Increase in Life-Expectancy and Saving Behaviour*

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Abstract

The age structure of the French population has been experiencing dramatic changes over the past decades and is likely to do so in a near future. The increasing proportion of the elder people may modify the savings behaviour of households. The level of savings, as well as its composition, may be altered by the ageing of the French population. This paper investigates the relationship between an increasing life expectancy and saving behaviour. We set up a life-cycle model in which the increase in life-expectancy is modelled as an increase in the probability of death at older ages. We introduce uncertainty as a consumption shock to stylise the fact that individuals may face an (uncertain) increase in expenditure for long term care (such as Alzheimer disease). We then show that, contrary to the standard life-cycle message, an increase in individual life-expectancy does not imply a decrease in saving or a more risk averse behaviour.

Keywords: Life-expectancy, saving.

JEL: D91, G11, G22

Contents

1 Introduction

The retirement of the large baby boom cohorts is likely to have many consequences on financial accumulation. What will be the saving behaviour of baby boomers when they reach retirement age? Will they still save for life-cycle purposes as their life-expectancy increases? Will they change their portfolios' structure, with more risk-free assets? Some projections of the French Institute of Statistics and Economic Studies (INSEE) show that population ageing could entail a decline of household saving as the baby boomers retire. Besides, the Patrimoine 1998 Survey conducted by INSEE shows that the demand for life insurance follows a hump-shaped curve, increasing with the age until 60 and declining from then on.

Our paper investigates the relationship between an increasing life expectancy and saving behaviour. We set up a life-cycle model in which the increase in life-expectancy is modelled as an increase in the probability of death at older ages. We introduce uncertainty as a consumption shock to stylise the fact that individuals may face an (uncertain) increase in expenditure for long term care (such as the Alzheimer disease). We then show that, contrary to the standard life-cycle theory, an increase in individual life-expectancy does not imply a decrease in saving or a more risk-averse behaviour.[1]

The paper is structured as follows. The next section presents the theoretical framework. The third section introduces the contraints. The fourth section studies the individual saving behaviour with no dependency when life-expectancy increases. The fifth section introduces the dependency risk. Section six tries to evaluate the robustness of previous results. Section seven concludes.

2 The framework

We consider a life-cycle model in which individuals live at most three periods ($t = 1, 2, 3$). They work in the first period and they retire in the following

[1] See also Artus (2002) [1] for a theoretical framewework with similar conclusions.

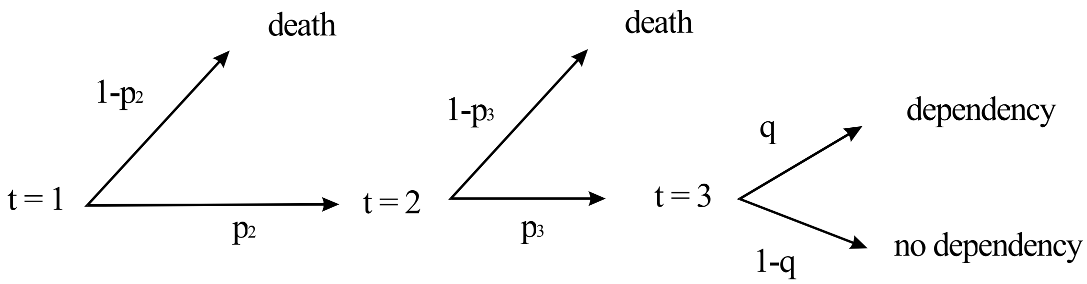

Figure 1: The timing.

two periods. Individuals are exposed to two risks: death and long-term care (see figure 1). At $t = 1$, each agent has a survival probability equal to p_2, and a probability to die equal to $1 - p_2$. At the beginning of period 2, each agent survives with a probability equal to p_3 till the end of period 2, and the complementary probability to die equal to $1-p_3$. If he survives, he may suffer from long-term care with probability equal to q. For the sake of simplicity, we assume that individuals only care for consumption at different periods and in different states of nature, that there is no altruism. Let us denote c_1 and c_2 consumptions in periods 1 and 2, while consumption in period 3 is indexed on the state of nature : c_3^d and c_3^{nd}(d for dependency and nd for no dependency). Preferences are described through a time-separable expected utility function :

$$u(c_1) + \beta u(c_2) + \beta^2 u(c_3) \tag{1}$$

where u refers to an instantaneous strictly concave and increasing utility function, and a factor of time preference β - $0 < \beta < 1$. In order to give explicit solutions, we assume a constant elasticity of intertemporal substitution and so :

$$u(c_t) = \frac{1}{1-\sigma}(c_t)^{1-\sigma} \tag{2}$$

To deal with risks and to save, individuals trade-off between three financial assets : a riskless asset with return r, annuities, and a long-term care

insurance contract. Annuities perceived in period t are contracts signed in period $t-1$, that provide a gross return ρ if the agent is still alive in period t. In period 1, individuals perceive an income equal to $y_1 > 0$. They consume c_1, and save a_2 in annuities. In period 2, they retire and have no exogenous income. They consume c_2 and save a_3 in annuities. In period 3, they live their last retirement period. They are exposed to the risk of dependency, which incurs a potential extra-consumption (long-term care expenditure).

3 Constraints

We assume a perfect competition insurance market. So, if there is no loading factor, the gross return ρ_t on annuities thus depends on the riskless interest rate, and the survival probability, and is given by :

$$\rho_t = \frac{1+r}{p_t} \tag{3}$$

If we assume a loading factor, the return on the annuity market is a more complex function of the interest rate and the survival probability :

$$t(p_t)\rho_t = 1+r$$

where $t()$ is a loading function. When the elasticity of return with respect to survival probability, γ, is held constant, we may assume the following relationship:

$$\rho_t = (1+r)\, p_t^{-\gamma} \tag{4}$$

In period 3 (and only in this period), individuals may suffer from a dependency risk which implies more consumption (long-term care expenditures). As shown in figure 1, this risk occurs with probability q and may be covered on a perfect competition insurance market (no loading).

All these assumptions lead to the following budget constraints. In period 1, the representative agent consumes and saves through an annuity contract:

$$c_1 + a_2 = y_1 \tag{5}$$

In period 2, if the agent has survived, his budget constraint is similar, his income being now $\rho_2 a_2$:

$$c_2 + a_3 = \rho_2 a_2 \tag{6}$$

In the last period, the agent has to determine his desired consumptions in both the dependency state and the nondependency state, and his desired coverage I of the health risk:

$$c_3^d + \eta I = \rho_3 a_3 + I - \delta \tag{7}$$

$$c_3^{nd} + \eta I = \rho_3 a_3 \tag{8}$$

I being the insurance indemnity, and η the unit premium.

Using the completeness property and reasoning backward, we characterize the set of feasible consumptions in each period. From (8) we get:

$$I = \frac{1}{1-\eta}\left[c_3^d + \delta - \rho_3 a_3\right]$$

Substituting this result in the second budget constraint of the last period, we get the second period constraint :

$$c_3^{nd} + \frac{\eta}{1-\eta}\left[c_3^d + \delta - \rho_3 a_3\right] = \rho_3 a_3$$

or :

$$\eta c_3^d + (1-\eta)\, c_3^{nd} = \rho_3 a_3 - \eta\delta \tag{9}$$

This budget constraint can be in turn introduced in the second period budget constraint and so on. After the relevant substitutions, we finally get the intertemporal budget constraint:

$$c_1 + \frac{1}{\rho_2} c_2 + \frac{\eta}{\rho_2 \rho_3} c_3^d + \frac{1-\eta}{\rho_2 \rho_3} c_3^{nd} = y_1 - \frac{\eta}{\rho_2 \rho_3}\delta$$

In our framework, the representative agent must choose the assets a_2, a_3, and I, which enable him to reach his optimal consumption level in each period. Let P_t denote the survival probability in period t (as of the beginning

of the first period). The optimal consumption levels are solutions of :

$$\begin{cases} \max_{c_t} u(c_1) + \beta P_2 u(c_2) + q P_3 \beta^2 u(c_3^d) + (1-q) P_3 \beta^2 u(c_3^{nd}) \\ \\ \textit{subject to}: \\ \\ c_1 + \frac{1}{\rho_2} c_2 + \frac{\eta}{\rho_2 \rho_3} c_3^d + \frac{1-\eta}{\rho_2 \rho_3} c_3^{nd} = y_1 - \frac{\eta}{\rho_2 \rho_3} \delta \\ \\ c_t \geq 0, \ t = 1, 2, 3 \end{cases}$$

The first order conditions are :

$$\begin{cases} \beta P_2 \left(\frac{c_1}{c_2} \right)^{\sigma} = \frac{1}{\rho_2} \\ \\ \beta^2 q P_3 \left(\frac{c_1}{c_3^d} \right)^{\sigma} = \frac{\eta}{\rho_2 \rho_3} \\ \\ \beta^2 (1-q) P_3 \left(\frac{c_1}{c_3^{nd}} \right)^{\sigma} = \frac{1-\eta}{\rho_2 \rho_3} \\ \\ c_1 + \frac{1}{\rho_2} c_2 + \frac{\eta}{\rho_2 \rho_3} c_3^d + \frac{1-\eta}{\rho_2 \rho_3} c_3^{nd} = y_1 - \frac{\eta}{\rho_2 \rho_3} \delta \end{cases} \tag{10}$$

How does the increase in life-expectancy (i.e. the increase in P_2 and/or P_3) modify the demand for assets a_2, a_3 and I? What is the impact of dependency risk on these demands? We answer these questions in two steps: first, in a standard framework à la Yaari (that is with no loading and no dependency risk); second, in a framework with dependency risk; and finally, with the introduction of a loading factor.

4 The impact of an increase in life-expectancy on saving

With no loading and no uncertainty on future consumption, the sequential constraints are :

$$c_1 + a_2 = y_1 \tag{11}$$

$$c_2 + a_3 = \frac{1+r}{p_2} a_2 \tag{12}$$

$$c_3 = \frac{1+r}{p_3} a_3 \tag{13}$$

The intertemporal budget constraint is thus :

$$c_1 + \frac{P_2}{1+r} c_2 + \frac{P_3}{(1+r)^2} c_3 = y_1 \tag{14}$$

Given the above assumed utility function, the program of the representative agent is :

$$\begin{cases} \max_{c_t} u(c_1) + \beta P_2 u(c_2) + P_3 \beta^2 u(c_3) \\ \\ \textit{subject to}: \\ \\ c_1 + \frac{P_2}{1+r} c_2 + \frac{P_3}{(1+r)^2} c_3 = y_1 \\ \\ c_t \geq 0, \ t = 1, 2, 3 \end{cases} \tag{15}$$

At the optimum, we get the following first order conditions:

$$\begin{cases} u'(c_2) = \frac{1}{(1+r)\beta} u'(c_1) \\ \\ u'(c_3) = \frac{1}{(1+r)^2 \beta^2} u'(c_1) \\ \\ c_1 + \frac{P_2}{1+r} c_2 + \frac{P_3}{(1+r)^2} c_3 = y_1 \end{cases} \tag{16}$$

If we assume an isoelastic utility function such as the power function, we get:

$$\begin{cases} c_2 = \left[(1+r)\,\beta\right]^{\frac{1}{\sigma}} c_1 \\ \\ c_3 = \left[(1+r)\,\beta\right]^{\frac{2}{\sigma}} c_1 \\ \\ c_1 + \frac{P_2}{1+r} c_2 + \frac{P_3}{(1+r)^2} c_3 = y_1 \end{cases} \tag{17}$$

and :

$$\begin{cases} c_1 = \frac{1}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2} y_1 \\ \\ c_2 = \frac{[(1+r)\beta]^{\frac{1}{\sigma}}}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2} y_1 \\ \\ c_3 = \frac{[(1+r)\beta]^{\frac{2}{\sigma}}}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2} y_1 \end{cases} \tag{18}$$

The completeness of markets and the complete diversification of risks imply that consumption levels only depend on the discount factor, the return on the risk-free asset, and the survival probabilities. The higher p_2 and p_3, the lower the returns on annuities, or equivalently, the higher the cost of consumption in each period. An increase in survival probabilities thus imply a monotonic decrease in the consumption levels.

The consumption functions and the budget constraints give a more detailed characterisation of the financial behaviour. With a little algebra, we derive the following demands for assets :

$$a_2 = \frac{P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2} y_1$$

$$\begin{aligned} a_3 &= \frac{1+r}{P_2} a_2 - c_2 \\ &= \frac{(1+r)\left(\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+p_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2\right)-[(1+r)\beta]^{\frac{1}{\sigma}}}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2} y_1 \\ &= \frac{(1+r)p_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2} y_1 \end{aligned}$$

Defining the demands as percentages of income, a_2/y_1 et a_3/y_1:

$$\frac{a_2}{y_1} = \frac{P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}$$

$$\frac{a_3}{y_1} = \frac{(1+r)p_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}$$

We have the following comparative static results :

- An increase in survival probabilities implies an increase in initial accumulation a_2/y_1;
- An increase in the survival probability P_2 decreases the third period accumulation a_3/y since it reduces the return of the early retirement saving, which in turn reduces further saving;
- An increase in p_3 leads to an increase in saving in the first period of retirement, which is fairly intuitive.

We thus obtain the standard results of the literature [4] : since the increase in survival probabilities lengthens the agent's life horizon, it increases the value of saving; the intertemporal smoothing motive induces an increase in the demand for annuities when the agent retires. The theoretical message of our basic framework is in line with the intuition of the standard life-cyle model (which is indeed not surprising since the only risk taken account at that stage in the survival risk). Beyond this period, an increase in the life horizon may have opposite effects on saving: with a given return on the risk-free asset, an increase of the survival probability P_2 diminishes the return of annuities, and thus reduces the amount than can be reinvested in the following period $\rho_2 a_2$.

5 Long-term care and financial behaviour

We now introduce a dependency risk at older age, which take the form of an extra-consumption in period 3 when the risk occurs. Intertemporal smoothing is no longer the only motive for saving : long-term care adds a precautionary motive in the representative agent's saving behaviour. Formally, without loading :

$$\rho_2 = \frac{p_2}{1+r},\ \rho_3 = \frac{p_3}{1+r},\ \eta = q$$

the intertemporal budget constraint is:

$$c_1+\frac{P_2}{1+r}c_2+\frac{qP_3}{(1+r)^2}c_3^d+\frac{(1-q)P_3}{(1+r)^2}c_3^{nd}=y_1-\frac{qP_3}{(1+r)^2}\delta$$

Since we assume an instantaneous power utility function, the first order conditions for an optimal behaviour are :

$$\begin{cases} c_2=\left[(1+r)\,\beta\right]^{\frac{1}{\sigma}}c_1 \\ \\ c_3^d=\left[(1+r)\,\beta\right]^{\frac{2}{\sigma}}c_1 \\ \\ c_3^{nd}=\left[(1+r)\,\beta\right]^{\frac{2}{\sigma}}c_1 \\ \\ c_1+\frac{P_2}{1+r}c_2+\frac{qP_3}{(1+r)^2}c_3^d+\frac{(1-q)P_3}{(1+r)^2}c_3^{nd}=y_1-\frac{qP_3}{(1+r)^2}\delta \end{cases} \tag{19}$$

We find the standard result of the demand for insurance in a perfect competition setting: the agent demands full insurance at fair odds. With full insurance, the costs of consumption profiles are held constant :

$$c_3=c_3^{nd}=c_3^d=\left[(1+r)\,\beta\right]^{\frac{2}{\sigma}}c_1$$

Long-term care has the only impact of reducing the agent's wealth. Optimal consumption behaviour is given by :

$$\begin{cases} c_1=\frac{1}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}\left(y_1-\frac{qP_3}{(1+r)^2}\delta\right) \\ \\ c_2=\frac{\left[(1+r)\beta\right]^{\frac{1}{\sigma}}}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}\left(y_1-\frac{qP_3}{(1+r)^2}\delta\right) \\ \\ c_3=\frac{\left[(1+r)\beta\right]^{\frac{2}{\sigma}}}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}\left(y_1-\frac{qP_3}{(1+r)^2}\delta\right) \end{cases} \tag{20}$$

The dependency risk occurring in period 3 increases accumulation in the first period of the life-cycle :

$$\begin{aligned} a_2 &= y_1-c_1 \\ &= \frac{P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\frac{\left[(1+r)\beta\right]^{\frac{1}{\sigma}}+\frac{q\delta}{y_1}}{(1+r)^2}}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2}y_1 \end{aligned}$$

the higher the survival probability P_3 the greater the amount of saving in period 1. The dependency risk also modifies the accumulation pattern in period 2.

$$\begin{aligned}
a_3 &= \frac{1+r}{P_2}a_2 - c_2 \\
&= \frac{\frac{P_3}{P_2}(1+r)\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2 + (1+[(1+r)\beta]^{\frac{1}{\sigma}})\frac{qP_3}{(1+r)^2}\frac{\delta}{y_1}}{1+P_2\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]+P_3\left[(1+r)^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}\right]^2} y_1
\end{aligned}$$

With a low probability of being dependent, the demand for annuities increases with the survival probability in period 3 other things being equal.

The impact of the survival probabilities P_2 and P_3, along with the dependency probability q, on the demand for annuities more difficult to forecast, since these probabilities are basically determined by the same variable (the health state of the population). An improvement of agent's health has thus two opposite effects on probabilities: it reduces the dependency probability (q) but increases life horizon (increasing P_2 and P_3).[2] We have theoretically shown that an improvement of "life perspectives" has the two saving motives playing in an opposite way, the intertemporal smoothing motive reducing accumulation, while the precautionary pushes it up.

6 Robustness

What is the effect of a loading factor in the pricing of annuities? We show that introducing an implicit loading factor does not qualitatively modify our previous results. The loading assumption is introduced through a constant elasticity of returns vis-à-vis the survival probabilities :

$$\rho_t = (1+r)p_t^{-\gamma}$$

[2]This is an empirical observation: the increase in life-expectancy seems to be positively correlated with the increase in life-expectancy in good health.

The preceding optimal conditions are modified as follows:

$$c_1 = \frac{y_1 - \frac{\eta}{\rho_2\rho_3}\delta}{1 + \rho_2^{\frac{1}{\sigma}-1}\left(\beta P_2\right)^{\frac{1}{\sigma}} + \frac{\eta}{\rho_2\rho_3}\left(\frac{\rho_2\rho_3\beta^2 q P_3}{\eta}\right)^{\frac{1}{\sigma}} + \frac{1-\eta}{\rho_2\rho_3}\left(\frac{\rho_2\rho_3\beta^2(1-q)P_3}{1-\eta}\right)^{\frac{1}{\sigma}}}$$

If we denote :

$$g(P_t) = (P_t)^{\frac{1+\gamma(\sigma-1)}{\sigma}}$$

then :

$$c_1 = \frac{y_1 - q(1+r)^2 P_3^{-\gamma}\delta}{1 + g(P_2)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{1}{\sigma}} + g(P_3)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{2}{\sigma}}}$$

As :

$$\begin{cases} ((1+r)\beta)^{\frac{1}{\sigma}} P_2^{\frac{1-\gamma}{\sigma}} c_1 = c_2 \\ \\ ((1+r)\beta)^{\frac{2}{\sigma}} P_3^{\frac{1-\gamma}{\sigma}} c_1 = c_3 \end{cases} \tag{21}$$

we obtain :

$$c_2 = \frac{(1+r)P_2^{-\gamma} g(P_2)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{1}{\sigma}}}{1 + g(P_2)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{1}{\sigma}} + g(P_3)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{2}{\sigma}}}\left(y_1 - q(1+r)^{-2}P_3^{\gamma}\delta\right)$$

$$c_3 = \frac{(1+r)^2 P_3^{-\gamma} g(P_3)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{2}{\sigma}}}{1 + g(P_2)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{1}{\sigma}} + g(P_3)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{2}{\sigma}}}\left(y_1 - q(1+r)^{-2}P_3^{\gamma}\delta\right)$$

We test the robustness of our previous conclusions on comparative statics. Assume a coefficient of relative risk aversion above 1, then the g function is strictly increasing.

$$g'(P_t) = \frac{1+\gamma(\sigma-1)}{\sigma}(P_t)^{\frac{(\gamma-1)(\sigma-1)}{\sigma}} > 0 \; if \; \sigma > 1$$

With no dependency risk, an increase in survival probabilities (P_2 and P_3) leads to a higher saving rate, other things being equal. Moreover, if we assume and $\gamma > 1$, the g function is strictly convex. We then get the same conclusions as Levhari and Mirman (1977) [3] : if the (P_t') probability distribution is second-order dominated by (P_t) :

$$\sum_{i=1}^{t} P_i > \sum_{i=1}^{t} P_i'$$

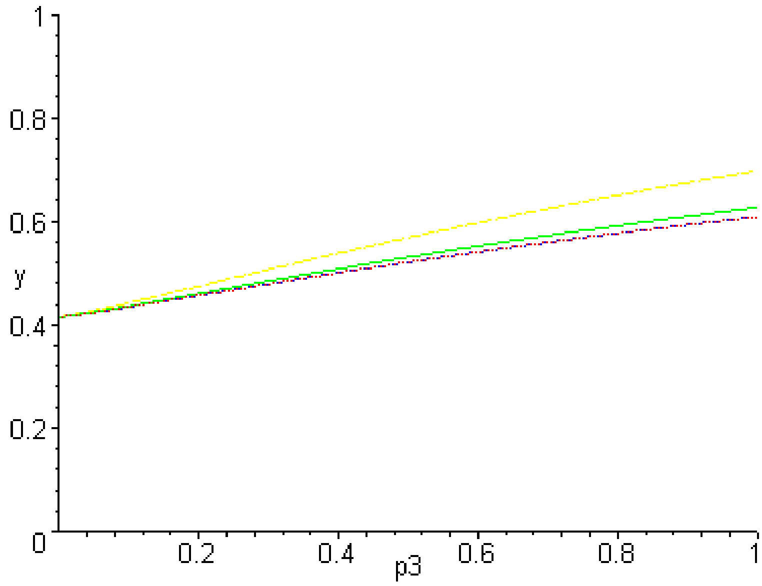

Figure 2: Impact of the dependency probability (p_3) on the first period saving rate (a_2/y_1)- with $q = 0$, 0.1, 0.2, 0.5.

then a_2/y_1 will be higher under (P_t). The saving behaviour in period 2 has the same property than in the perfect competition case, since :

$$\frac{a_3}{y_1} = \frac{(1+r)P_2^{-\gamma} g(P_3) \left[(1+r)^{1-\sigma}\beta\right]^{\frac{2}{\sigma}}}{1 + g(P_2)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{1}{\sigma}} + g(P_3)\left[(1+r)^{1-\sigma}\beta\right]^{\frac{2}{\sigma}}}$$

The demand for annuities in period 2 decreases with P_2, and increases with P_3.

If the dependency risk were taken into account, we would reach the same conclusions as before: the saving behaviour would be determined by the precautionary motive along with the intertemporal smoothing motive. This can be illustrated by numerical simulations, with

$$p_2 = 0.8, q = 0.1, r = 5\%, \beta = 0.8, \sigma = 5, \gamma = 1.2, \frac{\delta}{y_1} = 0.5$$

These simulations (figures 2, 3, 4, 5) show that the saving behaviour in each period is hardly modified when the dependency risk is introduced.

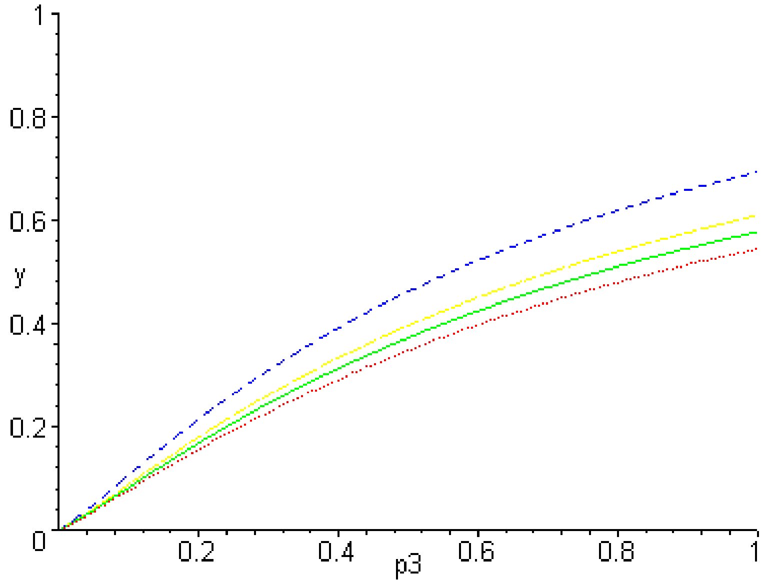

Figure 3: Impact of the dependency probability (p_3) on the second period saving rate (a_3/y_1)- with $q = 0$, 0.1, 0.2, 0.5.

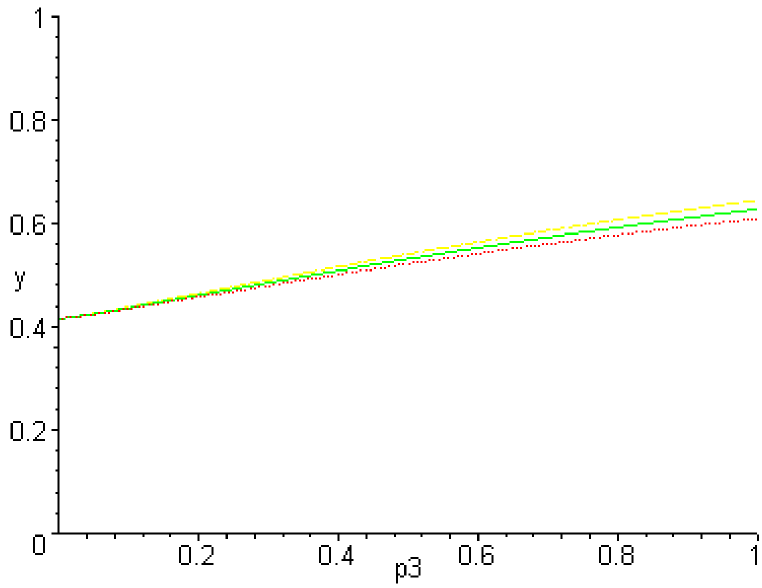

Figure 4: Impact of the dependency probability (p_3) on the first period saving rate (a_2/y_1)- with $\delta/y_1 = 0$, 0.5, 1

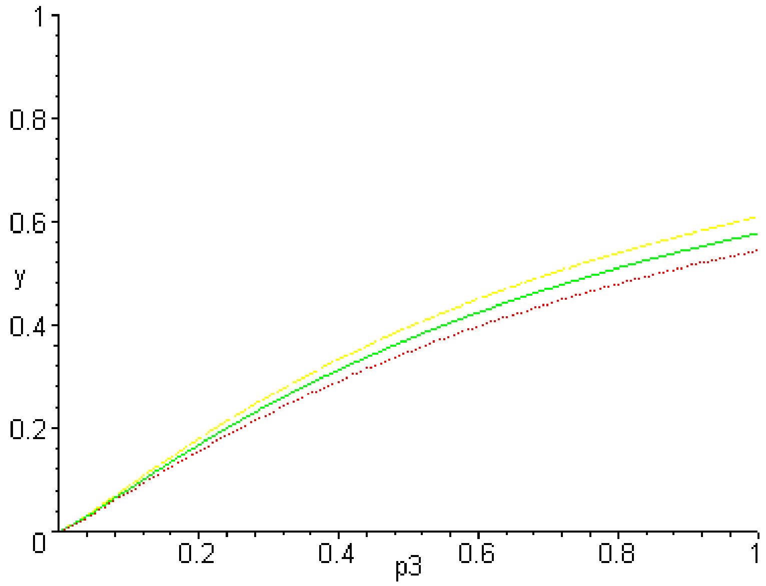

Figure 5: Impact of the dependency probability (p_3) on the second period saving rate (a_3/y_1)- with $\delta/y_1 = 0$, 0.5, 1

7 Conclusion

On a theoretical ground, there are several arguments in favour of an increase of retirement saving at old ages. Contrary to the standard life-cycle model, individual ageing does not necessarily imply dissaving or a reduction of financial risk taking. We have shown in this paper that an increase in life-expectancy may increase saving at older ages, when individuals have two saving motives, that is intertemporal smoothing and precautionary motives. We have computed simulations with plausible parameters related to risk-aversion, time preference and returns, and we have shown that the intertemporal smoothing motive is dominating.

These theoretical results are in line with empirical findings in the French case. Indeed, in a previous paper, we showed that French households still continue to save when they retire (Bernard et al. (2002) [2]) . Using a probit estimation on the demand for endowment insurance and retirement saving, we found that individuals belonging to the 60-74 age group have a signifi-

cantly higher proportion of their financial wealth in endowment insurance, annuities and voluntary retirement saving than people under 60. Using an OLS regression, we also found that the proportion of endowment insurance, annuities and voluntary retirement saving in financial wealth is higher for older individuals, other thing being equal. These empirical findings give some support to our theoretical framework.

References

[1] P. Artus, (2002). Profil de l'épargne pendant la retraite et équilibre financier. working paper, Caisse des Dépôts et Consignations, Paris, Mai 2002.

[2] Ph. Bernard, N. Mekkaoui de Freitas, A. Lavigne, and R. Mahieu, (2002). Ageing and the demand for life insurance : an empirical investigation using french panel data. working paper, Université Paris Dauphine and Université d'Orléans, December 2002.

[3] D. Levhari and L.J. Mirman, (1977). Consumption and saving with an uncertain horizon. *Journal of Political Economy*, 85:265–81, 1977.

[4] M. Yaari, (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, 32:137–50, April 1965.