Liquidity Contagion
The Emerging Sovereign Debt Markets example

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Abstract

Financial markets are today so interconnected that they are fragile to contagion. Massive investment funds with very short horizons in-and-out-flows can generate contagion effects between markets. Since 2010, investors are willing to get a liquid exposure to the EM sovereign debt. As a consequence, some asset management firms started to propose products to track the performance of this asset class. However in that case, the fund manager faces a mismatch of liquidity between assets and liabilities and needs some tools to manage the liquidity of his investments. The main contribution of this paper is the analysis of contagion looking at common market liquidity problems to detect funding liquidity problems. Using the CDS Bond Spread basis as a liquidity indicator and a state space model with time-varying volatility specification, we show that during the 2007-2008 financial crisis, there exist pure contagion effects both in terms of price and liquidity on the emerging sovereign debt market. This result has strong implication since the main risk for an asset manager is to get stuck with an unwanted position due to a dry-up of market liquidity.

JEL classification: G01, G15, C01, C32

Key words: Emerging Markets, Sovereign Debt Market, Liquidity Risk Management, Liquidity, Contagion Effects, Regime Switching models.

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1 Introduction

The term of *Emerging Markets* (EM hereafter) appears for the first time in 1981. Since then the World Bank classifies as EM any markets meeting at least one of the following criteria: (i) being located in a low or middle-income economy as defined by the World Bank, (ii) not exhibiting financial depth; the ratio of the country’s market capitalization to its Gross Domestic Product (GDP) is low\(^1\), (iii) existence of broad based discriminatory controls for non-domiciled investors, or (iv) being characterized by a lack of transparency, depth, market regulation, and operational efficiency. The creation of emerging markets is motivated by the need of developing countries to raise capital to finance their growth. Before the 2000’s developing countries borrowed either from commercial banks or from foreign governments multilateral lenders (International Monetary Fund or World Bank). Capital flows to emerging markets increased dramatically and commercial bank debt that was the dominant source of foreign capital has been replaced by portfolio flows\(^2\) or foreign direct investment [Bekaert and Harvey (2003)].

EM are today considered as an asset class per se by many investors. Emerging economies have passed an important stress test during the period 2008-2009 and are now the key drivers for global growth of the world economy. As pointed out by the JP Morgan recent study\(^3\), "Potential growth rates for emerging economies of 5.8% now overshadow potential growth of only 1.6% for advanced economies". This explains why these markets are associated with very interesting investment opportunities for any investor seeking both returns enhancement and diversification. Inflows into EM have reached a record of US$70 billion in 2010 and will continue to grow as EM yields stay attractive in the context of current global bond markets. Also interesting to notice, the proportion of EM sovereign debt in local currency now account for around 80% of the total EM sovereign debt. As a consequence, any simple mean-variance portfolio optimization suggests a high allocation to EM debt. Different client surveys made by banks show an increase in EM debt allocation from around 20% in 2009 to around 25% one year later. Therefore, EM investments appear as really interesting but they suffer from additional risk, such as liquidity risk and, in some cases, contagion effects, that are not taken

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\(^1\)World Bank define low GDP as less than 755 USD per capita.

\(^2\)Essentially composed of fixed income and equity.

into account in the basic mean-variance approach or more generally, by asset pricing models. For example, Brandon and Wang (2012) show in their paper that the performances of hedge funds are strongly impacted by the consideration of liquidity risk. This is especially the case for hedge funds invested in the asset class of EM. In this paper, we consider the example of index funds invested on Emerging Sovereign Debt Markets. The contribution of this paper is twofold. First, we propose a measure of liquidity for the sovereign debt market. Second, we present a method to identify pure contagion effects. Thus, we study the liquidity in a funding perspective analyzing the re-correlation in terms of market liquidity measure changes.

Investing on EM implies to focus on liquidity problems especially for a fund manager. While traders are funded by banks, fund managers are funded directly by investors. Indeed, we can distinguish the liability part of the fund which consists of the investor’s inflows and the asset part which contains the fund’s holdings. Consequently, the fund managers faces a liquidity mismatch between the asset and the liability sides. On the one hand, investors want more and more liquid exposures. As a result, the liquidity of the liabilities is contractually defined and usually very high. On the other hand, for the asset side of the fund, the liquidity is determined by the nature of investments and usually lower than that of the liabilities. As the behavior of these funding providers can largely differ, the fund managers need to monitor this liquidity mismatch. Increasing the cash balance of the fund is one way to minimize this problem. However, if the amount of cash is too large, it will be idle and not producing. Conversely, if it is too small, the fund will still be exposed to the liquidity risk, so that it would be useless.

In addition to the mismatch of liquidity that the fund managers suffer, we know that investors need funding to trade securities. When the funding liquidity conditions are bad, they cannot easily access to capitals which impair their trade capacities. If many investors are concerned by such a funding liquidity problem, trading is slowing down and market liquidity reduces. As investors’ funding also depends on assets’ market liquidity, market liquidity and funding liquidity problems can be mutually reinforced leading to liquidity spirals [see e.g., Gromb and Vayanos (2002), Morris and Shin (2004), Brunnermeier and Pedersen (2009), Menkveld and

\footnote{Brunnermeier and Pedersen (2009) distinguish funding liquidity from market liquidity. The first characterizes the possibility for traders to find funds while the second characterizes the ease to trade an asset on the market. Traders provide market liquidity and their ability to do so depends on their capacity of funding. Funding liquidity is binding market liquidity as traders can only provide liquidity if they can access to fundings.}
Wang (2011)]. Indeed, during financial turmoils, like in 2008, crises can spread across assets and markets as many investors were seeking for liquidity creating a contagion effects.

Considering the perspective of a fund manager, we have to deal with liquidity constraints defined in the characteristics of the fund. However, the manager should build a portfolio to benefit from the diversification principle. Thus, the main fear for the manager is a re-correlation of the assets. Indeed, this phenomena may lead to the cancelation of the diversification benefits increasing the risk of large losses. Taking the example of a fund manager tracking the performances of Emerging Markets, Figure 1 shows that a funding liquidity problem can largely impact the returns of a fund. Indeed, we see that the performances of two different index funds tracking the JP Morgan BGI-EM Index largely differ in October 2008. Pictet (in light gray) has experienced 800 millions outflows corresponding to 10% of the Assets Under Management (AUM) while Julius Baer (in dark gray) did acknowledged 1400 millions outflows representing more than 30% of the AUM. As we see, the latter exposes the difficulty that the managers have to track the index when they are exposed to liquidity problems.

In order to control for the liquidity risk, we have to define how to measure it but despite the large number of liquidity measures available\(^5\), measuring liquidity remains a difficult task. In fact, many liquidity measures require the use of high-frequency transactions and quotes data, which may not be available for some markets and even more so for emerging markets. However, in most cases, Goyenko et al. (2009) show that their performances are not better than the effective or realized bid-ask spread. Moreover, the poor availability of data for Emerging Markets encourages us to focus on liquidity measures based on price data. There exists few measures based on daily price data. Roll (1984) develops an implicit measure of the effective bid-ask spread based on the serial covariance of daily price changes. Hasbrouck (2004) uses a Bayesian estimation approach to estimate the Roll model and proposes a Gibbs measure of liquidity. Lesmond et al. (1999) use the proportion of zero return days as a proxy for liquidity. In the line of Levy (2009), we use the Credit Default Swap (CDS hereafter) Bond Spread basis as a liquidity indicator.

Besides liquidity problems, the fund managers investing on EM are exposed to contagion problems. In the academic literature, financial contagion is definitely a burning issue and a

\(^5\)Aitken and Winn (1997) report more than 68 measures for market liquidity.
major concern, particularly since the end of 1990’s. Indeed, during the past 20 years, financial markets became more and more interconnected and, as a consequence, more and more fragile to contagion. For example, massive investment funds’ in- and out- flows on very short horizons can be a source of contagion effects. Since the summer 2011, we observe some contagion phenomena that play a crucial role in exacerbating the sovereign debt problems in the euro area. But before this, the issue of financial contagion was mainly concerning EM with the Asian crisis 1997-1998, the Russian crisis 1998, the Brazilian crisis 1999, Turkish crisis in 2001 and Argentinean crisis in 2002. Every fund manager that faces a liquidity mismatch or a risk constraint, as it is the case in the industry, needs a model taking into account the possible re-correlation or contagion phenomena. Standard linear models such Markowitz approach can not tackle these problems. Thus, one need to consider non-linear models based on moments beyond the first two moments of the return’s distribution.

In this paper, we use an arbitrage relation to extract a liquidity measure of the sovereign
debt market that solely relies on price data. Based on this liquidity measure, we propose a pure contagion effects identifying tool. Such a tool allows us to study if there exist pure contagion effects on EM Sovereign Debt markets and we investigate the re-correlation phenomena that the sovereign debt market experiences during liquidity turmoils using a dynamic model based on a regime switching. Our results show similar regimes between the Bond market, the CDS market and our liquidity indicator-the CDS Bond Spread Basis. From Garleanu and Pedersen (2009), Fontana and Scheicher (2010) and Bai and Collin-Dufresne (2011), we know that the basis is related to the credit risk of a bond. In other words, a larger deviation from parity is found for lower rated bonds because it is more costly to finance the arbitrage trade. We show that when the correlations between CDS are high and investors are funding constrained, the basis deviates from parity. Namely, the re-correlation problem is even more severe during crisis periods especially when liquidity problem occurs.

The remainder of the paper is organized as follows. Section 2 introduces the liquidity measure and the regime switching dynamic correlation model to identify pure contagion events. Section 3 describes data and presents descriptive statistics and graphic results on the liquidity measure. Section 4 analyzes the dynamic of the smoothed probabilities showing the behavior of the sovereign bonds or credit default swaps and their liquidity. Section 5 concludes the paper.

2 Liquidity measure and contagion model

In this section, we explore the CDS Bond Spread Basis and discuss its ability to accurately measure the liquidity of the sovereign debt market. Although some other factors in addition to liquidity contribute to the level of the Basis, we explain why they do not have an impact on the dynamic of the liquidity indicator. We also present a way to compute the Profit and Loss distribution (P&L hereafter) of CDS and Bonds in order to get them comparable. As they are issued in two different currencies, we cannot directly subtract the Bond yield issued in local currency from the CDS Premium expressed in US dollar. In a second part, we expose contagion’s definitions and the two main issues for measuring contagion effects: the treatment of heteroscedasticity and the fixing of crisis dates. The former allows to separate pure contagion from interdependence effects while the second avoids spurious results due to a wrong exoge-
nous determination of crisis dates. To tackle these problems, we present the Regime Switching Dynamic Correlation model (RSDC hereafter). This model gives a simple interpretation of correlation switches, corresponding exactly to the most commonly used contagion’s definition.

2.1 CDS Bond Spread Basis

CDS were created in 1994 by J.P Morgan & CO. Since its creation the CDS market is rose until 2008 and has stagnated since. CDS became in a few years a standardized financial product used by most of the market major participants (banks, hedge funds, mutual funds...). Nowadays, it is one of the most popular tool for transferring credit risk. The CDS contract is defined as a bilateral contract that provides protection on the par value of a specified reference asset. The protection buyer pays a periodic fixed fee or a one-off premium to a protection seller. In return, the seller will make a payment on the occurrence of a specified credit event [Choudhry (2006), Mengle (2007)]. Then, CDS provides to buyer a protection against the risk of default by borrowers, named the entities. The default, also named credit event is contractually defined by the two parties and could be bankruptcy, failure to make a schedule payment, obligation default, debt moratorium, financial or debt restructuring and credit downgrade. This is important to precise that rating agencies have not influence in triggering CDS. Their actions may, but not need, taken into account. The protection buyer has to pay an amount of fees (also named CDS premium or CDS spread) to protection seller and receives a payoff if the underlying bond experiences a credit event. At the deal inception, the two parts define which kind of settlement they want. The CDS contract could be settled in one of two ways: cash or physical settlement. Most of the time, contracts are physically settled (about 75-85%). Although the CDS contract has a given maturity, it may terminate earlier if a credit event occurs. In this case, the protection seller has to pay an amount called the protection leg.

The basis is nothing else but correcting the CDS from the sovereign bond (CDS bond spread basis). This is a way to cancel out the global macro effects when analyzing the commonality of sovereign risk. In other words, we focus on the long term liquidity. The basis is defined as the difference between the asset itself and its synthetic version. The no arbitrage theory of pricing

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6 The main part of CDS are documented using the 2003 ISDA Credit Derivatives Definitions, as supplemented by the July 2009 Supplement.
CDS implies that the basis should be zero. As both of these two assets should price the same default risk of the country, from the law of the one price, they should be equal. In practice, this situation is almost never verified. The breaking case highlights a liquidity problem on one or the other market. In addition to the liquidity, the level of the basis could fluctuate for many reasons that could be split into two categories: technical and market factors. We mainly find in the technical factors the delivery option and counterparty risk. To characterize the first, we have to define what *deliverable options* means. CDS contracts usually allow buyer and seller to agree on a panel of alternative assets that the buyer can deliver in case of a credit event. It allows to the buyer to deliver the cheapest obligation that he possesses in his eligible basket of assets. This option does not add value systematically even in the case of sovereign debt market. As we see in Ammer and Cai (2007), the Cheapest-to-Deliver (CtD) option could be valuable for the emerging sovereign debt market. However, our model is based on the existence of frictions interfering with exact arbitrage between CDS and bonds. One of these frictions we are particularly interested in is the liquidity of the sovereign debt market. In this context, it becomes really difficult to model and evaluate the CtD option. Indeed, Ammer and Cai (2007) propose to measure the spread part that could be attributed to the CtD option. Their model requires two strong assumptions allowing to measure the CtD option: the recovery rate is independent of time-to-default and the CtD option is the only friction. This second assumption is not realistic in our case and this is empirically proved that market liquidity is one of the main frictions interfering in the arbitrage relation between the CDS premium and the bond yield spread over the risk free rate. As the CtD option, although valuable, is sometimes null we neglect it in our model to focus on the market liquidity. The second is the counterparty risk. On the one hand, the protection seller can default and do not settle the protection buyer in case of a credit event. On the other hand, the buyer can also default and stop paying the CDS premium to the seller. However, some mechanisms like the counterparty clearing system allow to reduce these risks (almost half of CDS are treated by clearing). Moreover, as showing in Levy (2009), if the default probability of the underlying bond and the default probability of the counterparty are not correlated, the two effects may cancel each other out. Furthermore, counterparty risk is a joint event of two defaults. Thus, the excess premium associated is weighted by a product of two probabilities and should be really small, or negligible. Our aim being the analysis of the dynamic of the emerging market
liquidity, we consider that the main part of CDS are issued by companies in countries which are outside our sample of EM. As a consequence, if the counterparty risk changes, its impact is approximately the same for all countries and does not alter the dynamic of correlations that we study. Based on a demonstration proposed by Levy (2009), we focus on the liquidity premium induced by the movements of the basis on emerging markets.

CDS includes two legs corresponding to the premium payments and the default payment. The pricing of a CDS depends, among others, on the recovery amount (a recovery rate of par value and accrued interest). Duffie (1999) or Hull and White (2000) expose two approaches for the pricing of CDS premium. The first, that we call "no arbitrage" approach, follows the idea that an investor can buy a CDS and the underlying bond to replicate the risk free rate. The second is based on a reduced-form model with random stopping time. In order to demonstrate the impact of liquidity on the CDS Bond spread basis, we use the first one. As a result, buying a risky bond and its CDS with the same maturity allow to the investor to eliminate the default risk associated with the bond. Assuming that there is no arbitrage opportunities, this portfolio should be equal to the value of the risk free bond with the same maturity. As in Zhu (2006), we price CDS premiums and Bonds separately. We construct a portfolio that replicate the CDS contract and we obtain the CDS Spread Basis. In this context, we assume a risk neutral world with three assets: a risk-free bond, a risky bond and a CDS contract.

Following Levy (2009), under the risk neutral valuation, we note the CDS premium, \( b \) satisfies:

\[
\sum_{t=1}^{T} b e^{-rt} F(t) = \int_{0}^{T} (100 \cdot RV_t) e^{-rt} f(t) dt,
\]

where \( T \) is the number of times till maturity or default, \( r \) is the risk-free rate, \( RV_t \) is the recovery value at time \( t \), \( f(t) \) is the probability of default at time \( t \) and \( F(t) \) is the survival probability\(^7\). The left hand side is called the Premium leg and the right hand side is called the Insurance leg.

The value of the risky bond is expressed as:

\[
F(t) = 1 - \int_{0}^{t} f(x) dx
\]

\(^7\)
\[ Y = \sum_{t=1}^{T} Ce^{-rt}F(t) + 100e^{-rT}F(T) + \int_{0}^{T} RV_{t}e^{-rt}f(t)dt, \]  
\text{(2)}

where \( C \) is the fixed coupon paid for each period.

And the value of a risk-free bond at risk-free rate \( r \) is expressed as:

\[ 100 = \sum_{t=1}^{T} re^{-rt} + 100e^{-rT}. \]  
\text{(3)}

Then, we construct a portfolio that shorts the risky bond and buys the risk free bond subtracting (2) to (3), we obtain:

\[ 100 - Y = \sum_{t=1}^{T} re^{-rt} + 100e^{-rT} - \sum_{t=1}^{T} Ce^{-rt}F(t) - 100e^{-rT}F(T) - \int_{0}^{T} RV_{t}e^{-rt}f(t)dt. \]  
\text{(4)}

But if we modify the risk-free bond equation to include default probability, we obtain:

\[ 100 = \sum_{t=1}^{T} re^{-rt}F(t) + 100e^{-rT}F(T) + \int_{0}^{T} 100e^{-rt}f(t)dt. \]  
\text{(5)}

Then, the value of our portfolio is:

\[ 100 - Y = \sum_{t=1}^{T} (r - C)e^{-rt}F(t) + \int_{0}^{T} (100 - RV_{t})e^{-rt}f(t)dt. \]  
\text{(6)}

However, rearranging with equation (1), we obtain:

\[ 100 - Y = \sum_{t=1}^{T} (b + r - C)e^{-rt}F(t) \]  
\text{(7)}

Finally, the CDS Bond Spread Basis is expressed as:

\[ b + (r - C) = \frac{100 - Y}{\sum_{t=1}^{T} e^{-rt}F(t)} \]  
\text{(8)}

The CDS Spread Basis is then equal to zero since the risky bond is traded at par, i.e. \( Y = 100 \). Moreover, the fixed coupon of a par bond is equal to the bond’s yield to maturity \( (y = C) \) and we have:
Furthermore, assuming that there are two traders: (i) with high liquidity (h) and (ii) with low liquidity (l). Note $b_i$ the CDS premium fair price for the trader $i$, $i = l, h$, $\tilde{S}$, the market price for this CDS and $p_i$ the probability to find in the first search a trader who are of type $i$. Then, we know that a trader who have liquidity problems, should pay an additional holding cost. Then, from equation (1) we obtain:

$$\sum_{t=1}^{T} b_h e^{-rt} F(t) = \int_0^T (100 - RV_t) e^{-rt} f(t) dt$$

for high liquidity traders (10)

$$\sum_{t=1}^{T} b_l e^{-(r+d)t} F(t) = \int_0^T (100 - RV_t) e^{-(r+d)t} f(t) dt$$

for low liquidity traders (11)

where $d$ is the additional holding cost.

From these two equations we can extract the CDS premium for each type of traders as:

$$b_h = \frac{\int_0^T (100 - RV_t) e^{-rt} f(t) dt}{\sum_{t=1}^{T} e^{-rt} F(t)}$$

for high liquidity traders (12)

$$b_l = \frac{\int_0^T (100 - RV_t) e^{-(r+d)t} f(t) dt}{\sum_{t=1}^{T} e^{-(r+d)t} F(t)}$$

for low liquidity traders (13)

Obviously, trade occurs only if $b_h < \tilde{b} < b_l$. Introducing the value of search process $V$, the trader has to be indifferent between searching alone or buying at a market maker. We get:

$$V = p_h b_h + p_l (V + C) = \frac{p_h b_h + Cp_l}{1 - p_l}$$

where $C$ is the search cost.

The market price $\tilde{b}$ that is equal to:

$$\tilde{b} = V = b_h + \frac{C p_l}{1 - p_l}$$

(15)

where $\frac{C p_l}{1 - p_l}$ is the additional spread for the asset (CDS and bond that we note respectively
\( S_{CDS} \) and \( S_{bond} \).

\( \tilde{b} \) is the market price for the CDS and is such that \( \tilde{b} = b + S_{CDS} \), \( \tilde{y} \) is the market price for the bond and is equal to \( y + S_{bond} \). Taking into account liquidity, equation (9) becomes:

\[
\tilde{b} = \tilde{y} - r - (S_{bond} - S_{CDS})
\]

To conclude, the parity between CDS and risky bond should hold only for the pure risk component that is priced into the two assets. As a consequence, we can expect a non-zero basis when liquidity differences exist.

### 2.2 Basis trade with multiple currencies

The CDS Bond Spread basis that compares CDS premium denominated in dollar to the local currency denominated sovereign debt is biased. To tackle this problem, we compute and correct the P&L, of an investment strategy corresponding to the basis. In other words, we buy/sell both instruments when the basis is negative/positive. The computation of the P&L is the way that traders refer to the daily change of the value of their trading positions. The P&L is generally defined as the difference between the value at time \( t + 1 \) and the value at time \( t \). In other words, the P&L of an asset is the profit or the loss that this asset makes between two dates. In this sense, we can split the P&L between two parts: the Mark-to-Market (MtM hereafter) part and the Carry part. The former is the gain (or the loss) realized when selling the asset. The latter, called the carry, is the gain (or the loss) i.e., the income you earn on the asset during the period you own it (in our case, one day). Schematically, we can express the P&L of an asset as the sum of the MtM P&L and the Carry P&L.

Both CDS and Generic bonds P&L are computed at a daily frequency, and thus, ignoring the carry. This component is very close to zero due to the daily investment horizon. As a consequence, the annual return of the asset is divided by 250, that could be neglected in the P&L computation.
Generic Bond P&L

The MtM component of the P&L of an asset is the same whatever the asset i.e. the difference between the prices at two distinct dates. In the case of a bond, it can be expressed as the variation of the yield-to-maturity (YtM hereafter) multiplied by the sensitivity of a one unit variation. Thus, we note:

\[
P\&L_{\text{bond}}^{t} = (YtM_{t} - YtM_{t-1}) \times \text{sensi}_{t}^{\text{bond}}. \tag{17}
\]

In order to define the sensitivity, we have to specify what is the duration. This latter is the weighted average maturity of cash flows, expressed as:

\[
D = \sum_{i=1}^{N} t_{i} \times \frac{CF_{i}}{\left(1 + YtM_{t}\right)^{t_{i}}} \times PB \tag{18}
\]

where \(t_{i}\) is the time in year until the next \(i^{th}\) payment, \(CF_{i}\) is the \(i^{th}\) cash flow, \(YtM\) is the Yield-to-Maturity and \(PB\) is the present value of the bond.

The sensitivity, \(\text{sensi}_{t}^{\text{bond}}\) is defined as the opposite of the modified duration, that is, in the case of periodically compounded yields, the duration over the Yield-to-Maturity:

\[
\text{sensi}_{t}^{\text{bond}} = \frac{D}{\left(1 + YtM_{t}\right)}. \tag{19}
\]

In the case where the CDS and the Bond are not expressed in the same currencies, we need to correct the P&L of the generic bond by the corresponding exchange rate. We call \(PB_{t}\) the present value of the bond in local currency and \(X_{t}\) the exchange rate at time \(t\). We know that the dollar price’s variation is expressed as:

\[
\left[PB_{t} - PB_{t-1}\right]_{X} = \frac{PB_{t}}{X_{t}} - \frac{PB_{t-1}}{X_{t-1}} \tag{20}
\]

Linearizing this expression, we can separate the MtM component of the bond’s P&L in two parts:
\[ [PB_t - PB_{t-1}] \simeq \frac{1}{X_{t-1}} (PB_t - PB_{t-1}) - \frac{PB_{t-1}}{X_{t-1}} \left( \frac{X_t - X_{t-1}}{X_{t-1}} \right). \] (21)

The first term represents the gain (or loss) due to the variation of the bond’s price and the second, the gain (or loss) due to the variation of the exchange rate. Crossing the expression of the P&L given in equation (17), we obtain:

\[ [PB_t - PB_{t-1}] \simeq \frac{1}{X_{t-1}} \left( (Y_{tM_t} - Y_{tM_{t-1}}) \times \text{sensi}^\text{bond}_t \right) - \frac{PB_{t-1}}{X_{t-1}} \left( \frac{X_t - X_{t-1}}{X_{t-1}} \right). \] (22)

**CDS P&L**

To compute the P&L of CDS, two informations need to be recalled. The first is that the trading horizon is a day while the second is that the price of a CDS strategy at the issuance is equal to zero. Consequently, the P&L of a CDS, assuming that we neglect the carry part, equals the selling price. In the case of a CDS, we are able to express the price at time \( t \) as the product of the premium’s variation between \( t \) and the issuance date, and the sensitivity to a change of 1bp of the CDS premium. Summing up, the P&L of the CDS can be expressed as:

\[ [PC_t - PC_{t-1}] = PC_t = [S_t - S_{t-1}] \times \text{sensi}^\text{CDS}_t, \] (23)

where \( PC_t \) is the price of the CDS contract at time \( t \). As we consider \( t-1 \) as the date when we start the contract, \( PC_{t-1} \) is null because the value of a CDS at the opening is equal to zero.

Using a continuous time Poisson model, the sensitivity of a 1bp premium variation is equal to:

\[ \text{sensi}^\text{CDS}_t = \int_0^T e^{-(r+\lambda)\theta} d\theta = \frac{1-e^{-(r+\lambda)T}}{r+\lambda}, \] (24)

where \( \lambda = \frac{S_t}{1-RR} \) with \( RR \) is the recovery rate.

Once we compute these time series of P&L, we can easily calculate the CDS Bond Spread Basis. Indeed, when the basis is negative, we buy both the CDS protection and the bond,
conversely when the basis is positive.

2.3 Contagion and RSDC model

After having fund a way to measure liquidity on the sovereign debt market, the second major concern on EM is the identification of contagion effects. Although the World Bank proposes three definitions, there is still no consensus about how to define and measure it. In this subsection, we detail the definitions of financial contagion and particularly the difference between interdependence and pure contagion. The treatment of the heteroscedasticity is at the very center of this distinction. We also point out the difficulty that the fixing of crisis dates may cause. Finally, we present a dynamic model that is able to tackle these two issues and allows us to detect pure contagion effects both in terms of prices and liquidity.

Interdependence and Pure Contagion

Financial contagion refers to the notion that financial markets move more closely together during turmoil (Bekaert and Harvey (2003)). Rigobon (2001) recalls that if economists agree about which events have constituted instances of contagion\(^8\), there is still no consensus on the definition of contagion. The three definitions of the World Bank are: (i) \textit{Broad definition}: contagion is the cross-country transmission of shocks or the general cross-country spillover effects., (ii) \textit{Restrictive definition}: Contagion is the transmission of shocks to other countries or the cross-country correlation, beyond any fundamental link among the countries and beyond common shocks. This definition is usually referred as excess co-movement, commonly explained by herding behavior, (iii) \textit{Very restrictive definition}: Contagion occurs when cross-country correlations increase during crisis times relative to correlations during tranquil times. The latter is the more restrictive definition but is also the most widely used. Contagion occurs when cross-country correlations increase during troubled periods relative to tranquil times. Forbes and Rigobon (2002) distinguish pure contagion from interdependence. This distinction is at the very center of the contagion’s analysis. Actually, the definition of contagion is related to the co-movements between markets after a shock. Nonetheless, these are conditional on market volatility. Inter-

dependence reflects high cross market linkages both during tranquil times and crises while pure contagion highlights a significant increase of correlations during financial turmoils conditionally to the increase of volatility.

To our knowledge, the first empirical study of financial market contagion has been made by King and Wadhwani (1990) who show that an increase in price volatility in the United States leads to a rise in the correlation of returns across markets. Some papers in the literature use the restrictive definition. Financial contagion is the propagation between countries, of shocks in excess of what should be expected by fundamentals and considering the co-movements triggered by the common shocks. However, although more restrictive, this definition implies another measurement problem. Indeed, as Bekaert and Harvey (2003) emphasize, there may have disagreement considering the definition of fundamentals. Following Forbes and Rigobon (2001), the main part of the literature uses the very restrictive definition explaining a contagion phenomena as the change in the transmission mechanisms that take place during a turmoil period. The third definition has the advantage of allowing to measure contagion effects as in Bertero and Mayer (1989), King and Wadhwani (1990) or Calvo and Reinhart (1996) that are interested in correlation shifts during turmoil times. Thus, measuring contagion effects resumes to estimating jumps in the correlation between financial time series, or, in other words, if the parameters of the econometric model shift when turmoils occur. In this way, Rigobon (2001) proposes a good survey of parameter stability tests, which are mainly based on Ordinary Least Square estimates, Principal Components, Probit models and correlation coefficient analysis. Rigobon (2000), Forbes and Rigobon (2001), Rigobon (2003a), Rigobon (2003b) propose to study the difference between covariances of two different periods. However, many studies point out a number of methodological problems and particularly the problem of heteroscedasticity and fixing of crisis dates.

Rigobon (2001) and Forbes and Rigobon (2002), who generalize the approach of Boyer et al. (1999), show that correlation coefficients are biased. In fact, they are conditional on market volatility. During turmoil periods, market volatility increases and estimates of cross market correlations are biased upward. Indeed, this can lead to accept the contagion hypothesis while false. The authors develop an adjusted correlation coefficient that correct the bias caused by heteroscedasticity. They distinguish pure contagion from interdependence and show that there
is no contagion but interdependence during the Mexican or the Asian crisis contrary to previous findings. Many studies, using different econometric methods tried to avoid the problem of heteroscedasticity. Among them, ARCH and GARCH (for Generalized AutoRegressive Conditional Heteroskedasticity) models are largely used in contagion analysis. Hamao et al. (1990) estimate conditional variance with a GARCH model and test the correlation between three different markets. Edwards and Susmel (2001) include regime switching in a ARCH modeling to take into account systematic changes. Their model points out large correlations during times of high market volatility, which confirms the existence of contagion effects. However, multivariate GARCH models provide more efficient tools for analyzing comovements and volatility spillover between financial assets. Wang and Nguyen Thi (2007), Chiang et al. (2007) or Naoui et al. (2010) also estimate contagion with Dynamic Conditional Correlation that allow for tracking the evolution of the correlation between two or more assets. Kenourgios et al. (2010) use an Asymmetric Generalized Dynamic Conditional Correlation model and study the correlations between the four BRIC countries (i.e: Brazil, Russia, India and China), the US and UK markets. The use of an asymmetric model allows to see that bad news increase dramatically the equity conditional correlations among the BRIC and developed markets.

In addition to the problem of heteroscedasticity, Boyer et al. (1999) point out that the methods previously described have an exogenous definition of the crisis periods that may lead to spurious conclusions. Using a state-space model with a time-varying volatility specification is the solution to tackle this problem. Billio and Caporin (2005) propose a multivariate Markov Switching Dynamic Conditional Correlation GARCH model to estimate contagion effects. This class of models allows for discontinuity in the propagation mechanism to assume that international propagation mechanisms are discontinuous. A markov chain is introduced to describe this discontinuity and allows the endogenous definition of crisis periods. Moreover, dynamic correlation permits to analyze the dynamics of contagion. Many other authors explore Markov switching models like Ramchand and Susmel (1998), Chesnay and Jondeau (2001) or Ang and Bekaert (2002) for example. Billio et al. (2005) show the Markov switching model abilities to estimate contagion. This approach defines contagion as a break that produces non-linearities in the linkages among financial markets. They emphasize that when using Markov switching models: (i) the heteroscedasticity problem is solved, (ii) the definition of the crisis periods is made
endogenously, (iii) the estimations are more efficient because of the full-information approach and (iv) the distinction between long and short run breaks in the factor loadings of the long and short run factors risk is allowed.

**RSDC model**

According to these results, our approach is in the line of Pelletier (2006) that has been used in the context of portfolio allocation [see Giamouridis and Vrontos (2007)]. It allows in particular to decrease the number of variance parameters to consider. Our model is a combination of a mixture model for the correlation matrix and a Threshold GARCH model [or TGARCH, Zakoian (1994)] to take into account asymmetric volatility dynamics. However, our estimation method imposes to assume that the heteroscedasticity is asset specific and not common across assets.

Note the \( K \) asset returns are defined by:

\[
rt = H_t^{1/2} Ut,
\]

where \( Ut | \Phi_{t-1} \sim iid(0, I_K) \), \( Ut \) is the \( T \times K \) innovation vector, and \( \Phi_t \) is the information available up to time \( t \).

The conditional covariance matrix \( H_t \) is decomposed into [Bollerslev (1990) or Engle (2002)]:

\[
H_t \equiv S_t \Gamma_t S_t,
\]

where \( S_t \) is a diagonal matrix composed of the standard deviation \( \sigma_{k,t} \), \( k = 1, \ldots, K \) and \( \Gamma_t \) is the \((K \times K)\) correlation matrix. Both matrices are time varying.

The conditional variance may follow a TGARCH(1,1) such that:

\[
\sigma_{i,t} = \omega_i + \alpha_i^- \min(\epsilon_{i,t-1},0) + \alpha_i^+ \max(\epsilon_{i,t-1},0) + \beta_i \sigma_{i,t-1},
\]

where \( \omega_i, \alpha_i^-, \alpha_i^+ \) and \( \beta_i \) are real numbers.

Under assumptions of: \( \omega_i > 0, \alpha_i^- \geq 0, \alpha_i^+ \geq 0 \) and \( \beta_i \geq 0 \), \( \sigma_{i,t} \) is positive and could be interpreted as the conditional standard deviation of \( r_{i,t} \). However, it is not necessary to impose
the positivity of the parameters and the conditional standard deviation is the absolute value of $\sigma_{i,t}$.

The correlation matrix is defined as:

$$\Gamma_t = \sum_{n=1}^{N} 1_{(\Delta_t = n)} \Gamma_n,$$

where $1$ is the indicator function, $\Delta_t$ is an unobserved Markov chain process independent from $U_t$ which can take $N$ possible values ($\Delta_t = 1 \cdots N$) and $\Gamma_n$ are correlation matrices. Regime switches are assumed to be governed by a transition probability matrix $\Pi = (\pi_{i,j})$, where

$$Pr (\Delta_t = j | \Delta_{t-1} = i) = \pi_{i,j}, \ i, j = 1, \cdots, N.$$  

This approach allows to discriminate between on the one hand the volatility dynamics through $S_t$ and on the other one the correlation dynamics through the state variable $\Delta_t$.

Finally, coming back to liquidity, we know that the contagion between markets drives their in and outflows, and liquidity moves consequently. In the line of the above approach, we can link contagion and liquidity moves by comparing the commonalities between the liquidity indicators introduced in the previous section. If the commonality is between liquidity and volatility, there is no contagion effect but only interdependence. On the contrary, if the liquidity shock has an impact on the correlation matrix, liquidity can be considered as a contagion channel.

### 3 First Results

In the case of financial turmoils, a crucial point is to take into account correlations more accurately. Indeed, the main risk is to consider that the crisis has ended while not true. As we will see in this section, the 2007-2008 crisis exhibits some of these stylized facts. The focus on CDS or Bond markets leads the fund manager to think that there is no more problems only few months after the Lehman Brothers collapse. However, this is absolutely not the case until the correlations between assets stay high and unable to build a portfolio. In this section, we
describe data and we show that our liquidity indicator add some precious informations about the nature of the problem.

3.1 Data description

We use data on sovereign bond yield spreads, sovereign CDS, interest rates and foreign exchange rates for 9 emerging markets: Brazil, Chile, Hungary, Mexico, Poland, Russia, South Africa, Thailand and Turkey. Our sample period is ranging from 01/01/2007 to 26/03/2012. The database is downloaded from Bloomberg.

The time series cover many of the recent crisis and allows us to explore emerging markets behavior during economic disturbances. We are interested to know if it is still possible to benefit from diversification principle and when these benefits could be higher. The CDS premiums are based on 5-year U.S. dollar contracts, for senior claims, and they assume a recovery rate of 25%. We use as risk-free rate the US Swap rate 30/360 paid semiannually.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>Relative Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>148.53</td>
<td>77.94</td>
<td>61.50</td>
<td>123.00</td>
<td>52.48%</td>
</tr>
<tr>
<td>Chile</td>
<td>86.93</td>
<td>59.28</td>
<td>12.50</td>
<td>74.79</td>
<td>68.19%</td>
</tr>
<tr>
<td>Hungary</td>
<td>261.24</td>
<td>172.27</td>
<td>17.34</td>
<td>250.58</td>
<td>65.94%</td>
</tr>
<tr>
<td>Mexico</td>
<td>141.38</td>
<td>90.31</td>
<td>28.17</td>
<td>119.92</td>
<td>63.88%</td>
</tr>
<tr>
<td>Poland</td>
<td>129.56</td>
<td>88.83</td>
<td>7.67</td>
<td>133.50</td>
<td>68.56%</td>
</tr>
<tr>
<td>Russia</td>
<td>209.91</td>
<td>181.84</td>
<td>36.88</td>
<td>153.60</td>
<td>86.63%</td>
</tr>
<tr>
<td>South Africa</td>
<td>164.00</td>
<td>103.12</td>
<td>24.57</td>
<td>147.80</td>
<td>62.88%</td>
</tr>
<tr>
<td>Thailand</td>
<td>123.30</td>
<td>66.10</td>
<td>31.84</td>
<td>113.01</td>
<td>53.67%</td>
</tr>
<tr>
<td>Turkey</td>
<td>230.46</td>
<td>94.12</td>
<td>118.61</td>
<td>195.72</td>
<td>40.84%</td>
</tr>
</tbody>
</table>

Table 1 provides descriptive statistics for the sovereign CDS premiums, in other words, for the sovereign default risk. The wide range of averages highlights the high degree of heterogeneity among countries with a minimum of 86.93 for Chile and a maximum of 356.95 for Thailand. This is confirmed by the relative STD, where its value for Russia (86.63%) is more than twice that of Turkey (40.84%). For example, the cost of credit protection for Russia increases from 36.88 to 1,113.38 basis points while it reaches only from 12.50 to 322.96 for Chile. We have similar results for the sovereign debt market as Table 2 shows us. Brazil has the higher mean with 1242.57

9Exchange rates are only used to deal with the problem of different currency issuance among CDS and bonds.
while Thailand exhibits the lowest average bond yield. The return on an investment into the sovereign debt of one of these countries allowed the investor to earn at least 2.56% annually, investing in Turkey and a maximum of 19% investing in Brazil. Comparing Table 1 and Table 2, we see that the ranking of countries widely differs. Brazil has the biggest bond yield while it is only the sixth country in terms of CDS premium. We have similar results for some other countries that exhibit different behavior of their bond yield relatively to their CDS premium. These tables show that the sovereign debt market is less volatile than the CDS market with a maximum of relative standard deviation equal to 26.94% while the CDS market has a minimum of 40.84%.

### 3.2 Liquidity Description

Our empirical study of the market liquidity indicators confirms some stylized facts and the collapse of Lehman Brothers just as the 2007-2008 financial crisis are strongly highlighted. Figure 2 presents, for each of the 9 emerging countries in our sample, the level of CDS Bond spread basis. Firstly, we can see that the basis is almost never equal to zero except for Brazil. We see that all countries experienced almost simultaneously a liquidity problem. Indeed, the graphics reveal a large increase of the basis for every countries at the end of September 2008. Even if the close relation between all the basis is obvious during this period, the increase of volatility for all the markets may be the only source of contagion. If this is true, the fund manager does not have to change the portfolio allocation. However, if the correlations increase after having control for volatility, corresponding to pure contagion effects, the fund manager is
exposed to a new risk and the allocation of his portfolio is not efficient anymore.

Figure 2 – CDS Bond spread basis for each of the 9 emerging countries from the 01/01/2007 to the 03/26/2012. The results are computed with daily observations, expressed or corrected in US dollars. The grey band shows a period with explicit common dynamic between all the basis arbitrary chosen.

Now, we explore the Brazilian example more deeply. The figures in Appendix B describe the behavior of the CDS Bond spread basis as well as its two components. However, in terms of profit and loss, the behavior is approximately identical to the other countries. The Brazil basis is the only country with a negative basis but the dynamic is similar to the others. This illustration demonstrates that the liquidity problem is identified by a movement of the basis rather than by its level. Indeed, we study the contagion in terms of P&L of an arbitrage strategy. We consider that the manager invests and keeps his arbitrage position at a daily horizon. Thus, a variation of the basis has an impact on the P&L resulting from a liquidity problem due to the slowness of the adjustment of one market to the other.
4 Pure contagion rather than interdependence

Interdependence reflects high cross market linkages both during tranquil times and crisis while pure contagion highlights a significant increase of correlations during financial turmoils. Therefore, if pure contagion occurs, we should have two distinct regimes for correlations in the RSDC model that are significantly different. The smoothed probabilities summarize the dynamic of the dependence structure emphasizing pure contagion effects. A large increase of the smoothed probability to be in the state of high correlations indicates when pure contagion effects happen. In this section, we present results both on price data and liquidity. Pure contagion in terms of price impacts the risk of the portfolio through the simultaneous re-correlation of its assets. But what is even worse for a fund manager is whether this contagion comes from a liquidity problem. We study contagion in terms of liquidity because this situation can impede managers to unwind a position when needed corresponding to the main fear for a fund manager. All our results are conditional on the ability to capture the heteroscedasticity allowing to concentrate on changes of correlations. Thereby, we perform a robustness check studying which univariate conditional model is able to filter the variations of volatility.

4.1 Results

The major part of the contagion measurement literature is devoted to test the parameter stability of an econometric model. Indeed, a shift of these parameters is interpreted as a change in terms of correlation, i.e. a contagion phenomena. However, following Pelletier (2006), we study the probability to be in a state of "low correlations" (state 1) versus "high correlations" (state 2). Our first results show that the former is assimilated to "normal" times while the latter is assimilated to "crisis" times. We start our study by analyzing the re-correlation phenomena in terms of prices before to study the contagion effects in terms of liquidity problems. Our aim is to reveal the existence of contagion effects in emerging markets during the last crisis, particularly in terms of financial flows. Although both of these two subjects are actively debated on the recent literature, there are only few papers that study both contagion effects and liquidity, especially together.

We give in Figure 3 the smoothed probabilities of being in the regime of high correlation
for the bond market. We see that the correlations appear to be dynamic and that we switch between regimes. At each date, there is an uncertainty about the regime of correlations. On the one hand, the process spends more time in regime two and spells in regime one are shorter on average than in regime two. This is explained by the estimate of the transition probability matrix. The probability of being in regime two at time $t$ conditional on being in regime two at time $t-1$, $p_{2,2}$ is around 0.9998. That means a very high level of persistence in the Markov chain. Indeed, we are almost in the case of an absorbing state. In comparison, for regime one this probability is around 0.9040. Despite the close proximity of these probabilities, as Pelletier (2006) shows, it results large differences. Indeed, after 5 periods, these probabilities are respectively approximately equal to 0.95 and 0.55\(^9\). On the other hand, we show that the magnitudes of almost all the correlations in regime one are smaller than in regime two. We give in the appendix the correlation matrices. Moreover, the smoothed probability to be in the state two largely increase at the end of September 2008. As a consequence, we can easily consider that there exists a re-correlation phenomena on the sovereign debt market and this phenomena occurs almost simultaneously with the Lehman Brother collapse.

The same applies to the CDS market (see Figure 4). Indeed, the shift in correlations appears at the same date. However, the smoothed probability to be in the state of high correlations for the CDS market is more volatile than for the bond market. This results appears through the transition matrix in which the values to stay in the same regime are smaller. The probability to be in the state of low correlations at time $t$ and $t+1$ is equal to 0.9414 which is much greater than in the case of the bond market. Thus, as both CDS and bond markets exhibit shifts in terms of correlations at the end of 2008, the benefits from the diversification principle have plummet and the risk of the portfolios dramatically increased. We have to know if this phenomena also comes from a liquidity problem, that could be an additional risk for the fund manager. Moreover, we state that even if the assets behaviors came back to normal after some months, the correlations between them stayed high. In this case, build a portfolio has not the expected returns due to the high correlations between sovereign debt markets.

To confirm our results, we have to define if the two correlation matrices are significantly different. In other words, we test the number of regimes. However, the Markov switching ap-\(^{10}\)
approach in our model does not allow us to apply standard methods. Under the null hypothesis, a nuisance parameter is not identified. Garcia (1998) shows that asymptotic theory works for Markov switching only assuming the validity of the score distribution. Nevertheless, the asymptotic distribution is not so far from the standard Chi-square distribution while our likelihood ratio statistic is much greater than the critical value of this distribution. We conclude that a two regimes model offers greater results and confirm the significance of the difference between the two correlation matrices. To compute this statistic, we have to compare the likelihood of our model, the RSDC with two regimes and the CCC model (for Constant Conditional Correlations) which is assimilated to the RSDC with only one regime.

Now, we focus on the liquidity contagion effects of sovereign debt market. Again, the worse case for the manager would be a re-correlation phenomena due to a liquidity problem precluding him to unwind his positions. We apply the same model as below on our liquidity indicator: the CDS-Bond spread basis. As we see on Figure 5, the behavior of the smoothed probabilities to....
Figure 4 – Smoothed Probabilities to be in the state of high correlations at a daily frequency. The results concern the CDS market between 01/01/2007 and 26/03/2012.

be in the state two is almost similar for the CDS market and the basis. Indeed, we see that the probability to be highly correlated is very low, almost null; with some peaks between the start of our sample period and October 2008. Thus, we can conclude that the Lehman Brother collapse had a great influence on the re-correlation phenomena that CDS premiums, bond yields and Basis experienced. Moreover, we see that the jump in probabilities, synonymous of a pure contagion event, occurs at the same time. Thus, the re-correlation problem on CDS premium and bond yields is due to a liquidity problem. Indeed, if the problem was due to a fundamental event leading to the re-correlation of the CDS premiums, we should have only the same dynamic on CDS and Bond markets and a distinct one for the basis. Because of the simultaneous shift include the liquidity indicator, we can conclude to the existence of funding liquidity problem leading to a pure contagion problem. In other words, when CDS are high and investors are funding constrained, the basis deviates from parity. As the deviations from zero of the basis are more correlated during the financial crisis, we can establish that the pure contagion effects are
the result of a liquidity problem.

4.2 Robustness Check

All the results are based on the RSDC model of Pelletier (2006) and its capacity to distinguish interdependence from pure contagion. This model is estimated in two steps and assumes that the first one is able to correct for heteroscedasticity. In this robustness check, we want to control whether the volatility is removed accurately allowing us to concentrate on pure contagion phenomena. In other words, the univariate conditional volatility model with which we construct the matrix $S_t$ in equation (26) has to capture all the dynamic of the univariate variance. We check for ARCH effects in the standardized residuals and determine what is the better model for the first step in the estimation. We start with the well known GARCH(1,1) model, then, if we reject the hypothesis of independence of the squared residuals, we estimate a TGARCH(1,1) taking into account the asymmetry of P&L time series. If there is still some remaining heteroscedasticity
of all time series, we propose an alternative model [TGARCH(2,2)].

We sum up the results in Table 6. First, we see that the GARCH(1,1) model captures all heteroscedasticity effects for all countries except for Hungary in case of the CDS and the Basis. However, in a second step, we see that even a TGARCH(1,1) can not capture all the remaining ARCH effects on these two special cases. We have to apply a TGARCH(2,2) on our time series to get all the standardized residuals homoscedastic. To test if the univariate volatility estimation fully captures the heteroscedasticity, we employ the LM test proposed by Engle (1982) that check the autocorrelation of squared residuals. Compared to a Chi-square distribution, we can conclude about the presence of ARCH effects in the residuals of our model.

However, whatever the univariate volatility model is, we detect pure contagion effects at the same date. Indeed, the smoothed probabilities have a similar dynamic no matter which model is applied. We can conclude that even if the first step is not able to fully capture the heteroscedasticity but only the main part of it, the impact on the results is really small. Thus, the RSDC model is robust to the volatility specification and allow us to concentrate on pure contagion effects resulting from a shift in the correlation structure.

5 Conclusion

EM have experienced many financial crisis with contagion problems but, they are today the key drivers for global growth of the world economy. They propose very attractive investment opportunities for asset managers who consider them as an asset class. Nevertheless, the main risk for an asset manager is to loose the diversification benefits of his/her portfolio.

Firstly in this paper, we study the liquidity of the sovereign debt market showing the ability of the CDS Bond spread basis for accurately measuring liquidity. Secondly, we use a non linear model to detect contagion effects both in terms of prices and liquidity. Whereas interdependence is not a main concern for a fund manager, pure contagion phenomena may be problematic. The RSDC model is able to separate interdependence from pure contagion in order to focus on the second one. A such phenomena is highlighted by a shift in the probability to be in the state where the link between time series is stronger.

As a result, we detect pure contagion effects during the 2007-2008 financial crisis for a sample
of 9 emerging countries. The results are similar for bond market, CDS market and the liquidity indicator. We conclude that the pure contagion events result from a liquidity problem rather than from fundamentals. As the impact on the diversification benefits of a contagion event is largely bigger when it comes from liquidity, fund managers have to monitor this additional risk. The approach developed in this paper is able to dynamically watch the preponderance of such risk.
References


A Estimation of RSDC

The estimation of this model is made using a two-step procedure: (i) the univariate estimation of standardized residuals with TGARCH model and maximum likelihood and, (ii) the estimation of correlation matrices and probabilities to be in state \( n = 1, \cdots, N \) with an EM algorithm (Dempster et al. (1977)). Using this method is preferable when the number of observed series is more than a few. Indeed, the number of parameters could become very large and the one-step likelihood maximisation becomes untractable.

We should introduce \( \theta \), the complete parameter space, that we split in two parts with: \( \theta_1 \) that corresponds to the parameter space of the univariate volatility model and \( \theta_2 \) that corresponds to the parameter space of the correlation model. Firstly, we compute the log-likelihood taking a correlation matrix equal to the identity matrix. In other words, we estimate univariate TGARCH model for each asset.

A.1 First Step

To model the full covariance matrix, we estimate the standard deviations and the correlations separately. This first step focus on the estimation of standard deviations.

The parameters of univariate TGARCH model are estimated with maximum likelihood, taking the case of a TGARCH(1,1), as presented in section 1. We have to specify the distribution of \( U_t \) in order to estimate the likelihood function that we want to maximize. In our case, \( U_t \) are \( iid \) and normally distributed \( [U_t \sim \mathcal{N}(0,1)] \) allowing to consider gaussian likelihood. However, we don’t make the assumption that is the true law of \( U_t \).

Note \( \theta_1 = (\omega, \alpha^-, \alpha^+, \beta) \). Thus, the gaussian likelihood is:

\[
L(\theta_1) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\tilde{\sigma}_t^2}} \exp \left( -\frac{r_t^2}{2\tilde{\sigma}_t^2} \right)
\]

with \( \tilde{\sigma}_t \) are obtained recursively (\( \forall t \geq 1 \)) as:

\[
\tilde{\sigma}_{i,t} = \omega_i + \alpha_i^- min(\epsilon_{i,t-1}, 0) + \alpha_i^+ max(\epsilon_{i,t-1}, 0) + \beta_i \tilde{\sigma}_{i,t-1}
\]
Taking the logarithm and simplifying the expression, we have to minimize the log-likelihood \( \tilde{l}_t(\theta_1) \) that is defined by:

\[
\tilde{l}_t = \tilde{l}_t(\theta_1) = \frac{r_t^2}{\tilde{\sigma}_t^2} + \log(\tilde{\sigma}_t^2)
\]

Thus, \( \hat{\theta}_1 \) is the solution of:

\[
\hat{\theta}_1 = \arg \min_{\theta_1} \frac{1}{T} \sum_{t=1}^{T} \tilde{l}_t(\theta_1)
\]  \hspace{1cm} (30)

After the estimation of parameters, we get the standardized residuals, noted \( \tilde{U}_t \) as:

\[
\tilde{U}_{i,t} = \frac{r_{i,t}}{\tilde{\sigma}_{i,t}}
\]

In the next step, we use it to estimate the correlation matrices. We introduce a regime switching to add dynamic in correlations. It measures the probability to be in the state \( n \) (in our case \( n = 0, 1 \) corresponding respectively to liquid and illiquid states).

\section*{A.2 Second Step}

In this second part of estimation of our model, we use the Expectation Maximization algorithm (EM thereafter). The main advantage is the possibility to taking into account high number of parameters coming from each \( \Gamma_n \).

\textbf{EM Algorithm}

This algorithm is presented in Hamilton (1994, chapter 22). We have to estimate the vector of parameters \( \theta_2 \):

\[
\hat{\theta}_2 = \arg \min_{\theta_2} \left[ \frac{1}{2} \sum_{t=1}^{T} Klog(2\pi) + log(||\Gamma_t||) + \tilde{U}_t'\Gamma_T^{-1}\tilde{U}_t \right]
\]  \hspace{1cm} (31)

Unlike the first step, we have to use Hamilton filter because in this part of the estimation, \( \Delta_t \) is unobserved. Moreover, the number of parameters increases at a quadratic rate with the
number of asset returns. Thus, to realize these estimation, we use EM algorithm that has no restrictions on the number of parameters.

Then, Hamilton (1994, chapter 22) expose that Maximum Likelihood estimates of the transition probabilities (i) and the correlation matrices (ii) satisfy:

\[(i) \quad \tilde{\pi}_{i,j} = \frac{\sum_{t=2}^{T} P[\Delta_t = j, \Delta_{t-1} = i|\tilde{U}_t; \tilde{\theta}_2]}{\sum_{t=2}^{T} P[\Delta_{t-1} = i|\tilde{U}_t; \tilde{\theta}_2]} \tag{32}\]

\[(ii) \quad \tilde{\Gamma}_n = \frac{\sum_{t=1}^{T} (\tilde{U}_t \tilde{U}_t') P[\Delta_t = n|\tilde{U}_t; \tilde{\theta}_2]}{\sum_{t=1}^{T} P[\Delta_{t-1} = n|\tilde{U}_t; \tilde{\theta}_2]} \quad \text{for } n = 1, 2 \tag{33}\]

Estimates of transition probabilities are based on the smoothed probabilities. We could see that \(\tilde{\Gamma}_n\) is not directly a correlation matrix. It must be rescaled because their diagonal elements are not constrained to be equal to one. Off-diagonal elements are between \(-1\) and \(1\). This step is needed because the product of standardized residuals is not constrained to have elements between \(-1\) and \(1\). Then we rescale \(\Gamma_t\) at each iteration as:

\[\Gamma_t = D_t^{-1} \tilde{\Gamma}_t D_t^{-1} \tag{34}\]

where \(D_t\) is a diagonal matrix with \(\sqrt{\tilde{\Gamma}_{n,n,t}}\) on row \(n\) and column \(n\).

The algorithm starts with initial values \(\tilde{\theta}_2^{(0)}\) for the vector \(\theta_2\). With \(\tilde{\theta}_2^{(0)}\) we can compute a new vector \(\tilde{\theta}_2^{(1)}\) based on equations (32) and (33). The algorithm works until the difference between \(\tilde{\theta}_2^{(m)}\) and \(\tilde{\theta}_2^{(m+1)}\) is less than a defined threshold.

**Computation**

We develop in this subsection the method to compute the EM algorithm. The elements of the transition probabilities matrix, \(\tilde{\pi}_{i,j}\) are defined as the ratio of consecutive probabilities \(P[\Delta_t = j, \Delta_{t-1} = i|\tilde{U}_t, \theta_2]\) and the probabilities to be in state \(j\) at time \(t\). They are obtained iteratively from \(t = 1\) to \(T\).

Note that, conditional probability is defined by [see Hamilton, (22.3.7)]:
\[ P[\Delta_t = j|\bar{U}_t, \theta_2] = \frac{\pi_j \times f(\bar{U}_t|\Delta_t = j, \theta_2)}{f(\bar{U}_t|\theta_2)} \]  
\[(35)\]

where \(f(\bar{U}_t|\Delta_t = j, \theta_2)\) is the probability density of the multivariate normal distribution with zero mean and \(\Gamma_j\) as covariance matrix, evaluated for \(\bar{U}_t\).

With equation (35), we compute probabilities at time \(t = 1\). Then, we compute consecutive probabilities recursively:

\[ P[\Delta_t = j, \Delta_{t-1} = i|\bar{U}, \theta_2] = P[\Delta_{t-1} = i|\bar{U}, \theta_2] \times P[\Delta_t = j|\bar{U}, \theta_2] \times \pi_{i,j} \]  
\[(36)\]

where \(P[\Delta_t = j|\bar{U}, \theta_2] = f(\bar{U}|\Delta_t = j, \theta_2)\).

Then, conditional probabilities to be in state \(j\) at time \(t\) are obtained making the ratio of the sum of the two consecutive probabilities of being in state \(j\) at time \(t\) and the sum of all consecutive probabilities.

Introduce the notation \(\xi_{t|\tau}\), the \((N \times 1)\) vector whose \(j^{th}\) element is \(P[\Delta_t = j|\bar{U}_\tau, \theta_2]\). This notation allows to present two cases of \(\xi_{t|\tau}\):  
(i) for \(t > \tau\) it represents a forecast about the regime and (ii), for \(t < \tau\) it represents the smoothed inference (about the regime in date \(t\) based on data obtained through some later date \(\tau\)). We focus on smoothed probabilities that is defined by:

\[ \tilde{\xi}_{t|\tau} = \tilde{\xi}_{t|t} \odot \{\Pi^t \cdot [\tilde{\xi}_{t+1|T}(\hat{\Xi})\tilde{\xi}_{t+1|t}]\} \]  
\[(37)\]

Smoothed probabilities are obtained iterating on backward for \(t = T, T - 1, T - 2, \cdots, 1\). We come back from equation (36) to compute consecutive probabilities with smoothed probabilities. Then, we compute \(\theta_2^{(m)}\) with equation (32) and (33) rescaling at each iteration the correlation matrix with equation (34).

The breaking rule of the algorithm is defined by the fact that the correlation matrix computed by the last iteration is almost equal to the previous correlation matrix. We have to define a threshold under which, we consider that matrices are equal.
Initialisation of the Algorithm

To start the algorithm, we have to choose the space of initial parameters, $\theta_2^{(0)}$. In this space, we input correlation matrices for each state of our model (in our case, two). The algorithm starts with one matrix of correlations of the state (1) equal to identity matrix. For the second state, we use the Gramian matrix method (Holmes (1991)) to generate random correlation matrix. Note that a correlation matrix has to be defined semi-positive with diagonal elements that are equal to one and off-diagonal elements that are between $-1$ and 1. We use the Gramian matrix $T' T$ where $T := (t_1, \ldots, t_K)$ and $t_i$ is the $i^{th}$ column. Then, we normalize the matrix as: $t_i = \tau_i/||\tau_i||$.

For a K-variate process, we generate K independent pseudo-random vectors normally distributed, $\tau_i$. 
B The Brazilian example

Figure 6 – Illustration of the bond yield, the bond PnL and the Net Asset Value starting from 100 between 01/01/2008 and 31/08/2009 for the Brazilian case. The data have a daily frequency.

Figure 7 – Illustration of the CDS premium, the CDS contract PnL and the Net Asset Value starting from 100 between 01/01/2008 and 31/08/2009 for the Brazilian case. The data have a daily frequency.
Figure 8 – Illustration of the basis level, the basis PnL and the Net Asset Value starting from 01/01/2008 and 31/08/2009 for the Brazilian case. The data have a daily frequency.

C Correlations Matrices

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Chile</th>
<th>Hungary</th>
<th>Mexico</th>
<th>Poland</th>
<th>Russia</th>
<th>South Africa</th>
<th>Thailand</th>
<th>Turkey</th>
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Table 3 – Correlations matrices of the two regimes for the CDS market. The blue part of the matrix corresponds to the regime 1 and the black part corresponds to the regime 2.
Brazil 0.277 0.427 0.516 0.341 0.262 0.454 0.056 0.430
Chile 0.313 0.283 0.255 0.313 0.233 0.303 0.084 0.286
Hungary 0.555 0.381 0.416 0.709 0.554 0.554 0.032 0.520
Mexico 0.569 0.394 0.564 0.309 0.182 0.474 0.073 0.381
Peru 0.608 0.393 0.860 0.640 0.634 0.423 0.111 0.457
Poland 0.358 0.361 0.529 0.384 0.517 0.032 0.105 0.329
South Africa 0.596 0.383 0.678 0.620 0.708 0.478 0.056 0.554
Thailand 0.210 0.198 0.283 0.245 0.282 0.255 0.281 0.094
Turkey 0.591 0.405 0.694 0.607 0.714 0.482 0.693 0.299

Table 4 – Correlations matrices of the two regimes for the sovereign debt market. The blue part of the matrix corresponds to the regime 1 and the black part corresponds to the regime 2.

Brazil -0.012 0.047 -0.004 -0.191 0.015 -0.175 0.057 -0.139
Chile 0.188 0.276 0.409 0.027 0.339 0.205 0.346 -0.178
Hungary 0.121 0.418 0.341 -0.029 0.413 0.283 0.211 -0.261
Mexico 0.044 0.602 0.496 -0.034 0.651 0.247 0.299 -0.215
Peru -0.315 -0.044 0.089 -0.051 -0.127 0.269 -0.047 0.380
Poland 0.166 0.496 0.766 0.514 0.041 0.357 0.350 -0.290
South Africa -0.089 0.287 0.596 0.362 0.323 0.659 0.197 0.130
Thailand 0.074 0.252 0.347 0.269 0.069 0.353 0.282 -0.210
Turkey -0.382 -0.181 -0.057 -0.154 0.513 -0.095 0.202 -0.078

Table 5 – Correlations matrices of the two regimes for the basis considered in terms of profits and losses generated by such strategy. The blue part of the matrix corresponds to the regime 1 and the black part corresponds to the regime 2.

D Robustness Check
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Table 6 – ARCH effect test on standardized residuals from GARCH(1,1) model and alternative models in case of heteroscedasticity. The test is realized for a risk threshold of 5% and a number of lags equal to 12. In other words, we test whether all $\rho$ are equal to 0 in the following equation: $u^2_t = \alpha + \sum_{p=1}^{P} \rho^p u^2_{t-p} + \epsilon_t$, $\forall p = 1, \cdots, 12$. 

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