monopolies, as exclusive concessions.

8 References


Appendix

A.1. The benchmark of complete information

The unregulated firm is under complete information on $q_a$ when it chooses its own quantity. It simply maximizes $\pi^b = (d_b - sq_a - \theta^b)q_b - (q_b)^2$, which gives $q^b(q^a) = \frac{1}{2}(d_b - \theta^b - sq_a)$.

The quantities under complete information and complete regulation When regulation is complete, the regulator is able to choose the quantities produced by both firms. It maximizes over $q_a$ and $q_b$ the following objective:

$$SW(\theta^a, \theta^b) = GS(q^a, q^b) - \theta^a q^a - \theta^b q^b = (d^a - \theta^a)q^a + (d^b - \theta^b)q^b - \frac{1}{2}((q^a)^2 + (q^b)^2) - sq^a q^b.$$  

The first-order conditions yield a system of 2 equations for 2 unknowns, whose solution is the one given in the text.

The quantities under complete information and incomplete regulation The problem of the regulator when he cannot tax firm $F_b$ is very similar to the program for complete regulation, except that maximization is done on $q_a$ only, but taking into account that $\frac{dq^b}{dq_a} = -\frac{s}{2}$. The first-order condition gives $(d^a - \theta^a) - \frac{s}{2}(d^b - \theta^b)q^b - q^a - \frac{s}{2}q^b(q^a) = 0$. Replacing $q^b$ by its expression gives the solution, $q^a(\theta^a)$.

A.2. Regulation in the absence of collusion

We show below that there exists a profile of ex post gains such that the four constraints stated in the main text are satisfied as equalities. This is equivalent to finding a profile of ex post gains
\((\pi^a_{11}, \pi^a_{12}, \pi^a_{21}, \pi^a_{22})\), such that
\[
\begin{pmatrix}
p_{11} & p_{12} \\
p_{12} & p_{22}
\end{pmatrix}
\begin{pmatrix}
\pi^a_{11} \\
\pi^a_{12}
\end{pmatrix}
= \Delta \theta \begin{pmatrix}
0 \\
(p_{12} + p_{22})q^a(\bar{\theta})
\end{pmatrix}
\]
and
\[
\begin{pmatrix}
p_{11} & p_{12} \\
p_{12} & p_{22}
\end{pmatrix}
\begin{pmatrix}
\pi^a_{21} \\
\pi^a_{22}
\end{pmatrix}
= -\Delta \theta \begin{pmatrix}
(p_{11} + p_{12})q^a(\bar{\theta}) \\
0
\end{pmatrix}.
\]

A necessary and sufficient condition for such rents to exist is that the determinant of the matrix, 
\(p_{11}p_{22} - (p_{12})^2 = \rho\), be different from zero, i.e. the degree of correlation be non null: \(\rho \neq 0\).
Moreover, if the expected rent of both types of regulated firm are zero (that is, when both Bayesian individual rationality constraints are binding), then the optimization of the expected social welfare function with respect to quantities leads to the quantity profile \(\{q^a(\theta), q^b(\bar{\theta})\}\) which are the socially optimal quantities when \(R^a\) is informed on \(\theta^a\) but not on \(\theta^b\).

A.3. Coalition formation

Let \(\hat{\Pi}^k(\theta^k)\) be the non cooperative expected profit for firm \(F^k\), \(k = a, b\), when it has type \(\theta^k\) and no coalition forms.

We will use the following simplifying notations: \(\phi_{11} = \phi(\theta^a = \theta, \theta^b = \bar{\theta}), \phi_{12} = \phi(\theta^a = \theta, \theta^b = \bar{\theta}), \phi_{21} = \phi(\theta^a = \bar{\theta}, \theta^b = \bar{\theta})\) and \(\phi_{22} = \phi(\theta^a = \bar{\theta}, \theta^b = \bar{\theta})\). Similarly, \(q^a_{11} = q^b(\theta^a = \theta, \theta^b = \bar{\theta}), q^a_{12} = q^b(\theta^a = \theta, \theta^b = \bar{\theta}), q^a_{21} = q^b(\theta^a = \bar{\theta}, \theta^b = \bar{\theta}), q^a_{22} = q^b(\theta^a = \bar{\theta}, \theta^b = \bar{\theta})\), etc. Moreover, the rents of the firms from the point of view of the third party, i.e., without the side transfers, are denoted as follows, for \(j = 1, 2\):

- For the regulated firm:
  \[
  \pi^a_{1j} = [P^a(q^a(\phi_{1j}), q^b_{1j}) - \bar{\theta}]q^a(\phi_{1j}) - t^a(\phi_{1j}),
  \pi^a_{2j} = [P^a(q^a(\phi_{2j}), q^b_{2j}) - \bar{\theta}]q^a(\phi_{2j}) - t^a(\phi_{2j}).
  \]

- For the unregulated firm \(F^b\):
  \[
  \pi^b_{j1} = [P^b(q^a(\phi_{j1}), q^b_{j1}) - \bar{\theta}]q^b_{j1}
  \pi^b_{j2} = [P^b(q^a(\phi_{j2}), q^b_{j2}) - \bar{\theta}]q^b_{j2}.
  \]

\(^{24}\)The constraints of a \(\theta\)-type regulated firm are multiplied by \(p(\theta)\), with \(p(\bar{\theta}) = p_{11} + p_{12}\) and \(p(\bar{\theta}) = p_{21} + p_{22}\).
A.3.1. The program of the third party

Since side transfers are budget balanced in each state of nature, the program of the third party can be written as follows

\[
\max_{\{\phi(.), q(.), y(.)\}} \sum_{i,j=1,2} p_{ij}[\pi_{ij}^a + \pi_{ij}^b]
\]

subject to

\[
p_{11}[\pi_{11}^a - y^a(\theta, \theta)] + p_{12}[\pi_{12}^a - y^a(\theta, \theta)] \geq 0
\]

\[
p_{11}[\pi_{21}^a + \Delta \theta q^a(\phi_{21}) - y^a(\theta, \theta)] + p_{12}[\pi_{22}^a + \Delta \theta q^a(\phi_{22}) - y^a(\theta, \theta)] \geq BIC_T^a(\theta),
\]

\[
p_{11}[\pi_{11}^b - y^b(\theta, \theta)] + p_{12}[\pi_{21}^b - y^b(\theta, \theta)] \geq 0
\]

\[
p_{11}[\pi_{12}^b + \Delta \theta q^b(\phi_{12}) - y^b(\theta, \theta)] + p_{12}[\pi_{22}^b + \Delta \theta q^b(\phi_{22}) - y^b(\theta, \theta)] \geq BIC_T^b(\theta),
\]

\[
p_{11}[\pi_{11}^a - y^a(\theta, \theta)] + p_{12}[\pi_{12}^a - y^a(\theta, \theta)] \geq 0
\]

\[
p_{11}[\pi_{21}^a + \Delta \theta q^a(\phi_{21}) - y^a(\theta, \theta)] + p_{12}[\pi_{22}^a + \Delta \theta q^a(\phi_{22}) - y^a(\theta, \theta)] \geq BIR_T^a(\theta),
\]

\[
p_{11}[\pi_{11}^b - y^b(\theta, \theta)] + p_{12}[\pi_{21}^b - y^b(\theta, \theta)] \geq 0
\]

\[
p_{11}[\pi_{12}^b + \Delta \theta q^b(\phi_{12}) - y^b(\theta, \theta)] + p_{12}[\pi_{22}^b + \Delta \theta q^b(\phi_{22}) - y^b(\theta, \theta)] \geq BIR_T^b(\theta),
\]

\[
\sum_{i=a,b} y^i(\theta^a, \theta^b) = 0, \quad \forall (\theta^a, \theta^b) \in \Theta^b \quad BB_T(\theta^a, \theta^b).
\]

We denote by \(\delta^T\) the multiplier associated to \(BIC_T^a(\theta)\), \(\nu^T\) the multiplier associated to \(BIR_T^a(\theta)\), \(\nu^\theta\) the multiplier associated to \(BIR_T^b(\theta)\) and \(p_T(\theta^a, \theta^b)\) the multiplier associated to \(BB_T(\theta^a, \theta^b)\). Computations can be immediately adapted if the Bayesian incentive compatibility of inefficient (regulated and unregulated) firms are incorporated in the program of \(T\).

A.3.2. Optimality conditions for the side transfers

Optimizing the Lagrangean associated to the previous problem with respect to the side-transfers gives the following conditions:
• First order conditions with respect to $y^a(\bar{\theta}, \bar{\theta})$ and $y^b(\bar{\theta}, \bar{\theta})$

$$-p_{11} \delta^a_T - p_{11} \nu^a_T + \rho_T(\bar{\theta}, \bar{\theta}) = 0$$
$$-p_{11} \delta^b_T - p_{11} \nu^b_T + \rho_T(\bar{\theta}, \bar{\theta}) = 0.$$ 

• First order conditions with respect to $y^a(\bar{\theta}, \bar{\theta})$ and $y^b(\bar{\theta}, \bar{\theta})$

$$-p_{12} \delta^a_T - p_{12} \nu^a_T + \rho_T(\bar{\theta}, \bar{\theta}) = 0$$
$$p_{11} \delta^b_T - p_{12} \nu^b_T + \rho_T(\bar{\theta}, \bar{\theta}) = 0.$$ 

• First order conditions with respect to $y^a(\bar{\theta}, \bar{\theta})$ and $y^b(\bar{\theta}, \bar{\theta})$

$$p_{11} \delta^a_T - p_{12} \nu^a_T + \rho_T(\bar{\theta}, \bar{\theta}) = 0$$
$$-p_{12} \delta^b_T - p_{12} \nu^b_T + \rho_T(\bar{\theta}, \bar{\theta}) = 0.$$ 

• First order conditions with respect to $y^a(\bar{\theta}, \bar{\theta})$ et $y^b(\bar{\theta}, \bar{\theta})$

$$p_{12} \delta^a_T - p_{22} \nu^a_T + \rho_T(\bar{\theta}, \bar{\theta}) = 0$$
$$p_{12} \delta^b_T - p_{22} \nu^b_T + \rho_T(\bar{\theta}, \bar{\theta}) = 0.$$ 

We can deduce immediately the following relations between the multipliers:

$$\begin{cases}
\delta^a_T + \nu^a_T = \delta^b_T + \nu^b_T, \\
\delta^a_T + \nu^a_T = \nu_T^b - \frac{p_{11}}{p_{12}} \delta^b_T, \\
\delta^b_T + \nu^b_T = \nu_T^a - \frac{p_{11}}{p_{12}} \delta^a_T, \\
\nu_T^a - \frac{p_{11}}{p_{22}} \delta^a_T = \nu_T^b - \frac{p_{11}}{p_{22}} \delta^b_T.
\end{cases}$$
A.3.3. Optimality conditions for the manipulation function

Using the previous conditions on the multipliers, it is immediate to derive the optimality conditions on the manipulation function $\phi(.)$.

\[ \phi^*_{11} \in \arg \max_{\phi_{11}} \{ \pi^a_{11} + \pi^b_{11} \}, \]

\[ \phi^*_{12} \in \arg \max_{\phi_{12}} \{ \pi^a_{11} + \pi^b_{12} \}, \]

\[ \phi^*_{21} \in \arg \max_{\phi_{21}} \{ \pi^a_{21} + \pi^b_{21} - \frac{p_{11}}{p_{12}} e^\epsilon \Delta \theta q^a(\phi_{21}) \}, \]

\[ \phi^*_{22} \in \arg \max_{\phi_{22}} \{ \pi^a_{22} + \pi^b_{22} - \frac{p^2_{12} e^\epsilon}{p_{12} p_{22} + e^\epsilon} \Delta \theta q^a(\phi_{22}) \}, \]

where $e^a = \frac{\delta^a}{1 + \nu^a + s^a}$. For further reference, we define $f(\epsilon) = -\frac{p^2_{12} e^\epsilon}{p_{12} p_{22} + e^\epsilon}$. 

A.3.4. The quantity of the unregulated firm with active collusion

Similar computations yield immediately the following conditions, for the quantity of the unregulated firm when there is active collusion.

\[ q^b_{11} \in \arg \max_{q^b_{11}} \{ -sq^a_{11} q^a(\phi_{11}) + [d^b - q^b_{11} - sq^a(\phi_{11}) - \bar{q} q^b_{11}] \}, \]

\[ q^b_{12} \in \arg \max_{q^b_{12}} \{ -sq^a_{12} q^a(\phi_{12}) + [d^b - q^b_{12} - sq^a(\phi_{12}) - \bar{q} q^b_{12}] - \frac{p_{11}}{p_{12}} e^b \Delta \theta q^b_{12} \}, \]

\[ q^b_{21} \in \arg \max_{q^b_{21}} \{ -sq^a_{21} q^a(\phi_{21}) + [d^b - q^b_{21} - sq^a(\phi_{21}) - \bar{q} q^b_{21}] \}, \]

\[ q^b_{22} \in \arg \max_{q^b_{22}} \{ -sq^a_{22} q^a(\phi_{22}) + (d^b - q^b_{22} - sq^a(\phi_{22}) - \bar{q}) q^b_{22} - \frac{p^2_{12}}{p_{22}} f(\epsilon^b) \Delta \theta q^b_{22} \} \]

where $e^b = \frac{\delta^b}{1 + \nu^b + s^b}$.
A.4. Collusion-proof equilibrium allocations

The program of the regulator is the following:

\[
\max_{\{q_i^a, q_i^b\}_{i,j=1,2}} \quad p_{11} \left( (d^a - \theta)q^a(\phi_{11}) - \frac{1}{2}(q^a(\phi_{11}))^2 + \frac{1}{2}(q^b(\phi_{11}))^2 - \pi_{11}^a \right) \\
+ p_{12} \left( (d^a - \theta)q^a(\phi_{12}) - \frac{1}{2}(q^a(\phi_{12}))^2 + \frac{1}{2}(q^b(\phi_{12}))^2 - \pi_{12}^a \right) \\
+ p_{21} \left( (d^a - \theta)q^a(\phi_{21}) - \frac{1}{2}(q^a(\phi_{21}))^2 + \frac{1}{2}(q^b(\phi_{21}))^2 - \pi_{21}^a \right) \\
+ p_{22} \left( (d^a - \theta)q^a(\phi_{22}) - \frac{1}{2}(q^a(\phi_{22}))^2 + \frac{1}{2}(q^b(\phi_{22}))^2 - \pi_{22}^a \right)
\]

subject to

\[
\begin{align*}
\pi_{11}^a + \pi_{11}^b &\geq \pi_{21}^a + \pi_{21}^b + \Delta \theta q^a(\bar{\theta}) & \text{CPC}_1 \\
\pi_{12}^a + \pi_{12}^b &\geq \pi_{22}^a + \pi_{22}^b + \Delta \theta q^a(\bar{\theta}) & \text{CPC}_2 \\
\pi_{21}^a + \pi_{21}^b &\geq \pi_{11}^a - \Delta \theta q^a(\bar{\theta}) + \pi_{11}^b - \frac{p_{11}}{\pi_{12}} \theta q^a(\bar{\theta}) - q^a(\bar{\theta}) & \text{CPC}_3 \\
\pi_{22}^a + \pi_{22}^b &\geq \pi_{12}^a - \Delta \theta q^a(\bar{\theta}) + \pi_{12}^b - \theta \Delta q^a(\bar{\theta}) - q^a(\bar{\theta}) & \text{CPC}_4 \\
p_{11}\pi_{11}^a + p_{12}\pi_{12}^a &\geq p_{11}\pi_{21}^a + p_{12}\pi_{22}^a + \Delta \theta [p_{11}q^a(\phi_{21}) + p_{12}q^a(\phi_{22})] & \text{BIC}^a(\bar{\theta}) \\
p_{21}\pi_{21}^a + p_{22}\pi_{22}^a &\geq p_{21}\pi_{11}^a + p_{22}\pi_{12}^a - \Delta \theta [p_{21}q^a(\phi_{11}) + p_{22}q^a(\phi_{12})] & \text{BIC}^a(\bar{\theta}) \\
p_{11}\pi_{11}^a + p_{12}\pi_{12}^a &\geq 0 & \text{BIR}^a(\bar{\theta}) \\
p_{21}\pi_{21}^a + p_{22}\pi_{22}^a &\geq 0 & \text{BIR}^a(\bar{\theta}).
\end{align*}
\]

Assume that \(\text{CPC}_1\) and \(\text{CPC}_2\) are binding.

Then, direct computations show that \(\text{CPC}_3\) and \(\text{CPC}_4\) will be satisfied if \(q^a(\bar{\theta}) \geq q^a(\bar{\theta})\).

Therefore, playing on the ex post rents of an inefficient firm \(F^a\) does not enable to satisfy these constraints more easily.

Assume then that \(\pi_{21}^a = \pi_{22}^a = 0\), i.e., a \(\theta\)-type regulated firm gets no ex post rent. The ex post rents of an efficient regulated firm are then given by

\[
\begin{align*}
\pi_{11}^a &= \pi_{21}^a - \pi_{11}^b + \Delta \theta q^a(\bar{\theta}), \\
\pi_{12}^a &= \pi_{22}^a - \pi_{12}^b + \Delta \theta q^a(\bar{\theta}).
\end{align*}
\]

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Recall that \( \pi^b = (q^b)^2 = \frac{1}{4}(d^b - \theta^b - sq^a)^2 \). This implies that
\[
\frac{\partial \pi^a_{11}}{\partial q^a(\theta)} = \frac{s}{2}(d^b - \theta - sq^a(\theta)), \\
\frac{\partial \pi^a_{11}}{\partial q^a(\theta)} = -\frac{s}{2}(d^b - \theta - sq^a(\theta)) + \Delta \theta, \\
\frac{\partial \pi^a_{12}}{\partial q^a(\theta)} = \frac{s}{2}(d^b - \theta - sq^a(\theta)), \\
\frac{\partial \pi^a_{12}}{\partial q^a(\theta)} = -\frac{s}{2}(d^b - \theta - sq^a(\theta)) + \Delta \theta.
\]

Then, replacing the values of the regulated firm’s rent in the expected social welfare, optimizing with respect to \( q^a(\theta) \) and \( q^a(\bar{\theta}) \) and rearranging terms yields the quantities given in the proposition.

\( CPC_3 \) and \( CPC_4 \) are satisfied as long as \( q^a(\theta) \geq q^a(\bar{\theta}) \). If this condition is not satisfied, then bunching occurs in equilibrium.

When \( q^a(\theta) \geq q^a(\bar{\theta}) \), we have \( \pi^b_{21} - \pi^b_{11} \geq 0 \) and \( \pi^b_{22} - \pi^b_{12} \geq 0 \). Therefore, \( BIR^a(\theta) \) and \( BIC^a(\bar{\theta}) \) are trivially satisfied.

Finally, in equilibrium we have
\[
BIC^a(\bar{\theta}) \iff 0 \geq p_{12}[\pi^a_{11} - \Delta \theta q^a(\theta)] + p_{22}[\pi^a_{12} - \Delta \theta q^a(\theta)].
\]
Simple manipulations show that \( \pi^a_{11} - \Delta \theta q^a(\theta) \leq 0 \) and \( \pi^a_{12} - \Delta \theta q^a(\theta) \leq 0 \).