Need and determination of safety stocks in deterministic universe due to lot-sizing problem in a supply chain dedicated to customized mass production

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In a supply chain designed for the production of goods of wide variety, the production schedule creates an ordered list of all the alternate components that will be mounted on a same post of the chain. The supply of these components by the supplier may induce transportation-related constraints. There from systematically follows a time-gap between orders and deliveries. This mechanism, commonly called "rank-change", may be analyzed with the help of the multinomial distribution associated to these alternate components; only a simulation approach has relevance as of that matter, for an analytic approach is not possible. Such an analysis allows assessing the importance of the need to form safety stocks because of the lot sizing, even though requirements are certain.

INTRODUCTION

The subject will be an elementary supply chain consisting in a production unit (indicated in this article as the customer) configured as an assembly line allowing mass production of various products (an automotive production line, for example) and another unit (indicated here as the supplier) producing alternate components (e.g. car engines) or optional components (e.g. sunroof) assembled on a work station of this line and contributing to the required diversity (Anderson and Pine, 1997). In the context of our study, optional components are a particular type of alternate components.

For the few weeks to come, the customer daily production (= n) is predetermined by the opening duration of the line (stable on this horizon) and the line's cycle time. The total daily demand of alternate components to order from the supplier is thus known. From the final demand (vehicle orders), the industrial customer determines his production schedule on a horizon of K days, which in turn determines, on this horizon, the ordered list of the alternate components to assemble on the assembly line work station where they are mounted. Beyond this horizon K, the customer only has information about the average structure of the demand. At the beginning of every t day, the schedule of days t to t+K-1 is kept and the new orders of day t+K are sequenced according to the revolving planning logic. The ordered list of all alternate components to mount determines the order of demand of these components. For the purpose of our study, it is assumed that the requirements are daily. They are designed to fulfill the sequenced demand of alternate components. In our case, the lead-time $λ$ is shorter than K, to stay in a context where the sequence of the alternate components to mount is known when daily orders are sent.

In this context, the requirements that are submitted must be considered as firm. We will assume that their delivery was guaranteed. The impact of batch constraints on the supply chain control does not seem to be evoked in literature. The lot-sizing influence on capacity due to scheduling is analyzed for a long time. The capacity is reduced by the setup time of a reference, possibly depending on the sequence of references to produce (White and Wilson, 1977). The optimal sizes for the batches result from the minimization of a cost function that includes possession and setup costs. The determination of the size of the batches to produce can be constrained by conditioning or by the limited storage near a workstation that mounts alternate components. The conditioning also plays a role in certain issues regarding transportation: it influences the transportation means capacity, but not the orders assessment to submit to the supplier. Therefore we will broach the impact of lot-sizing

constraints on the management of the various logistical flows, assuming that deliveries match with the orders and made in time.

This problem will be analyzed in the context of a supply of alternate components to an assembly line, from which results that the procedure is the same when constraints on alternate components influence a production schedule. The first section will explain how the integration of the lot-sizing constraint can create rank-changes; a second section will detail the procedure to create safety stocks in a number that will allow facing those rank-changes.

1. ANALYSIS OF THE RANK-CHANGES CAUSED BY THE LOT-SIZING

In a first paragraph, we will explain the concept of rank-changes, then we will outline the rank-change function. We will focus here on the integration of the lot-sizing constraints on supply. Further on, we will see that the same mechanism applies to production when the scheduling relies on batch deliveries of alternate components that are to be mounted on a same work station of the line, for the diversity of the supply is impossible to ensure at the line side.

1.1. Overview of the rank-change mechanism

The demand of alternate components to mount at a workstation on the assembly line is expressed by the ordered list S_1, periodically updated. The customer orders M engines every θ days and receives, with the same periodicity, a shipment of M engines, ordered λ days before (λ being the supplier's lead time). Here is supposed that conditioning constraints force the grouping of delivered references by lots of m identical products, which leads to a delivery of $\gamma = M/m$ batches. It is assumed that deliveries are completed on time and perfectly match the demand. Lot sizing has two consequences:
- At the arrival of each delivery one must perform a reconstruction of the sequence of M alternate components respecting the order of assembly.
- The batch of M delivered components has practically no chance to coincide with the batch that will be assembled; the number of missing components corresponds to the number of surplus components.

The existence of a rank-change mechanism is known in the case of the handling of a quality issue on a vehicle production line, taking effect through the removal of the vehicles to reprocess, for a variable time before their reinsertion on the line (Giard *et al.*, 2001a, 2001b;

Giard, 2003). They refer to rank-changes as the "difference, positive, negative or nul, between the final rank of a unit at the exit of the work station and its rank prior to entering this station (a negative rank-change thus corresponds to a situation of advance)". The Figure 1 facilitates the understanding of the rank-change mechanism in our example.

Figure 1 Origin of rank-changes
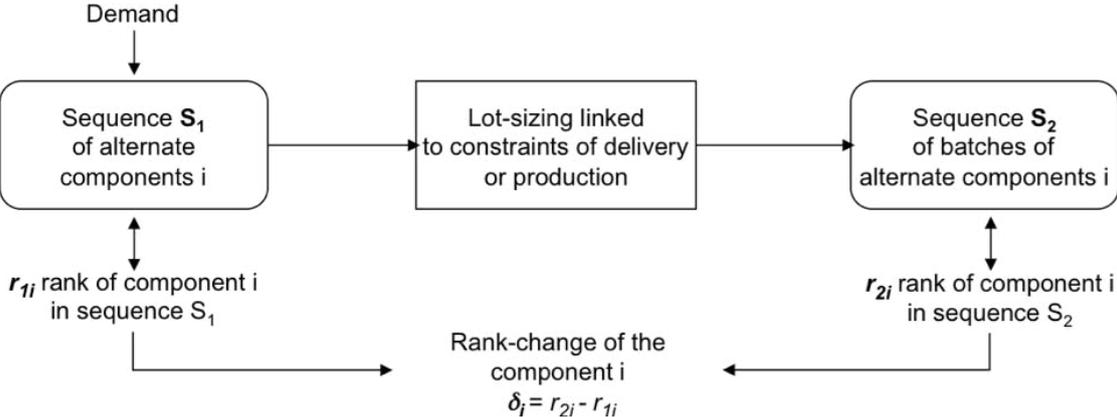
To define the sequence S_2, we need to start from sequence S_1 and take into account the evolution of the projected stock level affected by the following demands of S_1. If the consideration of the demand of rank i into sequence S_1 leads to a negative stock level, we need to avoid stock-out with a batch delivery of the concerned reference. Two main methods can be adopted:

- Waiting upon completion of a batch of firm demands to position the aforesaid batch of products into S_2, which means a temporarily negative stock level, which can further be nullified when it equals $-m$, that is to say when a batch is launched.
- Deciding to launch a batch of m components as soon as their projected stock level becomes negative (then amounting $m-1$).

Following the first method, restricted to firm demands and with a focus on product i that was not yet launched and whose projected stock level just turned negative, the probability that a batch of m units of the same reference cannot be completed with the remaining demands of ω components is obtained by the Negative Binomial distribution $\mathscr{NB}\,(m\text{-}1,\,\omega,\,p_i)$. This probability can be significant and lead to a stock-out. For instance, the risk that a batch of six products, with a probability of demand $p = 0.9\%$ and $\omega = 1,000$ unaddressed requirements, should not be completed amounts 11.3%. In a context where a stock out is hardly acceptable because of a high line stoppage cost, the second lot-sizing method is necessarily privileged. Now that the concept of rank-change is defined, we shall study its function.

1.2. Study of the rank-change function

To understand the following results, we shall base our approach upon the probability of rank-change of a product, with a variable δ_i corresponding to the number of ranks that a product won or lost at delivery, compared to its consumption. There is no analytic way to determine the distribution of the rank-change probability. Only an approach by simulation, using the Monte Carlo method, allows drawing relevant conclusions, in an environment defined by precise specifications (multinomial distribution, K, λ, θ, m).

The study of a rank-change function was conducted on the basis of a vehicle assembly line on which a workstation mounts the engine selected by the final customer, who can choose between 19 engines P_i (alternate components). The engines are produced in another plant in which there is no setup time between references on the production line. The engines are then shipped to assembling plants in packages of six identical engines. Let assume that the distribution of the demand for each engine p_i is that of Table 1.

Table 1 Distribution of the probability by product

Product	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%
Product	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	
p_i	0.4%	0.9%	3%	6.5%	6.1%	3.5%	6.1%	11.3%	2.6%	

We proceeded to the simulation of a randomly generated sequence of 3 billion products following the distribution of probability from Table 1. The 12,000 first components are excluded from the simulation since they represent the transient state coming before the steady state that is our interest (this transient state was willingly overestimated). The packages can hold up to six identical products. Thus the size of the batches is six. Concerning the handling of the product of rank y in sequence S_1, the rule to create S_2 is as follows:

- If the required product is in the stock, it is taken out of there, thus its stock level is lowered by one unit;
- Else, the production of a batch of six products is launched, and the batch is included in the stock, which level is incremented by five;
- Move on to product $y+1$ in S_1.

Table 2 summarizes the simulation results for each product P$_i$ having the probability of use p_i, their average $\overline{\delta_i}$ of rank-changes, the standard deviations $\sigma_{\delta i}$ of the rank-changes for each product.

Table 2 Rank-change expectations –standard variations of rank-changes– per product

| Product	P$_1$	P$_2$	P$_3$	P$_4$	P$_5$	P$_6$	P$_7$	P$_8$	P$_9$	P$_{10}$
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%
$\overline{\delta_i}$	-98.7	6.99	-65.2	-230	-230	39.4	29.44	-145.7	-230	-576
σ_{δ_i}	134	36.8	103	257	258	9.13	16.75	178.1	259	585.7
Product	**P$_{11}$**	**P$_{12}$**	**P$_{13}$**	**P$_{14}$**	**P$_{15}$**	**P$_{16}$**	**P$_{17}$**	**P$_{18}$**	**P$_{19}$**	
p_i	0.4%	0.9%	3%	6.5%	6.1%	3.5%	6.1%	11.3%	2.6%	
$\overline{\delta_i}$	-586	-229	-36.5	8.95	7.52	-24.64	6.40	25.20	-48.2	
σ_{δ_i}	588	256	76.3	34.6	36.1	65.44	36.80	20.18	87.62	

According to the notes in Figure 1, a negative value for rank-change δ_i means that the recently arrived product is advanced regarding its positioning in the initial demand; in the example, product 10 is systematically in advance. A positive value means that the product arrived with delay compared to its initial positioning, as product 6 may illustrate. The weighted sum of average rank-changes $\overline{\delta_i}$ by probability of use p_i is null[1]. The rank-change curves of the products are not identical: the mathematical expectation of the rank-change of products varies in the same way as their probability of use (Figure 2), whereas the standard deviations of product rank-change vary in inverse order of their use probability (Figure 3). Therefore, those products in low demand are delivered somewhat ahead of schedule, whereas the delivery of products in high demand is quite delayed.

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[1] In steady state, there is necessarily compensation between won and lost ranks. On a sample, the observed average (here $\overline{\delta_i}$) coincides only exceptionally with the mathematical expectation (0).

Figure 2 Evolution of the mathematical expectation of
rank-change of products according to their probabilities p_i
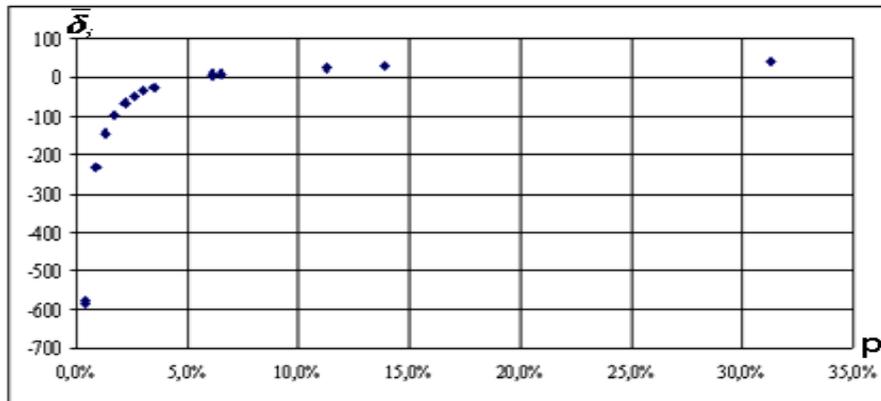
Figure 3 Evolution of the standard variation of
rank-change of products according to their probabilities p_i
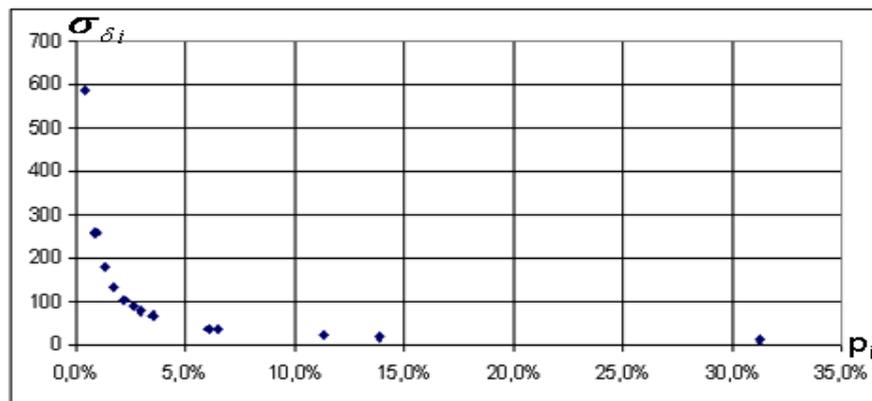
The rank-change function of a product i depends both on the number l of products ($l = 1,..,l$) and on the distribution of probabilities p_i. Figure 4 details the occurrences of rank-change for a product with low demand, and Figure 5 is for the product with the highest demand; outlined with different scales. In the case of a product with low demand, the rank-change is essentially negative, with widely spread negative values (up until 1,000 won ranks, even 2,000 at certain points) and a very heterogeneous occurrence of values: on there is a peak for a small amount of lost ranks, then the occurrence of rank-changes decreases as the number of won ranks increases. In the case of a product with high demand, rank-changes are positive, but the number of lost ranks remains quite low though (approx. no more than 75). The occurrence of rank-changes is symmetrical to an approximate value of 40 lost ranks, with no high difference between the occurrences.

Figure 4 Curve of rank-changes for product P_{12} ($p_{12} = 0.9\ \%$; $\overline{\delta_{12}} = -229$)
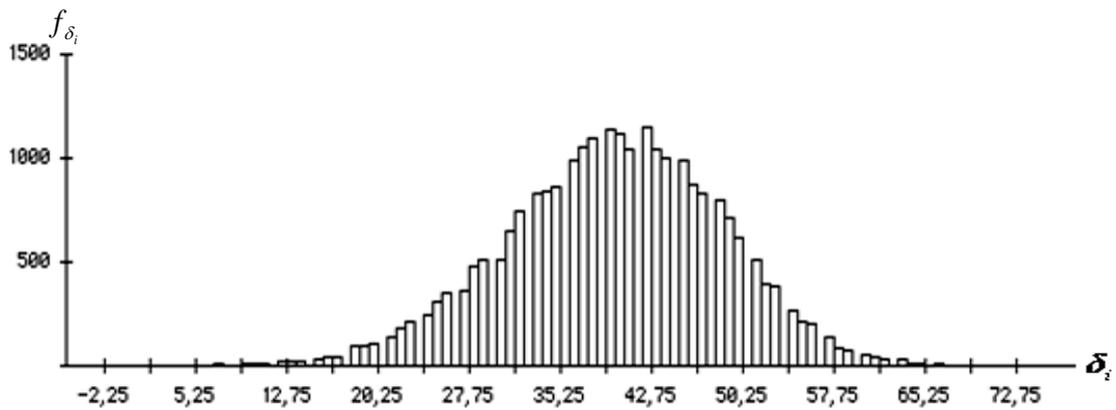
Figure 5 Curve of rank-changes for product P_6 ($p_6 = 31.3\ \%$; $\overline{x_i} = -39.4$)
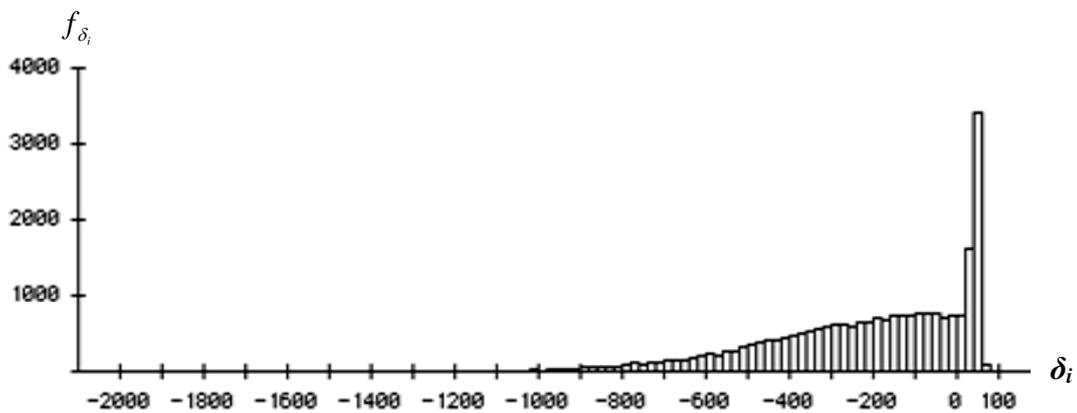
Lot-sizing creates rank-changes, which in turn can create stockout. The customer must thus form safety stocks (at the vehicle assembling plant) to avoid the side of lot-sizing on supply.

2. NEED OF A SAFETY STOCK TO COUNTER THE SIDE EFFECTS OF RANK-CHANGES INDUCED BY THE LOT-SIZING

We shall first explain the need of a safety stock to face the lot-sizing in a deterministic universe, and then its determining factors will be analyzed.

2.1. Explanation of the need

We have seen that the probability that the delivered products match with those to assemble is very low, because of the rank-changes induced by the lot-sizing. To avoid an interruption of supply, one needs to maintain a safety stock for every complete product. These safety

stocks are fed by early products, which consequently lower the amount of products left to create the safety stock. Late products refill the safety stocks from which products where taken to ensure punctual assembly. We have seen that products with low demand are ordered (and thus delivered) somewhat in advance, but their high dispersal force the creation of safety stocks. On the contrary, the strongly demanded engines are delivered with average delay, but their dispersal minimizes the need of safety stocks.

It is important to precise that every rank-change does not systematically create an interruption of supply and, consequently, the creation of a safety stock: the supplier's deliveries concern γ delivered batches with a lead time θ. Thus it is not sequence S_2 that determines interruptions of supply but the constitution of a group of γ delivered products, while changes in this group are still possible without bearing any consequence.

2.2. Analysis of the factors determining the safety stock

In this type of case, it is not possible to obtain analytic results from the rank-change function of a reference on the basis of the multinomial distribution. Only a simulation can be performed. This procedure allows highlighting "tendencies" of the impact of certain factors, as every set of hypothesis is likely to modify the relative impact of the considered factors.

Every simulation was performed on a demand of 6 billion products, in order to empirically obtain safety stocks SS_i with an insignificant stock-out probability. At the beginning of the simulation, the amount of available products was set to a value W. The first delivery is made immediately before the first product is taken. This first delivery corresponds to the θ first products of sequence S_2 and the first withdrawn product corresponds to the first product of sequence S_1. Each safety stock level is calculated as the difference between W and the lowest inventory level during the simulation, since W must be high enough to avoid an empty stock.

In deterministic universe, safety stocks depend on the rank-changes and on the lead time θ. The rank-changes depend in turn on the range I of products and on their probabilities p_i. We will evaluate the safety stock level by a simulatory approach of the steady state. We shall study the impact of various factors on the safety stock, in a context of a lot-sizing caused by transportation constraints. First of all, we shall analyze the impact of probabilities on the safety stock. On the basis of the data in Table 1, we obtain by those simulations the following safety stock amounts (Table 3).

Table 3 Safety stocks related to rank-change

Product	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%
SS_i	5	10	6	4	4	33	17	5	4	3
Product	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	
p_i	0.4%	0.9%	3%	6.5%	6.1%	3.5%	6.1%	11.3%	2.6%	
SS_i	3	5	8	11	11	8	10	16	8	

The safety stock of products varies in the same way as their probabilities (Figure 6), which was not obvious *a priori*. In this industrial example, an approximately linear relation can be noticed ($\hat{\rho}^2 = 0.983$; $SS_i = 0.95889\, p_i + 3.008$).

Figure 6 Evolution of the safety stock SS_i according to the probabilities p_i
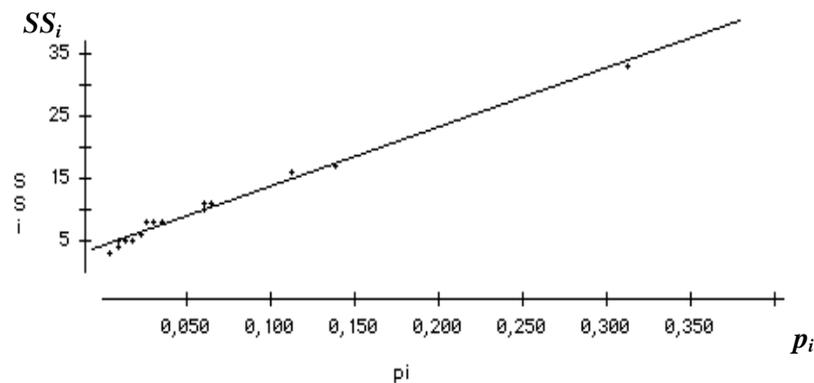

```
Dependent variable is:    SSi
No Selector
R squared = 98,4%      R squared (adjusted) = 98,3%
s =  0,9364  with  19 - 2 = 17  degrees of freedom

Source      Sum of Squares   df    Mean Square    F-ratio
Regression  891.094           1    891.094        1.02e3
Residual    14.9061          17    0.87683

Variable   Coefficient   s.e. of Coeff   t-ratio    prob
Constant   3.95323        0.2669         14.8       ≤ 0,0001
pi         95.8887        3.008          31.9       ≤ 0,0001
```

We shall thereafter study the impact of the dispersal of probabilities on the safety stock level, on average this time. The various products can have equal probabilities: this is a case of equal probabilities of demand. They can also be extremely different, all situations are possible. We thus proceeded to new simulations, replacing the observed probabilities by distribution $p_i = p_1 \times k^{i-1} + b$, with a constraint of $\sum_i p_i = 1$. We shall outline various possible curves, descending and of exponential nature. We state that b = 0,005; it is a limit corresponding the lowest probability for a product. k is the factor for controlled decrease.

The closer it is to 1 k, the more equal probabilities of component demands are. Figure 7 illustrates the distribution of these probabilities.

Figure 7 Example of curves of probability distribution
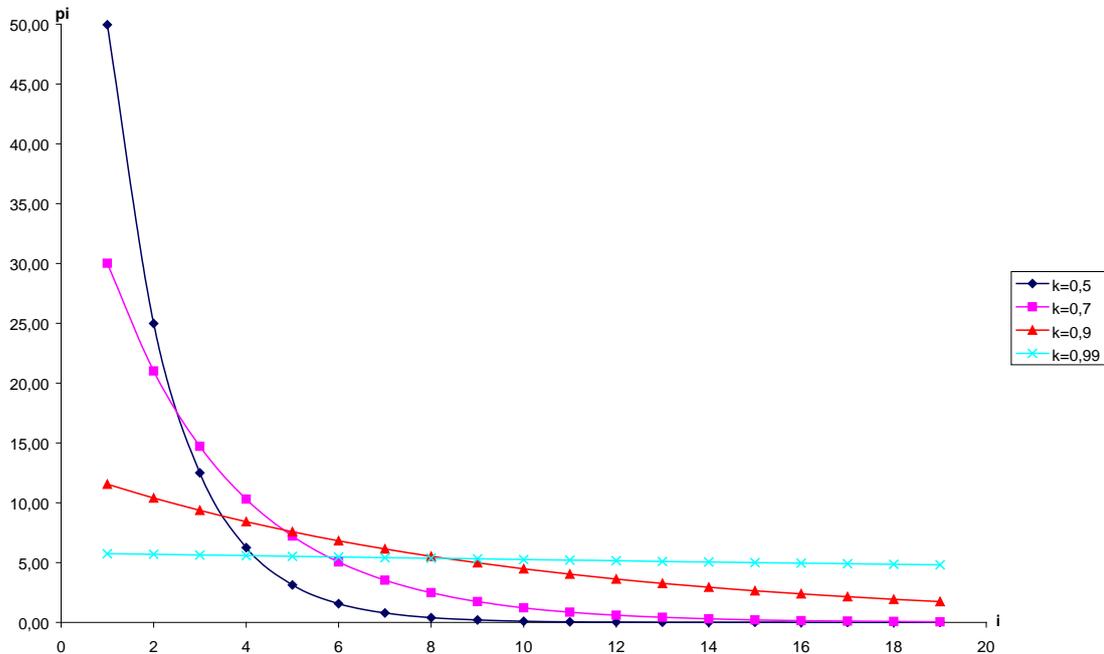
An approximately linear relation can be noticed ($\hat{\rho}^2 = 0.991$; $\overline{SS} = 0.0438\ k + 0{,}1579$)

Figure 8 Evolution of the average safety stock $\overline{SS}$, depending on k

Dependent variable is:　**SSi bar**
No Selector
R squared = 99,2%　　R squared (adjusted) = 99,1%
s = 0,09063　with　8 − 2 = 6　degrees of freedom

Source	Sum of Squares	df	Mean Square	F-ratio
Regression	6.31521	1	6.31521	769
Residual	0.0492802	6	0.00821336	

Variable	Coefficient	s.e. of Coeff	t-ratio	prob
Constant	6.5549	0.1196	54.8	≤ 0,0001
k	4.37857	0.1579	27.7	≤ 0,0001
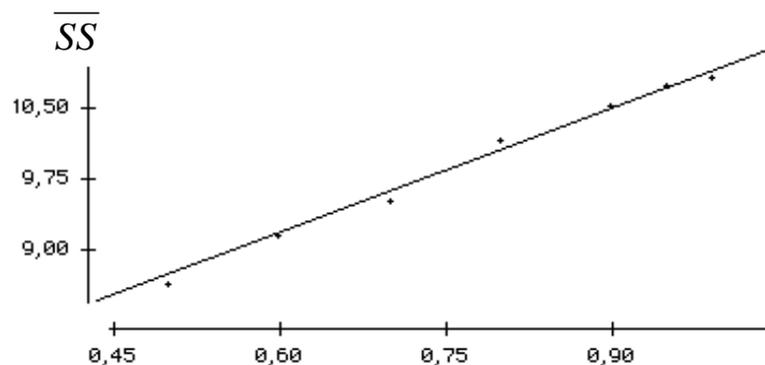
The following factor of variation corresponds to the supply periodicity θ. We proceeded to an analysis of its impact by repeating the same simulation with different values for θ. This analysis was performed using three structures of demand, with 5, 10 and 19 possible products, plus an equal probability of demand between those products (yielding the worst case for safety stock). We note that when the lead time increases, the average safety stock decreases in a fairly constant way (Figure 9).

Figure 9 Average safety stock $\overline{SS}$ depending on θ for various values of I
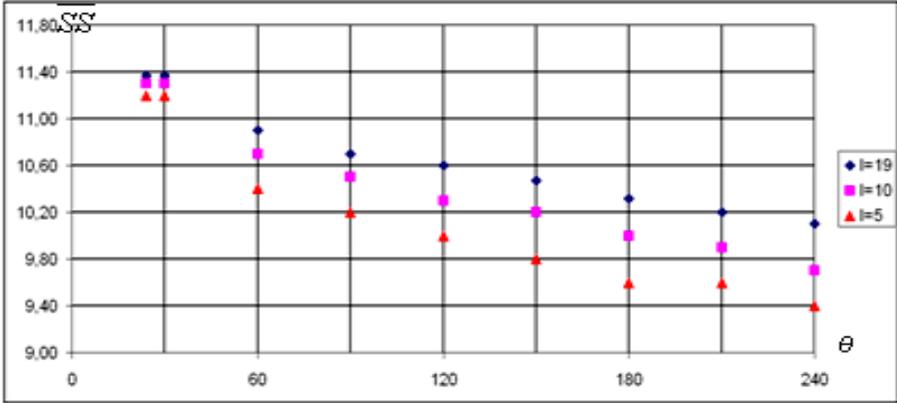
The use of three different structures of demand allows highlighting two phenomenons. The first impression, on which we will further elaborate, is that when the variety of products increases, the average safety stock decreases, which seems logical because when the variety is lower, lot-sizing induces rank-changes of necessarily lesser importance. The second remark is that of a similarity between the variation profiles of the safety stock compared to the lead time for various values of I: the series of points for various values of I really look alike.

We proceed to a more in-depth analysis of the impact of variety I on the average safety stock for equal distributions, as illustrated in Figure 10. The more products, the greater the average safety stock needs to be in order to counter rank-change side effects. However, this increase is not proportional to the increase of variety: the increase is strong for values of I under 8 included, then slightly less pronounced for I between 8 and 12, the average safety stock closing in to an asymptote for I greater than 12. This means that increase of the variety of products has a strong impact on the average safety stock for $I \leq 12$ in our example, but beyond those values I, adding one or several alternate components bears no consequence on the average safety stock lever (if properly calculated).

Figure 10 Average safety stock $\overline{SS}$ depending on variety I
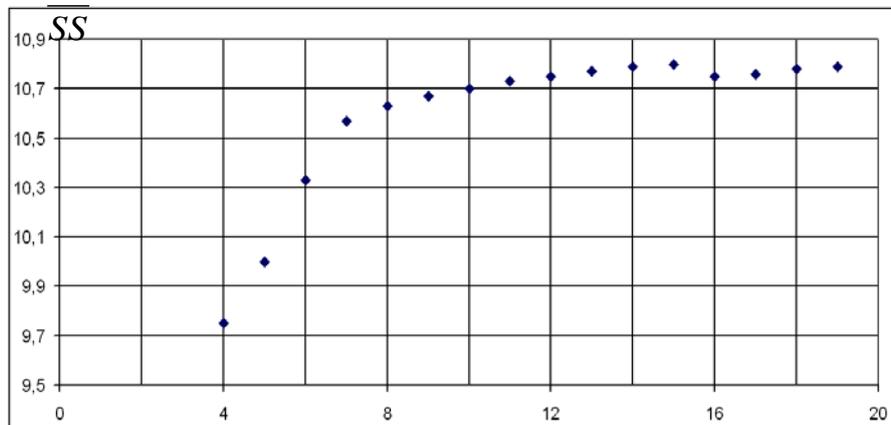
Looking more closely at the distribution of demand in our industrial case, we notice that four products have very few orders: engines 10 and 11 have a probability of 0.4% and engines 9 and 12 of 0.9%. There is neither delay nor setup cost related to a reference change on the production line. Removing these four products makes no change on the average safety stock: it is the same, whether 15 or 19 references are produced. Thus removing the offer of these four products is no advantage to either the customer (average safety stock does not decrease) or the supplier (no reference change cost).

Safety stocks are also created to counter other hazards: quality problems in a production context, variation of transportation time or modification of the firm sequence of final demand. These are explicitly excluded from the analysis, but it is obvious that the integration of a combination of disruptions leads to a mutualization of risks. The safety stock required to face multiple disruptions is thus less than the sum of all required safety stocks if the disruptions are considered separately.

We illustrate the impact of a combination of disruptions on the safety stock from an example of rank-changes caused by lot-sizing issues at production. The analysis of this type of lot-sizing is similar to that of delivery constraints. The submitted information concerns batches between the selected unit (engine manufacturing plant) and its provider. Several technical constraints, such as limited space at line side on a station handling big components, can lead to the production scheduling in a sequence of successive batches. The difference with the previous situation (procurement) is that these batches may not concern the same

product, but products sharing a common specification, like containing a same component (in our case, a same crankcase used by several different engines).

This lot-sizing is slightly more complex. On the one hand, it implies submitting information about batches of component to deliver to the supplier. But on the other hand, it leads to the creation of a second lot-sizing at the production of the designated unit, which is not homogeneous, for the batch thus created contains various products. Table 4 gives an illustration of this. We simulate the impact on the safety stock of a separate, then simultaneous integration of:

- The rank-change related to production constraints: lot-sizing of the engines by batches of 24 identical crankcases.
- Quality issues: percentage of rejection after production amounting 2%.

Once again, the safety stock is held by the customer (engine manufacturing plant) that has to own a safety stock of engines. The simultaneous integration of both disruptions allows decreasing the safety stock required to avoid stock-out by $\dfrac{161-141}{161} = 12.4\%$.

Table 4 Decrease of the safety stock induced by a mutualization of disruptions

Product	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%
$SS_i^{rankchange}$	6	6	6	6	6	18	12	6	6	6
$SS_i^{quality}$	1	2	1	1	1	3	2	1	1	0
$SS_i^d + SS_i^q$	7	8	7	7	7	21	14	7	7	6
$SS_i^{d\&q}$	6	8	6	6	6	18	12	6	6	6
Product	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	**Total**
p_i	0.4%	0.9%	3%	6.5%	6.1%	3.5%	6.1%	11.3%	2.6%	100%
$SS_i^{rankchange}$	6	6	6	6	6	6	6	12	6	138
$SS_i^{quality}$	0	1	1	2	1	1	1	2	1	23
$SS_i^d + SS_i^q$	6	7	7	8	7	7	7	14	7	161
$SS_i^{d\&q}$	6	6	6	6	7	6	6	12	6	141

2.3. Managerial implications

The supply chain is subjected to multiple risks. To avoid the stock shortages, the factories constitute safety stocks of the components to be assembled. When the factory can assemble on order and its supplier can produce on order, the synchronous production is

possible. When the differentiation of the components is made in the supplier's plant, it is often necessary to adapt the containers to the shapes of the products to be delivered. The batch constraints due to efficiency search in transport then oblige to constitute safety stocks even if the problem is in deterministic universe. Most of the time, the need and the importance of these stocks are not perceived because they were created to face various risks. From a managerial point of view, the existence of this kind of stock related to batching considerations is important to understand that the stocks reduction by a better control of the risks has limits. In addition, it poses the problem of the localization of the differentiation. Made in the supplier, one will have to multiply the types of containers. Made at the customer, the containers can easily be standardized but that has an impact on the design of the assembly line. This article highlights the existence of new tradeoff to be taken into account in the supply chain.

CONCLUSION

We have studied synchronization and decoupling of the control of the last two links of a supply chain dedicated to customized mass production. Taking into account lot-sizing in deterministic universe entails the creation of safety stocks. The explanatory factors of their importance were analyzed.

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