A mean Field Game approach to technological transition

Imen Ben Tahar · René Aïd

Abstract We develop a model to assess the diffusion of a new-technology among a population of potential adopters. Our main objective is to analyze the effect of the strategic interaction of the firms which supply this new technology in a context where production costs decline with cumulated production (*learning by doing effect*), and where a firm's learning or experience benefits its rivals (*learning spillover effect*). To produce a tractable model in a dynamic setting, we adopt a mean-field-game approach.

Keywords diffusion of a new technology · spill over effect · strategic interaction · mean field game approach

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

In response to critical environmental challenges, it is widely admitted that a deep and large-scale transformation of current technological systems is necessary. Complex issues are entangled in the transition from a reference technology to more environmentally safe yet no-mature one. Such a transition usually calls for important public policies involvement, for instance via subsidies or tax-measures, to encourage the demand for the new technology and bring it to some critical level insuring a sufficient moment for its development and diffusion. In order to design an efficient policy it is important to understand, not only the demand side, but also the supply side dynamics.

In this paper we develop a quantitative model for the diffusion of a new technology with the objective to asses the effect of the strategic interaction of its supplying firms. Our model is in lines with the model of Stoneman and Ireland [2]:

- We consider a Davies [1] probit type model for the demand side: a potential user actually adopts the new technology if his *willingness to pay*, w, exceeds the current price p_t. The criterion w is assumed

Imen Ben Tahar
CEREMADE, University Paris Dauphine and Finance for energy market Research Center
Tel.: +33-(0)1 44 05 48 82
Fax: +33-(0)1 44 05 45 99
E-mail: imen@ceremade.fr

Ren Aïd
EDF R & D and Finance for energy market Research Center
E-mail: rene.aid@edf.fr

to be distributed according to a cumulative distribution function $F(\cdot)$, which allows to to deduce the inverse demand function

$$p_t = G\left(\frac{1}{n}d_t\right) \quad \text{where} \quad G:\ x \mapsto F^{-1}(1-x)\ .$$

Here n represents the size of the potential adopters.

– We assume that N firms produce this new technology. We denote by x_t^i the cumulative production volume of firm i up to time t and $x_t^{j\neq i} := (x_t^1, \cdots, x_t^{i-1}, x_t^{i+1}, \cdots, x_t^N)$. Each firm i maximizes its inter temporal profit

$$\max_{x^i} \int_0^T e^{-rt}\left(p_t \dot{x}_t^i - C^i(t, \dot{x}_t^i, x_t^i, x_t^{j\neq i})\right) dt \tag{1}$$

Here $C^i(t, \dot{x}_t^i, x_t^i, x_t^{j\neq i})$ represents the cost of producing at time t the volume $q_t^i = \dot{x}_t^i$, given the cumulative production level x_t^i and $x^{j\neq i}$.

As in [**?**], we consider that production costs decline with cumulated production, this is the *learning by doing effect*. Our main contribution is that we explicitly take into account the *learning spillover effect*, that is the fact that a firm's learning or experience benefits its rivals. To produce a tractable model, we consider a cost function of the form:

$$C^i(t, q, x^i, x^{j\neq i}) \ = \ \Psi_1(x_t^i, \bar{\mathbf{x}}_t)q + \Psi_2(x_t^i, \bar{\mathbf{x}}_t)q^2 \ \text{ where } \bar{\mathbf{x}}_\mathbf{t} = \frac{1}{N}\sum_j x_t^j \tag{2}$$

and adopt a mean-field game approach to analyze the strategic interaction of the firms.

References

1. S. Davies. *The diffusion of process innovations.* Cambridge University Press, Cambridge, 1979.
2. P. Stoneman and N.J. Ireland. The role of supply factors in the diffusion of new process technology. *The Economic Journal*, 93:66–78, 1983.