CAHIER DU LAMSADE

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February 17, 2010

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C. Camisullis, V. Giard, G. Mendy-Bilek

The causes and determination of safety stocks in upstream supply chains for mass production of customized products

C. Camisullis[1], V. Giard[2], G. Mendy-Bilek[3]

[1] IRG, Université Paris-Est,– France (carole.camisullis@univ-paris12.fr)
[2] Université Paris Dauphine – France (vincent.giard@dauphine.fr)
[3] IAE, Université de Pau et des Pays de l'Adour – France (gisele.mendy-bilek@univ-pau.fr)

Abstract: *In an upstream supply chain dedicated to the mass production of customized products, many sources create production instability: the level and structure of production in the final assembly line, variability of lead times, quality issues, packaging and loading constraints on transportation, demand anticipation, and the synchronization of the flows of components sent, received, and produced. For periodic replenishment systems, each member of the supply chain must have two different safety stocks to prevent some sources of fluctuations: a safety stock of produced components to meet the demand of downstream links and a safety stock of supplied components to ensure its own production. Procedures must take the organizational framework of information and products exchanges into account. The relevance of supply and production rules depends on the relevance of structural information broadcast along the supply chain.*

Keywords: *Global supply chain management, bullwhip effect, information value, safety stock, supply chain.*

1. Introduction

For the mass production of customized products (Anderson & Pine, 1997) using a build-to-order supply chain (BTO-SC, Gunasekarana & Ngaib, 2005, 2009), as exemplified by the automotive industry, differentiation results from the combination of *n* optional components (e.g., sunroofs) or alternative components (e.g., gearboxes). These components are taken from *n* different sets and are assembled on *n* different workstations of an assembly line. The upstream supply chain (USC) then consists of units that contribute to the production of the vehicles assembled by the company. The various links of the chain are connected by flows of products and information. Eventually, the production decisions made by the last production link (i.e., the assembly line) pull production from the USC.

Two obstacles prevent centralized control of the USC by assembly lines: Many units belong to independent companies, and a link might belong to several supply chains (e.g., the supply chains monitored by PSA and Renault have many common links). The behavior of the chain, and thus its global performance, depends on the information and product exchanges that take place among the links, as well as the control rules used to make provisioning and production decisions.

The pernicious effects of decisions based on local information are well-known in the downstream supply chain, especially for low-cost, standardized, mass products (e.g., beer). They propagate oscillations of increasing amplitude along the supply chain, as highlighted by Forrester (1958) and later identified by name as the bullwhip effect.

Oscillations of production occur in USCs dedicated to the mass production of customized products too, but their causes and mechanisms are relatively unknown. For example, a car is assembled at the request of final customers or car dealers committed by commercial objectives. Because of the high price, each car is subject to individual tracking, which means the amplification effects observed in the distribution networks of low-cost, mass products are unlikely to occur. Upstream, production is organized to meet the demands of a single customer, final assembly line. However, studies of the bullwhip effect generally ignore this type of situation, understanding the mechanisms that promote oscillations along the USC is a prerequisite for improving supply chain performance in terms of efficiency and effectiveness.

Whatever the stochastic sources of oscillation in the USC (e.g., random demand, quality problems, random lead times, packaging constraints, restricted transport capacity), an exploitation of demand characteristics is possible. Therefore, the appropriate local application of the principles of periodic replenishment policies, based on a judicious exploitation of the known and stochastic characteristics of demand, keeps stock-outs under control for each link of the USC. Thus, appropriate safety stocks divide the supply chain into independent subsystems, and each entity retains its managerial autonomy, which prevents the propagation of disturbances.

Depending on the position of an order penetration point (OPP), a link can produce to order. If it produces a set of alternative components, using a synchronous production approach (Giard and Mendy 2008) can provide robust rules to face (even fast) modifications of the demand structure. It also offers better control over the problems caused by defective quality in production processes.

This article contains a formal analysis of the exchange mechanisms for information and goods between two links of the USC. This exchange is similar to a customer–supplier relationship, such that each actor is autonomous in its decision making. The production of each link is random, which does not pose a problem if

the links of the supply chain all have sufficient capacity (Camisullis and Giard, 2009).

In this article, we investigate the steady state of periodic provisioning policies in the supply chain. The second article presented at this conference, reveals the conditions in which it is possible to use pertinent information to adapt steady-state policies, preserve their performance, and detect the transformations of the demand structure. Because ignoring such rules may disturb the functioning of the USC, decentralized management in the supply chain rests on information sharing and the use of consistent management rules across its various links.

In the type of USC studied herein, the costs induced by a stock-out are so high that the use of an analytical model based on some expected cost function is useless. This article examines the impact of parameters that arise from organizational decisions about provisioning policies. Specific factors distinguish the periodic replenishment policies of supplied components from those of produced components. However, they depend on the same general approach and common factors. We examine analytically how some parameters influence the appropriate probability distribution for determining the order-up-to level. The order-up-to level can be defined as the percentile associated to a stock-out risk This distribution should include, simultaneously, several random factors (e.g., demand, lead time, quality) and provision constraints (e.g., conditioning, transport capacity). A Monte Carlo simulation provides an answer to this complex issue, as we illustrate herein.

After a brief survey of existing literature on safety stocks in supply chains, we outline the common characteristics of all safety stocks. Then we successively analyze the periodic replenishment policies of supplied and produced components, before we conclude with some operational implications.

2. Literature Review

Prior research offers several reasons to maintain safety stock in a supply chain. In general, safety stock mitigates stock-out risks due to random variations of the demand or production that occur between replenishments. Uncertainty due to production reflects various events that might occur, such as breakdowns and setup and repair times (Ho et al. 1995; Vargas and Dear, 1991). Provision uncertainly might relate to delivery lead times (Chiu and Huang, 2003) or quality (Koh et al. 2002). Uncertainty about demand relates to its random patterns (Guide and Srivastava, 2000), potential forecast errors (Lowerre, 1985, Krupp, 1997), and the occurrence of occasional peaks (Miller, 1979).

In addition, several approaches can determine the order-up-to level value of a periodic replenishment policy. Implicitly, the order-up-to level defines the safety stock.

Analytical models rely on cost functions and provide as many optimal formulae as there are decision variables in the model. Most of these studies relate to downstream supply chains; for example, Axsater and Zhang (1999)

deal with the relationship between the warehouse and distributors, and Diks and De Kok (1998) and Rau et al. (2003) consider multistage systems. In an upstream supply chain study, So and Zheng (2003) consider two contributors to variability in ordered quantities: the supplier's delivery lead time, which is assumed to be variable and depend on the quantities ordered, and updates of the demand forecast.

The simulation approach instead applies to systems that are too complex to derive an analytical solution. Some authors study supply chains that consist of more or fewer levels. For example, Liberopoulos and Koukoumialos (2005) simulate a two-level production supply chain with scheduled lead times, whereas Ng et al. (2003) work with an *n*-stage system that contains random lead times.

Finally, some studies mix analytical and simulation approaches to achieve resource pre-calibration and search for potential efficiency or effectiveness improvements in the same study. For example, Mohebbi et al. (1998) use stochastic lead times to study customer–suppliers relations in an USC, and Moinzadeh (2002) uses constant lead time. Tang and Grubbström (2003) also analyze a two-stage assembly system, with stochastic lead times to minimize total carrying and stock-out costs.

Traditionally, these studies distinguish three decision times: short, medium, and long run. Yet many articles assert demand is purely stochastic (e.g., Graves 1996; Cachon and Fisher 2000), so they consider the long run from the point of view of structure stability but the short term for defining operational decision rules. In some cases, demand is partly known (firm orders) and partly stochastic. For example, in Bourland et al.'s (1996) study, customers place their orders each week for the three next weeks and also forecast for the five or six following weeks. These two levels need safety stocks to offset the uncertainty that affects both orders and deliveries.

3. General Analysis of Safety Stock Needs

First, we introduce the reasons to create safety stocks by adopting a demand propagation perspective along the supply chain (Camisullis and Giard, 2008). Second, we consider the characteristics of the distribution to calibrate the safety stock and specify the factors that influence it. Third, we reveal the relationship between safety stock and stock-out probability. Fourth, we study the analytical relationship between safety stock and expected stock prior to delivery.

3.1. Location of and justification for safety stocks

A replenishment policy includes safety stock to address unknown demand. The stock might include products ordered to meet the needs of a production (or distribution) unit, as well as those manufactured by a production unit when that unit cannot build to order completely because its customers' OPP does not go far

enough into its productive process (Giard and Mendy 2007).

A basic periodic replenishment policy places a periodic order equal to the difference between its order-up-to level R and the inventory position observed at the time of the order. The order interval is θ, and R depends on the target stock-out probability α prior to the delivery, which might be given by economic calculations. The α can lead to negative safety stock, which is meaningless from an operational point of view. Safety stock equals the difference between R and the average demand over the same period. Safety stock also appears in replenishment policies of the type "order quantity q – reorder point r," in which r behaves like R.

In periodic replenishment policies, the safety stock definition therefore depends on the order-up-to level R. Inventory models use cost functions to propose analytical optimal relations to determine R, which always corresponds to the percentile of the demand distribution associated with an optimal value of the stock-out probability α, which in turn depends on the cost structure in the cost function. Generally, for members of the USC, unsatisfied demand is delayed, and α is very low. Two observations emerge:

1. For the supplier, the ordered quantity, which corresponds to a sum of random demands, is a random variable. It reflects the demands that its customer must satisfy.
2. This property is not valid if some supplied parts are rejected for quality reasons. It is then necessary to add to the number of rejected components to the demand since the last order.

With a relevant calibration of safety stock in the various stages, a supply chain can operate without significant fluctuations. The quality of the calibration mainly depends on the propagation of appropriate information from downstream to upstream. We analyze the safety stocks of stage B in the sub network A → B → C in a supply chain. Stage B might hold two kinds of safety stocks, because of its upstream and downstream relationships.

First, *production safety stocks* include the components i produced by B to be sold to its customer (stage C) when the OPP of C in its production system does not allow B to entirely build-to-order. These safety stocks are held by the supplier. We assume customer C transmits an order of q_{it} components i to its supplier B at the beginning of day t. The delivery occurs at the beginning of day $t + D_i$, and the lead time is λ_i ($\lambda_i \leq D_i$). The next order is placed at the beginning of day $t + \theta$, after which supplier B has $D_i - \lambda_i$ days to fulfill the order. If this duration is lower than the manufacturing lead time F_i of component i, the supplier fills the order by taking the needed quantities from its stocks. The process depends on the random characteristics of customer demand, as well as the accepted stock-out risk. In the opposite case ($D_i - \lambda_i > F_i$), B can build to order. If the interval $D_i - \lambda_i$ is greater than the order interval θ, C uses the information from its production schedule. The anticipation of a requirement enables B to move

from a build-to-stock to a build-to-order production system. Furthermore, it can allow supplier A, producing for B, to build to order. This propagation increases effectiveness (fewer stock-outs) and efficiency (less safety stock) at the same time. However, the assertion cannot hold if product quality is not guaranteed.

Second, the *provisioning safety stocks* relate to component j acquired from a supplier (A) to be used in production by B. These stocks are held by the customer (here, stage B), in contrast with the preceding case. The order q_{jt} of the component j sent by B to its supplier A at the beginning of day t is delivered at the beginning of day $t + D_j$ with a delivery lead time λ_j ($\lambda_j \leq D_j$). If A and B synchronize their orders, B gains more knowledge of the requirements but does not need it. When B sends A an order, B has already defined the production program for using component j up until the beginning of day $t + P_j$. If the scheduling time P_j is higher than or equal to the provision time D_j, the order corresponds exactly to the forecast consumption, and no safety stock should be held. However, if the order is based entirely on statistical knowledge of needs, $D_j - P_j > \theta$, and if it relies partly on firm demand and partly on statistical knowledge of needs, $D_j - P_j < \theta$.

3.2. Determination of the probability distribution

Several factors combine for the determination of the probability distribution of a demand for one period. It requires the use of a simulation approach to calculate the distribution empirically.

An alternative component i always has the same probability p_i of being included in one of the products. Demand for that component therefore is a random variable that comprises three sources of variation:

1. The size of the set to consider, or the product of a daily production n by a number L of production days. L can be certain or random. *A priori*, L can be random only for the components from a supplier.
2. In the steady state, the demand to satisfy X_{iL} defined on L days. This demand follows a binomial distribution $\mathscr{B}(nL, p_i)$.
3. If a delivered product has a positive probability π_i of being defective, X_{iL} demand has little chance of being satisfied entirely.

The demand to consider therefore includes the four elements in Table 1, which we illustrate in Table 2.

Quality of delivered products	L_i constant	L_i random
Always good ($\pi_i=0$) → use of X_{iL}	Case 1	Case 2
Not always good ($\pi_i>0$) → use of Y_{iL}	Case 3	Case 4

Table 1. *Characterization of demand to take into account*

In addition, demand can be the sum of demands from several customers, each of which represents one of the four cases. The probability distribution of that compounded demand is more complex.

Consider case 1 (constant lead time, guaranteed quality). The binomial distribution is not continuous, nor is its cumulative distribution. The definition of a percentile $R_{iL\alpha}$ implies a convention: We retain the

lowest value of $R_{iL\alpha}$, such that demand X_{iL} has a probability lower than α of being exceeded.

If the lead time L is random and quality is guaranteed (case 2), the demand X_{iL} follows a binomial distribution $\mathscr{B}(nL, p_i)$. The number of trials is a random variable, so the determination of this probability distribution is analytically complex. It can be achieved empirically with the Monte Carlo method. These distributions present a multimodal pattern that becomes more accentuated as the probability p_i increases.

In cases 3 and 4, lead time is certain or random, but quality is not guaranteed, so a delivered component is defective with a probability π_i. To meet X_{iL} demand, it is necessary to attain an extra quantity z_{iL} and make $X_{iL} + z_{iL}$ components available. The number of defective parts in this case follows the binomial distribution $\mathscr{B}(X_{iL} + z_{iL}, \pi_i)$. The z_{iL} quantity is an occurrence of the random variable Z_{iL}, which follows the negative binomial distribution $\mathscr{NB}(X_{iL}, \pi_i)$, whose cumulative probability $P(Z_{iL} \le z_{iL})$ corresponds to the probability of a maximum of z_{iL} defective parts in a batch of $X_{iL} + z_{iL}$ parts. It is thus necessary to use the probability distribution of the number of parts available, $Y_{iL} = X_{iL} + Z_{iL}$, to determine the value of the percentile Y_{iL}^α that, with a probability α, cannot cover demand, taking into account the three sources of random factors.

The demand for a component c to cover L production days may be the sum of independent demands. Two cases require the use of a Monte Carlo simulation to define the reference probability distribution:

- $a_{i'}$ units of component c appear in a part i' that belongs to a subset $\mathscr{E}$ of alternative parts that can be mounted in a station of the car assembly line. This presence is the result of the mechanism of the BOM explosion. , well-known in MRP. For example, gears provided to replenish stock get used in the production line of gearboxes that are mounted on the car assembly line. Demand $V_{cLi'}$ for component c of an alternative part i' follows a binomial distribution $\mathscr{B}(a_{i'} \cdot n \cdot L ; p_{i'})$; the probability distribution of the total demand $V_{cL} = \sum_{i' \in \mathscr{E}} V_{cLi'}$ can be obtained by simulation. If $a_{i'} = a, \forall i'$, then V_{cL} follows the binomial distribution $\mathscr{B}(a \cdot n \cdot L, \sum_{i' \in \mathscr{E}} p_{i'})$.

- Demand for component c might come from several assembly lines. If each line l that uses component c has a daily production n_l, the generalization of the previous assertions is immediate: Demand $V_{cLi'_l}$ issued for alternative part i'_l follows the binomial distribution $\mathscr{B}(a_{i'_l} \cdot n_l \cdot L ; p_{i'_l})$, and the probability distribution of total demand $V_{cL} = \sum_l \sum_{i'_l \in \mathscr{E}_l} V_{cLi'_l}$ can be obtained by simulation.

If a problem of quality arises for component c, we use the distribution of $Y_{cL} = V_{cL} + Z_{cL}$, where Z_{cL} follows a negative binomial distribution $\mathscr{BN}(V_{cL}, \pi_c)$. A simulation again is mandatory.

3.3. Influences of stock-out probability and demand variability on safety stock

Generally, safety stock varies in the same direction as the coefficient of variation (ratio of the standard deviation of the demand distribution to its average) and in the opposite direction of accepted risk. Safety stock relations can be established when the order-up-to level is defined to meet the needs of an alternate or optional component in a BOSC, in the steady state. Then a binomial distribution $\mathscr{B}(nL, p_i)$ is used. In certain conditions (Giard 2003a), which exist when nLp_i has a sufficiently high value, an approximation of this distribution can be given by the normal distribution $\mathscr{N}$ $(nLp_i, \sqrt{p_i(1-p_i)nL})$. The definition of the standard normal variable t_α is associated with a stock-out probability α, so the percentile $R_{iL\alpha}$ relates to t_α according to the expected value of the demand distribution and its standard deviation by relation [1].

$$R_{iL\alpha} = nLp_i + t_\alpha \sqrt{nLp_i(1-p_i)} . \qquad [1]$$

The safety stock $SS_{iL\alpha} = R_{iL\alpha} - nLp_i$ is then defined by

$$SS_{iL\alpha} = t_\alpha \sqrt{nLp_i(1-p_i)} . \qquad [2]$$

In an industry, the concept of a safety coefficient is more common, that is, the constant to multiply with the demand average to calculate safety stock. We can express it as a function of the coefficient of variation, or $\sqrt{p_i(1-p_i)nL} / nLp_i = \sqrt{(1-p_i)/(nLp_i)}$. The value of this safety coefficient is $t_\alpha \sqrt{(1-p_i)/(nLp_i)}$, which leads to relation [3].

$$R_{iL} = nLp_i + t_\alpha \sqrt{nLp_i(1-p_i)} = nLp_i(1 + t_\alpha \sqrt{\frac{1-p_i}{nLp_i}}) . [3]$$

The frequent use of a rule that fixes safety stocks according to an empirical coefficient imposed for a large set of references mechanically leads to variable stock-out probability for the same production.

Table 2 illustrates the four cases numerically, with $n = 962$, $p_i = 54.46\%$, L fixed to 12 or random (discrete uniform $\mathscr{DU}(10, 14)$), and $\alpha = 0.01\%$. With compound demand, V is the sum of V_1 and V_2, where $V_1 \sim \mathscr{B}(962L, 0,5446)$ and $V_2 \sim \mathscr{B}(962.4.L, 0,0513)$, $L \sim \mathscr{DU}(10, 14)$, and $\pi = 1\%$.

	Case 1	Case 2	Case 3	Case 4	Compound Demand V
L	12	$\mathscr{U}$	12	$\mathscr{U}$	$\mathscr{U}$
π	0%	0%	1%	1%	1%
$\overline{Y}_{iL}$	6287	6287	6350	6350	8743
R_i	6486	7525	6553	7602	10461
SS_i	199	1238	203	1252	1718

Table 2. Numerical determinations of R_i and SS_i.

3.4. Safety stock and average residual stock prior to delivery

If the probability of stock-out prior to delivery is low, the expected value of the component shortage $I_r(R_{iL\alpha})$ also is low. The safety stock therefore approaches the expected value of residual stock prior to delivery. Generally speaking, the expectation $I_p(R_{iL\alpha})$ of residual stock at the end of a period with demand X is given by relation [4] (Giard 2003b):

$$I_p(R_{iL\alpha}) = R_{iL\alpha} - E(X_L) + I_r(R_{iL\alpha}) .\qquad [4]$$

The order-up-to level $R_{iL\alpha}$ has to meet demand X, and Equation 4 is valid, regardless of the probability distribution of X_L.

When stock-out probability is negligible, $I_r(R_{iL\alpha})$ is close to 0. The average residual stock prior to delivery $I_p(R_{iL\alpha})$ is close to safety stock $R_{iL\alpha} - E(X_L)$. The average residual stock and average stock-out vary in opposite directions and generate, respectively, carrying costs and stock-out costs for the company.

In the case of a normal distribution, the expectation $I_r(R_{iL\alpha})$ of a stock-out, without indices i, L, and α, is given by (Giard 2003b):

$$I_r(R) = \sigma[f(t_R) - t_R P(t > t_R)],$$

where $t_R = (R - \bar{x})/\sigma$, and $f(t_R) = e^{-t_R^2/2}/\sqrt{2\pi}$. $\qquad [5]$

For example, with the data from case 1, we find that when $\alpha = 0.01\%$, $R_{IL\alpha} = 6486$, $I_R(R_{IL\alpha}) = 0.001$, and $I_P(R_{IL\alpha}) = 199.14$

4. Safety Stock of Supplied Components

The order q_{jt} for component j sent by B to A at the beginning of day t gets delivered at the beginning of day $t + D_j$, because the "due date" is D_j, and the lead time is λ_j ($\lambda_j \leq D_j$). The stock level PS_{jt} at the beginning of day t is the sum of the stock observed S_{jt} and the k_j expected delivery, as well as backorders. If the stock-out probability is low, we can ignore the impact of unsatisfied demand (backorders or lost sales). No k_j expected deliveries exist if the periodic replenishment review period is longer than the due date time ($\theta_j > \lambda_j$), but this number is positive otherwise, such that $k_j = \arg\max(K|K \leq D_j/\theta_j)$. The inventory position when ordering is defined by the relation [6].

$$PS_j = S_j + \sum_{h=1}^{k_j} q_{j,t-h\theta} ,$$
where $k_j = \arg\max(K|K \leq D_j/\theta_j)$. $\qquad [6]$

If the planning time P_j of B is less than or equal to the due date time D_j negotiated with supplier A ($P_j \leq D_j$), the order is based on statistical knowledge of anticipated demand. Customer B has no interest in negotiating a due date superior to the lead time, because doing so, even with a similar risk, increases safety stock.

When $P_j > D_j$, if $\theta_j > P_j - D_j$, the sent order can be based partially on known demand. If $\theta_j \leq P_j - D_j$, the order is determined by known requirements, and no safety stock is needed.

If no batch constraint needs to be taken into account when determining the order to send, supply therefore is unit-based. Batch based supply is a little more complicated to define. In both case , transport capacity limitation may be to take into account.

In all cases, we suppose that the accepted stock-out risk α is predetermined and very low.

4.1. Unit-based supplies: Stochastic demand

The probability distribution is defined for the period $\theta + \lambda$, so it follows the law $\mathscr{B}[n(\theta_j + \lambda_j), p_j]$. If the normal approximation is possible, we use the distribution $\mathscr{N}[n(\theta_j + \lambda_j)p_j, \sqrt{n(\theta_j + \lambda_j)p_j(1 - p_j)}]$. In this context, we can adapt Equations 1 and 2 to define the order-up-to level and safety stock:

$$R_{j,\theta+\lambda_j,\alpha} = n(\theta_j + \lambda_j)p + t_\alpha\sqrt{n(\theta_j + \lambda_j)p_j(1 - p_j)} \quad [7]$$

$$SS_{j,\theta+\lambda,\alpha} = t_\alpha\sqrt{n(\theta_j + \lambda_j)p_j(1 - p_j)} . \qquad [8]$$

If the lead time is random or quality is not guaranteed, we need a distribution generated by the Monte Carlo method (from Section 3.1) to determine the order-up to level associated with risk α.

For example, suppose that $\theta = 2$, $\lambda = 10$, $n = 962$, and $p_i = 54{,}46\%$. For $\alpha = 0.01\%$, $R_i = 6486$.

From a supply chain control perspective, two important observations emerge. First, if all alternative components assembled on the same workstation of end customer C are supplied by B, the total daily demand of alternative components is constant (n), due to its multinomial distribution. Therefore, the quantities ordered periodically equal $\theta.n$. Plant B may need alternative components j for its production, but if they are all provided by the same supplier, the property of constant total requirement remains valid. A supply of alternative components shared across several suppliers instead induces a periodic random total demand for each. This periodic load fluctuation may make the capacity commitment difficult for the supplier and induce additional costs.

Second, B might supply C with only a subset of what it needs, which creates a random amount of parts ordered from that supplier. But B can also supply other clients with the same parts, which increases the variability of the demand to be satisfied by B. We cannot determine the demand distribution of component j used by different customers of B analytically, but we can solve it using Monte Carlo methods. Simulation is essential for cases 2, 3, and 4. Moreover, with a quality problem, the generation of the random variable Z must be based on the sum of the demands generated for these customers.

4.2. Unit-based supplies: Stochastic and deterministic demands

We now consider the situation defined by a planning time P_j for customer B, which exceeds the due date time D_j negotiated with supplier A of less than θ_j days ($\theta_j > P_j - D_j > 0$). The due date time D_j is constant and can be equal to lead time λ_j. There is no quality problem. When an order is placed at the beginning of day t, to be delivered at the end of day $t + D_j$, B knows with certainty its needs for periods $t + D_j - 1$ to $t + P_j - 1$, which are the needs of $P_j - D_j$ days.

The order sent on day t is the sum of (1) a known quantity $\sum_{h=t+D_j-1}^{t+P_j-1} x_{jh}$, equal to the demand of $P_j - D_j$ days, after delivery at the end of day $t + D_j$, and (2) a quantity equal to the difference between an order-up-to level determined to meet a random demand with risk α and the inventory position. This demand is defined for a period that covers D_j and increases by the period during which demand is not known, $(\theta_j - (P_j - D_j))$.

The fact that we have no firm information on the days prior to the delivery rather than after it, has no impact. The period to cover is $\theta_j + 2D_j - P_j$. The order-up-to level is the percentile $R_{j,\theta+2D_j-P_j,\alpha}$ of a distribution of probabilities defined over $\theta_j + 2D_j - P_j$, which leads to the distribution $\mathscr{B}\left(n(\theta_j + 2D_j - P_j), p_j\right)$. If the normal approximation is possible, we can use $\mathscr{N}[\,n(\theta_j + 2D_j - P_j)p_j, \quad \sqrt{n(\theta + 2D_j - P_j)p_j(1-p_j)}\,]$, and Equation 9. If we exclude the possibility of lost sales, this difference equals the sum of observed demand on the last $\theta_j - P_j + D_j$ days prior to day t, $\sum_{h=t-\theta_j+P_j-D_j}^{h=t-1} x_{jh}$. In Equation 10, we summarize the determination of the order to send, in the steady state, with a mix of stochastic and deterministic demand:

$$R_{j,\theta+D_j-P_j,\alpha} = n(\theta_j + 2D_j - P_j)p_j + t_\alpha\sqrt{n(\theta_j + 2D_j - P_j)p_j(1-p_j)}$$

, if a normal approximation is possible, and $\theta > P_j - D_j > 0$.[9]

$$q_{jt} = \sum_{h=t+D_j-1}^{h=t+P_j-1} x_{jh} + \sum_{h=t-\theta_j+P_j-D_j}^{h=t-1} x_{jh}. \qquad [10]$$

To take quality into account (case 3), we must use the simulation to obtain the probability distribution and thereby determine the percentile. To define variable Z_j, we assume the quality problem exists for all orders and sum random demand with firm orders. In a steady state, these firm orders are random variables that follow a binomial law with the same probability used to deal with the random needs taken into account in the order. It is therefore necessary to use the distribution given by Equation 11 to determine the percentile $R_{j,\theta_j+2D_j-P_j,\alpha}$:

$$Y_j = X_j + Z_j,$$

where $X_j \sim \mathscr{B}\left(n(\theta_j + 2D_j - P_j), p_j\right)$,

and $Z_j \sim \mathscr{BN}(X_j, \pi_j)$. [11]

For example, if $D_j = 10$, $P_j = 12$, and $\theta = 5$, an order sent at the beginning of day $t = 1$ will be delivered at the end of day $t = 10$. It satisfies needs during $t = 11$ to $t = 15$. The needs for days 11 and 12 are known, but those for the next three days are unknown. The reference period is 13 days, $R_{1;13;0.01\%} = 7091$, and $SS_{1;13;0.01\%} = 212$.

Next suppose that the lead time λ_j is random and that the quality of the delivered items is guaranteed (case 2). The order sent at the beginning of day t still covers the needs for days $t + D_j - 1$ to $t + \theta_j - 1$. The demands for $P_j - D_j$ days are perfectly known. In a steady state, these demands correspond to a sum of daily demands, following the same distribution used for the other days (i.e., $\mathscr{B}(n, p_j)$). The random period of the distribution reveals the order-up-to level as the sum of the random lead time λ_j and the period during which demand is not known between two deliveries, $(\theta_j - (P_j - D_j))$.

For example, if $\lambda_j \sim \mathscr{U}(4; 8)$, $q_j = 8$, $P_j = 14$, and $D_j = 12$. The random period to cover is $L_j = \lambda_j + 6 \sim \mathscr{U}(10; 14)$. With $n = 962$, $p_i = 54.46\%$, and $\alpha = 0.01\%$, we find that $R_{iL} = 7525$. If $\pi_1 = 1\%$, then $R_{iL} = 7602$.

4.3. Batch-based supplies

External supply often depends on batch constraints. Each order sent is a multiple of the number κ of units used in transport, as models of stock determination note. The analytic solution is characterized by a double inequality for the cumulative probability of two successive discrete values, multiples of κ. They enclose an optimal target probability, depending on the structure of the costs used in the objective function of the model. To supply an USC, we cannot exceed risk. The propositions we offer next are valid in the four cases in Table 1.

First, if the order sent corresponds to the smallest multiple of κ, respecting the condition of a stock-out probability inferior to α (i.e., $\kappa \cdot \arg\min(K|K \ge (R_j - PS_j)/\kappa)$), the protection against stock-out risk is excessive.

Second, if the order sent corresponds to the greatest multiple κ, respecting the condition of a stock-out probability superior to α (i.e., $\kappa \cdot \arg\max(K|K \le (R_j - PS_j)/\kappa)$), the protection against stock-out risk is insufficient.

The multiple to choose depends on a reasonable risk β, greater than α and sometimes due to batch limitations.

Based on case 1, we assume the transport container contains $\kappa = 18$ units. With $\alpha = 0.01\%$, we find $R_{iL} = 6486$. Suppose we accept risk $\beta = 0.015\%$. Then $P(X_i > R_i) \le 0.015\%$ is equivalent to $R_i = 6480$. Although $q - \kappa \cdot \arg\min(K|K \ge R_j - PS_j)/\kappa)$ is less than 6, the choice of $\kappa \cdot \arg\min(K|K \ge (R_j - PS_j)/\kappa)$ leads to risk not greater than β.

4.4. Incurred risk and transport capacity limitations

Means of transport usually have limited capacity, G. The ordered quantity cannot exceed this limit ($\Rightarrow \text{Max}(R_j - PS_j, G)$). The incurred stock-out risk thus will be higher than desired, unless there is virtually no chance that demand exceeds this capacity. In this case, transport may be less efficient. To preserve the target risk level α, it is necessary to increase the order-up-to level, which we can do easily with a dichotomy method.

Therefore, if $n = 962$, $p_i = 54.46\%$, $\theta = 2$, $\lambda = 10$, and $\alpha = 0.01\%$, we find $R_i = 6486$ (see Table 2). If $G = 1060$, a simulation of this periodic replenishment policy for 5 million iterations leads to a risk of 0.0774%. To keep $\alpha = 0.01\%$, it is necessary to fix R_i to 6530. The safety stock then increases from 199 to 233.

This simulation widens the scope of the analysis from a local optimization to a more global one that includes transport, carrying, set-up, and stock-out costs. A batch limitation is more complicated to take into account; the transport capacity limitation is necessarily a multiple of packaging size.

5. Safety Stock of Produced Components

We focus on the production of a product i by unit B, in response to demand from customer C. In this context, the lead time λ_i between the start of production and the component's appearance in stock is assumed to be constant. The lead time corresponds to cycle time or production time, as appropriate.

Customer C sends B its orders for i at the beginning of the day, every θ_i days. The order received at the beginning of day t must be sent at the end of day $t + D_i$. With a lead time λ_i, B has $D_i - \lambda_i$ days to produce and send the order ($D_i \geq \lambda_i$). Component i gets integrated into the production cycle of H days. The production of component i finishes F_i days after the beginning of the cycle. If this cycle includes only component i, it is obvious that $H = F_i$, and there is no reason to synchronize deliveries (all θ_i days) or launches in production (all H days).

We first consider a make-to-stock production, then study a make-to-order production, and finally a combined production process.

5.1 Make-to-stock production

We assume the production cycle H is shorter than the replenishment cycle θ ($\rightarrow H \leq \theta_i$) and that the order from customer C is immediately executable ($D_i - \lambda_i = 0$) or else that the time remaining before delivery is less than the replenishment cycle θ ($\rightarrow D_i - \lambda_i \leq \theta_i$). The distribution to determine the order-up-to level is the binomial law $\mathscr{B}(n\theta_i, p_i)$. In a steady state, the quantity that B launches into production is equal to the quantity sent to C; it is a random variable. However, if the supplier produces all alternative references assembled on the workstation of the customer assembly line at the same time ($\rightarrow \theta_i = \theta$), the sum of ordered quantities is constant ($n\theta$), because demand for alternative components follows a multinomial distribution.

If $H > \theta_i$ and the time remaining before delivery is less than the production cycle, we can distinguish two cases.

1. If product i is the only one to be manufactured in this production cycle, $H = F_i$. When the order is sent at the beginning of day t, the inventory position PS_{it} of component i increases by the quantity launched q_{it} ($= R_{it} - PS_{it}$) and should meet demand until the end of the next production cycle, on day $t + 2H$. The number of deliveries during a period of $2H$ days must be between $\eta_1 = \mathrm{argmax}(K|K < 2H/\theta)$ and $\eta_2 = \mathrm{arg\,min}(K|K > 2H/\theta)$. For example, with $H = 5$ and $\theta = 4$, the number of deliveries is either 2 or 3. If $2H$ is a multiple of θ, the number of

deliveries during a $2H$-day period is constant. The distribution to determine the order-up-to level is binomial $\mathscr{B}(n\eta\theta_i, p_i)$, with $\eta = \eta_1$ or $\eta = \eta_2$. It often can be approximated by $\mathcal{N}(n\eta\theta; \sqrt{2n\eta\theta p_i(1 - p_i)})$.

For example, if B receives orders every 4 days from C and $F_i = 5$ days, the distributions are $\mathscr{B}(962 \times 2 \times 4; 0.5446)$ and $\mathscr{B}(962 \times 3 \times 4; 0.5446)$. When $\alpha = 0.01\%$, the order-up-to level is 4373 (two deliveries) or 6486 (three deliveries).

A lack of synchronization between cycles H and θ leads to periodic variation in the production. The quantity to be launched in production by B equals the sum of quantities previously shipped to C only if the order-up-to level used for the production launch is the same as that used previously. The sum of the quantities launched varies strongly when the number of deliveries since the previous launch changes (due to the change in η). This sum equals $n\eta\theta$ only if the order-up-to levels have not changed since the previous launch and B produces all alternative components.

2. If several components successively launch into production, an order sent at the beginning of day t increases the stock level PS_{it} of component i by the quantity launched in production q_{it} ($= R_{it} - PS_{it}$), to be delivered at the beginning of day $t + H$. It should meet demand until the next delivery, at the beginning of day $t + H + F_i$. The number of deliveries during $H + F_i$ days must be between $\nu_1 = \mathrm{argmax}(K|K < (H+F_i)/\theta)$ and $\nu_2 = \mathrm{arg\,min}(K|K > (H+F_i)/\theta)$. If $F_i = 2$ and $H = 5$, the number of deliveries is either 1 or 2. If ($H + F_i$) is a multiple of θ, the number of deliveries over $H + F_i$ days is constant. The distribution for determining the order-up-to level is binomial $\mathscr{B}(n\nu\theta_i, p_i)$, with $\nu = \nu_1$ or $\nu = \nu_2$. The distribution can be approximated by $\mathcal{N}(n\nu\theta; \sqrt{2n\nu\theta p_i(1 - p_i)})$ in many cases. The previous remarks about the variability of production remain valid.

In our example, the distributions are laws, $\mathscr{B}(962 \times 2 \times 4; 0.5446)$ and $\mathscr{B}(962 \times 1 \times 4; 0.5446)$, which lead to order-up-to levels for $\alpha = 0.01\%$ of 4373 (two deliveries) or 2211 (one delivery).

5.2. Make-to-order production

We suppose the production cycle is shorter than the replenishment cycle ($\rightarrow H \leq \theta_i$) and the supplier anticipation is greater than the replenishment cycle ($\rightarrow D_i - \lambda_i > \theta_i$). Then, production can be made to order, and a production launch involves no more than one delivery. The maximum number of canceled consecutive launches is $\mathrm{arg\,max}(K|K < (D_i - \lambda_i)/H)$. For example, if $H = 5$, $\theta_i = 12$, and $D_i - \lambda_i = 14$, the maximum number of null consecutive launches is 2.

If $H > \theta_i$ and supplier anticipation is greater than twice the production cycle ($\rightarrow D_i - \lambda_i > 2H$), production also can be made to order. In this context, it is possible to launch a production quantity that should be delivered during the following production cycle, as well as one or more subsequent deliveries if anticipation is sufficient.

However, it is preferable to smooth the load and launch only the deliveries for the following production cycle. Again, the number of deliveries to consider can vary between two production cycles: at least 1 and no more than $\arg\max(K \mid K > (D_i - \lambda_i - H)/\theta)$. If $H = 5$, $\theta_l = 3$, and $D_i - \lambda_i = 11$, we would launch no more than the quantity of two successive deliveries.

The irregularity of the number of launches and the quantities launched leads to disorganization. However, it can be eliminated if the production cycle equals the periodic replenishment review period but remains less than suppliers' anticipation ($\rightarrow D_i - \lambda_i > \theta$). Quantities vary from order to order, because they correspond to quantities consumed, whose distribution is binomial. However, if the supplier produces all alternative components for the customer's assembly line, the sum of ordered quantities is constant ($n\theta$). If this supplier has multiple customers for all or some produced components, the condition extends to each customer.

5.3. Mixed production

If $(H \leq \theta_i)$, production is to made to stock if $D_i - \lambda_i \leq \theta_i$ and made to order otherwise ($D_i - \lambda_i > \theta_i$). If $H > \theta_i$, production is to made to order if D_i-$\lambda_i > 2H$. If $H > \theta_i$ and $H < D_i - \lambda_i \leq 2H$, production may be a combination. The analysis of these possible configurations is similar to that in Section 5.1, except that only μ deliveries to the customer on $D_i - \lambda_i$ days after the production launch of component i on day t are known. They can be covered by made-to-order production. The number μ is either $\mu_1 = \arg\max(K \mid K < (D_i - \lambda_i)/\theta)$ or $\mu_2 = \arg\min(K \mid K > (D_i - \lambda_i)/\theta)$. The difference, possibly null, between η and μ therefore corresponds to deliveries that can be covered by a made-to-stock production.

Suppose $D_i - \lambda_i = 7$, $\theta = 4$, and $H = 5$. The inventory position updates with a decision to integrate known firm orders from the customer and deliveries for later orders. In this example, quantities launched in production at the beginning of the second production cycle are made to order. Quantities launched in production cycles 1 and 4 are made to stock and include one delivery only. The production of the third cycle is partly made to order and partly made to stock (two deliveries produced).

As in Section 5.1, we again face a possible change in production across cycles, related to the variable number of orders from the customer. Safety stock decreases, or even disappears, when we can produce to order.

6. Conclusion

This study focuses on a supply chain dedicated to the mass production of customized products, controlled by periodic replenishment policies. The analysis of the relations between the successive units of the supply chain show that safety stocks of produced and supplied components are needed. The importance of these stocks depends simultaneously on the level and structure of the production of the final assembly unit, lead time variability, levels of quality control, lot sizing transportation rules, and transport capacity constraints.

In addition, the variability of the periodically exchanged quantities between two successive units depends on the demand anticipation available for decision making. This variability in turn depends on the level of synchronization of delivered, produced, and replenished flows that can lead to or avoid the creation of production cycles. A plant has more variable total activity if it does not produce all the diversity demanded at its level by the alternative components it assembles. The relevance of periodic replenishment rules or production launch rules depends on the structural information propagated along the supply chain. To highlight the conditions for efficient control, we postulate a stable, steady state, which may be attained even if the characteristics of the steady state are periodically disrupted. However, a high level of performance requires the circulation of information in time, all along the supply chain. This goal in turn demands a proactive attitude from the owner of the final assembly plants.

7. Appendix: Summary of Notations

Constants

θ, θ_i, θ_j	Replenishment cycle
α	Target stock-out probability (stock-out risk)
β	Stock-out maximum accepted risk
D_i, D_j	"Due date" time of component i, j
F_i	Production time of component i
P_j	Planning horizon of component j
n	Daily production
t_α	Standard normal variable value for risk α
κ	Packaging unit used in transport
G	Transport capacity
H	Production cycle time

Decision variables

$R_{iL\alpha}$	Order-up-to level for component i, on L days, with risk α
q_{it}, q_{jt}	Order for produced component i or supplied j at the beginning of day t
$SS_{iL\alpha}$	Order-up-to level for a component Safety stock for component i

Random variables

λ_i, λ_j	Lead time of component i, j
L	Number of days of demand to cover
p_i	Assembly probability of component i
π_i, π_j	Defective probability of component i, j
X_{iL}	Demand to satisfy of component i on L days
Z_{iL}	Components i held to cope with quality issues
Y_{iL}	Components i available
PS_{jt}	Inventory position of component i at the beginning of day t
S_{jt}	Observed stock of component j on day t
$SS_{iL\alpha}$	Safety stock of component i
k_j	Number of expected orders for component j

Functions and operators

$I_r(R_{iL\alpha})$ Expected stock-out
$E(X_L)$ Expected demand
$I_P(R_{iL\alpha})$ Expected residual stock

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