Survival of Hedge Funds: Frailty vs Contagion

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Abstract

The rather short lifetimes of a majority of hedge funds and the reasons of their liquidation explain the interest of investors and academics in hedge fund survival analysis. In this paper we are interested in the dependence between liquidation risks of individual hedge funds. This dependence can either result from common exogenous shocks (frailty), or be due to contagion phenomena, which occur when an endogenous behaviour of a fund manager impacts the Net Asset Values of other funds. We introduce dynamic models able to distinguish between frailty and contagion phenomena, and to test for the presence and magnitude of such dependence effects, according to the age and management style of the fund.

Keywords: Hedge Fund, Liquidation Correlation, Frailty, Contagion Dynamic Count Model, Autoregressive Gamma Process.

JEL classification: G12, C23.
1 Introduction

The rather short lifetimes \(^1\) of a majority of hedge funds (HF) and the reasons of their liquidation explain the interest of investors and academics in HF survival analysis. There exist different reasons for liquidating \(^2\) an hedge fund:

\(i\) The HF return is not attractive;

\(ii\) The incentive for the fund manager is not sufficient, for instance if she does not reach frequently the high water mark performance objective, and she prefers to liquidate this HF and create a new one with more attractive management fees [see e.g. Brown, Goetzmann, Liang (2004), Darolles, Gourieroux (2009) for the description of fees];

\(iii\) There is a need of liquid assets by the shareholders, especially in a situation of liquidity crisis;

\(iv\) Finally, liquidation can be the consequence of operational risk alone including fraud, unauthorized trading, misappropriation of investor funds, inadequate resources. Half of all failures can be attributed to such risks [Feffer, Kundro (2003), Brown (2010)].

From the investor’s point of view, liquidation is a risk, which concerns both the timing of the payoffs and the value of the fund at liquidation time. It is partially at the discretion of the fund manager, who can slow down or accelerate liquidation by an appropriate management of gates for instance. The liquidation risk is similar to default risk, prepayment risk, or lapse risk encountered on corporate bonds, credit derivatives, or life insurance contracts, respectively.

Liquidation risk is often neglected in standard portfolio management. This is a hidden risk, in opposition to the visible risk directly measured by the volatility of returns, i.e. the market risk. A fund with a small visible risk can have a large hidden risk. This explains why minimizing the volatility of a portfolio return is not necessarily a bright idea when allocating a fund of hedge funds.

\(^1\)The global annual mortality rate for HF between 1994 and 2003 has been around 8%-9%, which corresponds to a median lifetime of 6-7 years. However, this mortality rate is considerably varying according to the year and management style, with values between about 4% and 30% for this period [see e.g. Getmanski, Lo, Mei (2004), Chan, Getmanski, Haas, Lo (2007), Table 6.14]. Moreover, this rate depends significantly on the definition of mortality (liquidation or graveyard) and on the retained database. For instance the median lifetime is larger than 10 years in Bares, Gibson, Gyger (2001).

\(^2\)Ineichen (2001) has analyzed the reasons for liquidation of a list of selected hedge funds.
There exist different ways to measure the amount of hidden risk. An indirect approach consists
in comparing the extreme losses of a hedge fund during a specific period of time, such as a 2-
month crisis period, with the worst fund drawdown during a standard period. The percentage of
funds within a management style that double the drawdown during the specific period can provide
a crude measure of liquidation risk. [see e.g. Douady (????)].

In this paper, we follow the alternative approach which predicts the joint occurrences of liqui-
dation for hedge funds within and between management styles. The introduction of an appropriate
dynamic model gives a path dependent measure of liquidation risk, which is much more relevant
for portfolio allocation.

Liquidation risk is especially important for the following two types of investors:
i) Most of the new money flowing to HF in the beginning of the previous decade comes from insti-
tutional investors, including endowments, foundations and pension funds (if they have a minimal
capital) [Casey, Quirk, Acito (2004)]. They invest on a long term basis, are interested in the low
correlation of HF returns with traditional assets classes, like equity and bonds, and want to avoid
the consequences of a short term liquidity crisis.

ii) The funds of funds can be very sensitive to risk dependencies, and in particular to liquidation
correlation. An appropriate survival analysis can help to detect the funds of funds, which are too
sensitive to systematic risk on individual funds.

The fact that liquidation is often a consequence of unexpected bad results can bias the perfor-
mance analysis of the funds, which are still alive, and lead to an unappropriate allocation of funds
in a portfolio. There exist a rather large literature, trying to correct the survivorship bias on indi-
vidual HF performance [see e.g. Brown, Goetzmann, Ibbotson, Ross (1992), Fung, Hsieh (1997),
(2000), Ackermann, McEnally, Ravenscraft (1999), Amin, Kat (2003), Baquero, Horst, Verbeek
(2005)]. Without taking into account the liquidation correlation, the survivorship bias is typically
of 300 to 400 bps per year for hedge funds compared to 100 bps per year for mutual funds. 3

The analysis of HF lifetimes is generally based on a duration model, which can be either para-
metric, semi-parametric, or nonparametric. Typically a first step analysis studies how the mortality
intensity depends on the age of the HF, and/or on calendar time. This is done by appropriate aver-

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3This survivorship bias seems also to be positively correlated with the return volatility [Brown, Goetzmann, Ross
(1995)].
aging of observed mortality rates [see e.g. the use of Kaplan-Meier non parametric estimation of the hazard function in Baba, Goko (2006), Figure 1, for the description of age dependence, or the log-normal hazard function used in Malkiel, Saha (2005)]. In a second step, a parametric specification of the (discrete-time) mortality intensity can be selected, such as a logit, or a probit model to analyze the possible determinants of mortality. The explanatory variables which are introduced can be time independent HF characteristics such as the management style, the registration country (off-shore vs domestic), the minimum investment, variables summarizing the governance structure, such as the existence of incentive management fees, and their design (high water mark, hurdle rate), the announced cancellation policy (redemption frequency, lockup period), the experience and education level of the manager [Boyson (2002)]. Regressors can also include time dependent HF individual or market characteristics such as lagged individual HF return, realized return volatility and skewness, asset under management (AUM), the recent fund inflows, the competitive pressure measured by the total number of HF [see e.g. Boyson (2002), Agarwal, Daniel, Naik (2004), Getmansky (2004), Baquero, Horst, Verbeek (2005), Malkiel, Saha (2005), Chan, Getmansky, Haas, Lo (2007), Section 6.5.1]. Finally, the parametric and nonparametric approaches can be mixed in the proportional hazard model introduced by Cox (1972), (1975) as in Brown, Goetzmann, Park (2001), Gregoriou (2002), Rouah (2005), Baba, Goko (2006).

However, these models are not always appropriate for risk prediction, for evaluation of systematic risk, or for representing the observed liquidation clustering.

For instance, it is not recommended to introduce time dependent explanatory variables in the models. Indeed, the future liquidation risk can only be analyzed after being able to predict the future values of the time dependent explanatory variables. This requires a joint dynamic model for these variables and the indicator of mortality, which is a difficult task.

Moreover, liquidation clustering and systematic risk are due to risk dependencies. Since the duration models considered in the literature assume the independence of individual liquidation risks given the selected observed explanatory variables, liquidation correlation has not yet been included in the HF survival models. Two causes of risk dependencies can be distinguished.

As noted in Baba, Goko (2006), the proportional hazard Cox model is subject to the very restrictive assumption of proportionality of mortality rates with respect to age. This hypothesis is not fulfilled in practice as seen in Section 3.
There exist underlying factors, with common influence on the mortality intensities of the individual HF. In the credit risk literature, these factors are called systematic risks, or common frailties and can represent the effect of market return, or the business cycle. Such common exogenous shocks also exist in the joint analysis of HF lifetimes. Let us consider the balance-sheet of an hedge fund and assume that the asset component of the balance is invested in rather illiquid assets. Liquidation can arise if there is a quick decrease of the liabilities. Such a decrease can result from the withdrawal of some prime broker (the effect is amplified by the use of this debt to create the needed leverage effect), or from the outflow of investors. If these behaviours of either the prime broker, or the investors apply simultaneously to several funds, this will create a frailty phenomenon.

A risk dependency can also rise when an endogenous shock on one fund has an impact on the other ones. This is the so-called contagion effect. In the case of credit risk, contagion is generally due to the debt structures, when some of the banks or funds invest in some other ones. A well-known example is the default of the Russian government debt in August 1998, when Long Term Capital Management (LTCM) and many other fixed-income HF suffered catastrophic losses over the course of a few weeks. Then, the failure of one of these funds can increase the probability of liquidation of another one. At the limit, if several funds are simply feeder funds of Madoff, the failure of Madoff implies the joint failure of all its feeder funds. This is an endogenous contagion effect.

However, there exist other reasons for a contagion in the case of HF, which are related with the structure of the asset part of the balance sheet. Indeed, the HF portfolios are often invested in rather illiquid assets. A fund in difficulty can be obliged to sell a rather large volume of such assets, with a significant impact on asset prices due to the lack of liquidity. Under marked-to-market pricing, this will impact the Net Asset Values (NAV) of the other funds invested in these illiquid assets.

The analysis of underlying common risk factors and contagion is a first step before studying the impact of HF on systemic risk for the global financial markets, and the possible cascade into a global financial crisis. This impact is largely due to the leverage mechanism. "Leverage has the effect of a magnifying glass, expanding small profit opportunities into large ones, but also expanding small losses into larger losses" [Chan, Getmansky, Haas, Lo (2007)]. The aim of our paper is to introduce both frailty and contagion effects in HF survival models, and to measure the respective magnitude of these effects, according to the age of the fund and its management style.
We introduce in Section 2 the basic stratified binomial and Poisson model with both common frailty and contagion. The Poisson model with contagion and autoregressive gamma frailty is especially convenient, since it provides a joint affine dynamics for the frailty and the mortality counts. This facilitates nonlinear prediction of future risk as well as estimation of parameters. The dynamic properties of models with contagion and frailty are illustrated in Section 3 by means of simulations. These models are estimated on HF data in Section 4. Section 5 concludes. Technical proofs are gathered in appendices.

2 Mortality by generation and type

We introduce in this section a dynamic model for the joint distribution of mortality count histories of hedge funds of different types. The liquidation risk dependence and its dynamics are captured by introducing i) the autoregressive effect of lagged counts, that is the contagion effect, and ii) a dynamic frailty to represent unobservable exogenous common shocks.

2.1 The model

Let us partition the set of HF according to a qualitative individual characteristic, called type. This characteristic can be the management style, the registration country, the minimum investment, the type of governance, or the crossing of such variables. The type can take $K$ alternatives, $k = 1, \ldots, K$. Moreover, let $h$ denote the age of a fund, where $h = 1, \ldots, H$.

For each period $t$, we denote by $n_{k,h,t}$ the number of hedge funds with age $h$ and type $k$ at the beginning of the period. Among these $n_{k,h,t}$ hedge funds, $Y_{k,h,t}$ will die during this period. The parametric model specifies the joint distribution of the number of deaths $Y_{k,h,t}$, for $k, h, t$ varying. The model is defined in two steps. We first define the joint distribution of these counts given the common frailty history. Then the model is completed by specifying the frailty dynamics, which is needed to account for the unobservability of the common factors.

Let us denote by $F_t$ the value of the factor at date $t$, by $F_t = (F_t, F_{t-1}, \ldots)$ and $Y_{k,h,t} = (Y_{k,h,t}, Y_{k,h,t-1}, \ldots)$ the factor and mortality count histories, respectively, at date $t$. The factor $F_t$ can be multidimensional with dimension $m$. We make the following assumptions:
Assumption A.1: Conditional on $F_t$ and $Y_{t-1} = (Y_{k,h,t-1}, k, h$ varying), the variables $Y_{k,h,t}$, for $k, h$ varying, are independent with binomial distribution:

$$Y_{k,h,t} \sim B[n_{k,h,t}; p_{k,h,t}(\theta)],$$

where $p_{k,h,t}(\theta)$ denotes the discrete-time mortality stochastic intensity at period $t$ in category $k, h$, and $\theta$ is an unknown parameter.

The mortality is time varying and stochastic, since it depends on the unobservable stochastic frailty process $(F_t)$ and lagged counts. In general we consider transformations of the mortality rates by a quantile function to ensure the constraint that $p_{k,h,t}(\theta)$ is between 0 and 1. We get a logit (resp. probit) model when the quantile function corresponds to the logistic distribution (resp. standard Gaussian distribution). We consider below the quantile of the exponential distribution since this transformation provides the continuously compounded mortality intensity, \(^5\) which is specified as:

$$-\log [1 - p_{k,h,t}(\theta)] = a_{k,h}(\theta) + b_{k,h}(\theta)'F_t + c_{k,h}(\theta)'Y_{t-1}^*,$$

where $Y_{t-1}^* = (Y_{1,t-1}^*, \ldots, Y_{H,t-1}^*, \ldots, Y_{K,t-1}^*, \ldots, Y_{K,H,t-1}^*)'$ measures the mortality rates at the source of possible contagions. The common factor and sensitivity coefficients are assumed such that the RHS of equation (2.2) is positive. The variable $Y_{k,h,t-1}^*$ can be chosen as the lagged count $Y_{k,h,t-1}$, or lagged frequency $Y_{k,h,t-1}/n_{k,h,t-1}$, according to the degree of integration of the HF market. Thus, an effect of the lagged class size can pass through the choice of variables $Y^*$. The current size variable $n_{k,h,t}$ could also be introduced in the mortality intensity as a third type of explanatory variable to capture a competitive pressure effect. Loosely speaking, if $n_{k,h,t}$ is small, the default of an HF can improve the monopolistic power of the surviving HF in this category and thus diminish their mortality rates. The contagion effects can be both within and between categories. The vector of sensitivity coefficients $c_{k,h}(\theta)$ has components $c_{k,h,k',h'}(\theta)$, for $k', h'$ varying, measuring the degree (magnitude) of contagion from category $(k', h')$ to category $(k, h)$. This contagion effect is not necessarily symmetric. For example, we can have contagion from $(k', h')$ to $(k, h)$ and no contagion from $(k, h)$ to $(k', h')$.

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\(^5\)that is, an affine specification of the stochastic intensity in continuous time model [see equation (2.8)].
Assumption A.2: The common frailty path $(F_t)$ is unobservable. This is an exogenous Markov process with transition density:

$$g(f_t|f_{t-1}; \varphi),$$

(2.3)

where $\varphi$ is an unknown parameter.

Under Assumption A.2, the transition of factor process $(F_t)$ depends on the lagged factor value, not on lagged count values $Y_{t-1}$. Thus, the factor dynamic is exogenous and $F_t$ summarizes the exogenous shocks affecting jointly the mortality intensities of the different categories $(h, k)$ at the beginning of period $t$. For instance, an increase of mortality rates in the last decade can come from an inflow of mediocre funds, or from more limited investment opportunities [Amin, Kat (2003)]. A cluster effect can also be a consequence of common portfolio strategies. Typically, HF performances show an asymmetric sensitivity to S&P 500, especially for management style "Distressed", or "Managed Futures" [see the analysis of betas in Chan, Getmansky, Haas, Lo (2007), Section 6.6.1].

The notion of frailty has been initially introduced in duration models in Vaupel, Manton, Stallard (1979) and later used to define the Archimedean copulas [Oakes (1989)]. This is an unobservable individual variable introduced to account for omitted individual characteristic and correct for the so-called mover-stayer phenomenon [see e.g. Baba, Goko (2006) for the introduction of an individual static frailty in the HF literature]. In our framework $F_t$ is indexed by time and common to all HF, which justifies the terminology "common dynamic frailty" introduced by Duffie, Eckner, Horel, Saita (2009) for application in credit risk. The unobservable common frailty (also called systematic risk) has to be integrated out, which creates a contemporaneous dependence between the individual HF liquidation risks. The coefficient $b_{k,h}(\theta)$ gives the sensitivity of the continuously compounded mortality rate with respect to this common frailty. In general, this coefficient depends on both $k$ and $h$. For instance, we expect that the newly created HF are more robust, and thus less sensitive to exogenous shocks during the first 2 years, when they have survivorship available reserves. Then the mortality rates will increase between 2 and 3 years to diminish later on. However, we do not know yet how the liquidation correlation depends on the age, and how these different age effects vary with the type.

The serial correlation between mortality counts can be due to either frailty (systematic risk), or
contagion. This explains why it is important to test for the existence of such effects.\textsuperscript{6} In particular, specification (2.1)-(2.2) includes as special cases: i) the model without mortality correlation and contagion, if
\[ b_{k,h}(\theta) = 0, \quad c_{k,h}(\theta) = 0, \quad \forall k, h. \] (2.4)
ii) The model with frailty only, when \( c_{k,h}(\theta) = 0, \forall k, h. \)
iii) The model with contagion only, when \( b_{k,h}(\theta) = 0, \forall k, h. \) This submodel can be seen as a time discretized version of the self- and mutually-exciting point processes introduced by Hawkes (1971) a,b, Hawkes, Oakes (1974), and used in the analysis of corporate default by Giesecke (2008), Azizpour, Giesecke (2008) and contagion across stock markets by Ait-Sahalia, Cacho-Diaz, Laeven (2010). It can also be seen as the extension to homogenous population of the Freund model [Freund (1961)].
iv) The proportional hazard model with both mortality and contagion if \( a_{k,h}(\theta) = d_h(\theta)a^0_k(\theta), b_{k,h}(\theta) = d_h(\theta)b^0_k(\theta) \) and \( c_{k,h}(\theta) = d_h(\theta)c^0_k(\theta), \) say. Then, \( d_h \) is the baseline mortality intensity, defined up to a multiplicative scale.

Frailty and contagion are both creating correlation effects between the lifetimes. These effects can be difficult to distinguish in practice. In the limiting case of a static model, that is with independent frailties \((F_t)\) and simultaneous effects of observed counts, the two phenomena cannot be identified. This is the reflection problem highlighted in Manski (1993). In a dynamic framework, they can be disentangled, if the analysis uses appropriate summary statistics, for instance by considering the conditional overdispersion to capture the dynamic of extreme counts [see e.g. Gagliardini, Gourieroux (2011) and Section 3.3]. By applying a maximum likelihood approach, appropriate nonlinear summary statistics are implicitly used.

Let us now derive the likelihood function. Conditional on factor history, the joint density of count variables is:
\[ K \prod_{k=1}^K \prod_{h=1}^H \prod_{t=1}^T \left\{ \binom{n_{k,h,t}}{y_{k,h,t}} [p_{k,h,t}(\theta)]^{y_{k,h,t}} [1 - p_{k,h,t}(\theta)]^{n_{k,h,t} - y_{k,h,t}} \right\}. \] (2.5)

Since the common frailty history is unobservable, the likelihood function is deduced by integrating out the factor. This likelihood function is given by:

\textsuperscript{6}The contagion component is typically omitted when a test for serial correlation is considered as a test of systematic risk exposure [see Chan, Getmansky, Haas, Lo (2007), Sections 6.4.1-6.4.2].
\[ L(\theta, \varphi) \propto \int \prod_{k=1}^{K} \prod_{h=1}^{H} \prod_{t=1}^{T} \left[ p_{k,h,t}(\theta) \right]^{y_{k,h,t}} \left[ 1 - p_{k,h,t}(\theta) \right]^{n_{k,h,t} - y_{k,h,t}} \prod_{t=1}^{T} \left[ g(f_t|f_{t-1}; \varphi) \right] df_t, \] (2.6)

where \( \propto \) means equality up to a scale factor independent of parameters.

This likelihood function has a complicated expression, which involves a \( mT \)-dimensional integral, whenever a common frailty is introduced in the model. Despite the complicated likelihood function, the model is very appropriate for simulating factor and count histories, and can be estimated by simulation based methods. Indeed, this model is a special nonlinear state space model, for which there exist standard methods based on Gibbs sampling for computing numerically the values of the likelihood function [see e.g. Cappé, Moulines, Rydén (2005), and Duffie et al. (2009) for an application in credit risk]. But these approaches can be time consuming.

The unobservable factor has also to be integrated out, when predicting the future counts. More precisely, we have:

\[ P[Y_t = y_t | F_t, Y_{t-1}] = \prod_{k=1}^{K} \prod_{h=1}^{H} \left\{ \left( \frac{n_{k,h,t}}{y_{k,h,t}} \right) p_{k,h,t}^{y_{k,h,t}} (1 - p_{k,h,t})^{n_{k,h,t} - y_{k,h,t}} \right\}, \] (2.7)

which depends on the factor path and the count history through the current factor value and the most recent lagged count only.

We deduce that:

\[ P[Y_t = y_t | Y_{t-1}] = \int P[Y_t = y_t | f_t, Y_{t-1}] g(f_t|Y_{t-1}) df_t, \] (2.8)

where \( g(f_t|Y_{t-1}) \) denotes the filtering distribution of factor value \( F_t \). Thus, the effect of lagged counts on future counts can arise either as a consequence of the contagion phenomena, or by means of the filtering distribution of the factor.

### 2.2 The Poisson approximation

Instead of approximating the likelihood function with multiple integrals by Monte-Carlo, it is possible to use a Poisson approximation leading to closed form expressions of the predictive distributions of the count variables.

Indeed, for monthly data, the mortality rate is small whereas the size of a category is rather large. Then, the binomial distribution can be well-approximated by the Poisson distribution [see
To derive such an approximation let us assume that the category sizes are such that:

\[ n_{k,h,t} \approx \gamma_{k,h,t} n, \]  

(2.9)

where \( n \) tends to infinity and \( \gamma_{k,h,t} \) are fixed, that are, independent of \( n \). Let us also assume that the mortality intensities are such that:

\[ \lim_{n \to \infty} np_{k,h,t}(\theta) = \lambda_{k,h,t}(\theta) \geq 0, \text{ say}. \]  

(2.10)

Then, the conditional distribution of the counts can be approximated by:

\[ Y_{k,h,t} \sim P[\gamma_{k,h,t}\lambda_{k,h,t}(\theta)]. \]  

(2.11)

When model (2.2) is chosen for the mortality intensity, its expansion provides an affine form for the limiting mortality intensity:

\[ \lambda_{k,h,t}(\theta) = a_{k,h}(\theta) + b_{k,h}(\theta)' F_t + c_{k,h}(\theta)' Y^*_{t-1}, \text{ say}. \]  

(2.12)

The scale component \( \gamma_{k,h,t} \) is just a size adjustment, which can for instance be replaced by:

\[ \tilde{\gamma}_{k,h,t} = n_{k,h,t}/n_{k,h,t_0}, \]  

(2.13)

where \( t_0 \) is a given date.

This adjustment is especially required for the HF sector, which has experienced a large development before the recent crisis. [see Section 4.1]. By this adjustment, the distribution of the count \( Y_{k,h,t} \) depends on the current sample size \( n_{k,h,t} \).

Whereas equation (2.1) defines the finite population or microscopic mortality model, equation (2.7) defines its limit for large population size, or macroscopic model. As seen below, by neglecting the effect of finite sample size, we get a simplified dynamic and recover affine dynamic models, especially appropriate for nonlinear prediction and estimation purposes.

Let us now consider the conditional Laplace transform \(^7\) of the process and how it depends on the lagged values. When the Poisson approximation is applied, the conditional Laplace transform

\(^7\text{Recall that the conditional Laplace transform characterizes the conditional distribution of a nonnegative process [Feller (1966)].}\)
of the current count vector $Y_t$ given $F_t, Y_{t-1}$ is equal to:

$$
\psi_t(u) = E \left[ \exp(-u'Y_t) | F_t, Y_{t-1} \right] 
= \prod_{k,h} E \left[ \exp(-u_{k,h}Y_{k,h,t}) | F_t, Y_{t-1} \right] 
= \prod_{k,h} \exp \left\{ -\gamma_{k,h,t}\lambda_{k,h,t}(\theta)[1 - \exp(-u_{k,h})] \right\} 
= \exp \left\{ -\sum_{k,h} [1 - \exp(-u_{k,h})] \gamma_{k,h,t}a_{k,h}(\theta) - \sum_{k,h} [1 - \exp(-u_{k,h})] \gamma_{k,h,t}b_{k,h}(\theta)'F_t 
- \sum_{k,h} [1 - \exp(-u_{k,h})] \gamma_{k,h,t}c_{k,h}(\theta)'Y_{t-1}^* \right\}.
$$

(2.14)

This conditional Laplace transform is an exponential affine function of $F_t$ and $Y_{t-1}^*$. By selecting an affine dynamics for the latent factor, that is, by assuming that:

$$
E \left[ \exp(-u'F_t) | F_{t-1}, Y_{t-1} \right] = \exp \left\{ -\alpha(u)'F_{t-1} - \beta(u) \right\},
$$

(2.15)

for some positive functions $\alpha$ and $\beta$, we deduce also an affine dynamic for the joint process $(Y_t, F_t)$. More specifically, the joint conditional Laplace transform of process $(Y_t, F_t)$ is deduced from (2.14)-(2.15) by iterated expectation. We get:

$$
\psi_t(u,v) = E \left[ \exp(-u'Y_t - v'F_t) | F_{t-1}, Y_{t-1} \right] 
= \exp \left\{ -\sum_{k,h} \gamma_{k,h,t}a_{k,h}(\theta)[1 - \exp(-u_{k,h})] - \sum_{k,h} [1 - \exp(-u_{k,h})] \gamma_{k,h,t}b_{k,h}(\theta)'F_t 
- \alpha \left( \sum_{k,h} \gamma_{k,h,t}b_{k,h}(\theta)[1 - \exp(-u_{k,h})] + v \right)'F_{t-1} 
- \beta \left( \sum_{k,h} \gamma_{k,h,t}b_{k,h}(\theta)[1 - \exp(-u_{k,h})] + v \right) \right\}.
$$

(2.16)

The joint conditional Laplace transform is exponential affine in lagged values $(F_{t-1}, Y_{t-1})$, that is, of the form:

$$
\psi_t(u,v) = \exp \left\{ -\alpha_{1,t}(u,v)'F_{t-1} - \alpha_{2,t}(u,v)'Y_{t-1} - \beta_{1,t}(u,v) \right\},
$$

(2.17)

where the sensitivity coefficients $\alpha_t$ and $\beta_t$ dependent on the class size and are time depend in general. Thus, process $(F_t, Y_t)$ is a time-heterogenous affine process. The closed form exponential
affine expression of the conditional Laplace transform of \((F_t, Y_t)\) simplifies the computation of the predictive distributions at any prediction horizon [see e.g. Duffie, Filipovic, Schachermayer (2003) for continuous time, Darolles, Gourieroux, Jasiak (2006) for discrete time], the derivation of the likelihood function by applying a filtering approach in frequency domain, and then the derivation of the maximum likelihood estimates [see Bates (2006)].

Let us now consider the single factor case for expository purpose. Coefficients \(a_{k,h}, b_{k,h}, c_{k,h}\) and factor \((F_t)\) are assumed real positive.\(^8\) The simplest positive real affine process is the time discretized Cox, Ingersoll, Ross process [Cox, Ingersoll, Ross (1985)], called Autoregressive Gamma (ARG) process in the time series literature [Gourieroux, Jasiak (2006)]. The transition of this process corresponds to a noncentral gamma distribution and its conditional Laplace transform is given by equation (2.15) with:

\[
\alpha(u) = \frac{\eta u}{1 + \nu u}, \quad \beta(u) = \delta \log(1 + \nu u).
\]

(2.18)

where \(\nu, \nu > 0\), is a scale parameter, \(\delta, \delta > 0\), the degree of freedom of the gamma distribution and parameter \(\eta, \eta \geq 0\), gives the magnitude of serial dependence. The factor process is stationary if \(\eta < 1\) [see Appendix 2].

### 2.3 Term structure of mortality risk in a crystallized portfolio of funds

The dynamic model with frailty and contagion can be used to analyze the term structure of mortality risk at date \(T\) in a given portfolio. This portfolio has a given structure with respect to type and age, which will be crystallized in the analysis. This structure is denoted by \(\gamma_{k,h}\), for \(k, h\) varying, independent of time. Thus, the dynamic of future mortality risk in the crystallized portfolio corresponds to the process \((F_t, Y_t)\), for \(t = T, T + 1, \ldots\), with the conditional Laplace transform (2.16), where \(\gamma_{k,h}\) is time independent. This future dynamic corresponds to a time-homogeneous affine process. In particular, the transition at horizon \(\tau = 1, 2, \ldots\) is also associated with an exponential

\(^8\)It should also be possible to introduce qualitative underlying factor. For instance, Chan, Getmansky, Haas, Lo (2007) introduce a static regime indicator of "phase locking" periods, in which "uncorrelated actions suddenly become synchronized" (in a joint analysis of HF returns). This can be done by means of a Markov switching regime a la Hamilton (1989), (1990).
affine conditional Laplace transform:

$$
\psi_{t,\tau}(u, v) = \exp\left[-[\alpha_{\tau,1}(u, v)F_{t-1} - \alpha_{\tau,2}(u, v)Y_{t-1} - \beta_{\tau}(u, v)]\right],
$$

where the new functions $\alpha_{\tau,1}, \alpha_{\tau,2}, \beta_{\tau}$ satisfy recursive equations [see Darolles, Gourieroux, Jasiak (2006)].

3 Dynamic properties of the model

Let us now illustrate the dynamic properties of the Poisson count model with frailty and contagion.

3.1 The generating process

We consider two possible types ($K = 2$), a single frailty factor ($m = 1$), and constant subpopulation sizes $n_{k,h,t} = 20$, for all $k$, $h$ and $t$; thus the scale adjustment $\gamma_{k,h,t}$ in equation (2.7) is set equal to 1 without loss of generality. We assume that the lagged sufficient summary of mortality in class $k$ is $Y^*_{k,t-1} = \sum_{h=1}^{H} Y_{k,h,t-1}$, for $k = 1, 2$. Thus, the Poisson model becomes:

$$
Y_{k,h,t} \sim P [\lambda_{k,h,t}(\theta)],
$$

where

$$
\lambda_{k,h,t}(\theta) = a_{k,h}(\theta) + b_{k,h}(\theta)F_t + c^*_{k,h}(\theta)Y^*_{t-1}, \quad k = 1, 2, \quad h = 1, \ldots, H,
$$

and $Y^*_{t-1} = (Y^*_{1,t-1}, Y^*_{2,t-1})'$. Moreover, we assume that the frailty $F_t$ follows an ARG process [see (2.18)].

The model is completed by specifying the parametric form of the sensitivity parameters. We consider specification with mortality intensity constant in age $h$. Thus we get:

$$
\lambda_{k,h,t} = a_k^* + b_k^*F_t + c^*_{k,Y^*_{t-1}}, \quad k = 1, 2,
$$

where $c^*_{k} = (c^*_{k,1}, c^*_{k,2})$. For class $k$, the intensity involves 4 parameters $a_k^*, b_k^*, c^*_{k,1}$ and $c^*_{k,2}$.
For such a Poisson model with mortality intensity independent of age, we could aggregate the counts over the age and consider the aggregate model:

\[ Y^*_k,t \sim \mathcal{P}(a_k + b_k F_t + c_k Y^*_{t-1}), k = 1, \ldots, K, \] (3.4)

where \( Y^*_k,t = \sum_{h=1}^H Y_{k:h,t} \), \( a_k = Ha^*_k \), \( b_k = Hb^*_k \), \( C = HC^* \), the variables \( Y^*_k,t, k = 1, \ldots, K \) are independent conditional on \( F_t \), \( Y^*_{k,t-1} \) and process \((F_t)\) follows an ARG process. It is always possible to assume \( EF_t = 1 \) for identification reason. Thus, the autoregressive equation of the ARG process becomes:

\[ E_{t-1}(F_t) = 1 - \rho + \rho F_{t-1}, \] (3.5)

where \( E_{t-1} \) denotes the expectation conditional on lagged values of the factor and count processes and \( \rho = \nu \eta \). The first and second-order unconditional moments on process \((F_t, Y^*_t)'\) are computed in Appendix 1.

**Proposition 1**: When the process \((F_t, Y^*_t)'\) is stationary, we get:

\[ E(Y^*_t) = (Id - C)^{-1}(a + b), \]
\[ Cov(Y^*_t, Y^*_{t-1}) = \sigma^2 \rho bb'(Id - \rho C')^{-1} + CV(Y^*_t), \]

where \( V(Y^*_t) \) is solution of the linear system:

\[ V(Y^*_t) = CV(Y^*_t)C' + diag[(Id - C)^{-1}(a + b)] \]
\[ + \sigma^2 bb' + \sigma^2 \rho C(Id - \rho C)^{-1}bb' + \sigma^2 \rho bb'(Id - \rho C')^{-1}C', \]

where \( \rho = \eta \nu \) and \( \sigma^2 = V(F_t) = 1/\delta \).

In particular the autoregressive coefficient in the Seemingly Unrelated Regression (SUR) of \( Y^*_t \) on \( Y^*_{t-1} \) is given by:

\[ Cov(Y^*_t, Y^*_{t-1})V(Y^*_{t-1})^{-1} = C + \sigma^2 \rho bb'(Id - \rho C')^{-1}V(Y^*_t)^{-1} \] (3.6)

As expected, it as the sum of matrix \( C \) corresponding to the direct endogenous contagion and of the component, which is due to the frailty and its indirect effect through contagion.
Let us now consider the model above. The following sets of parameter values are considered for the frailty dynamic to investigate the effect of the factor persistence.

**Set F.1:** $\eta = 0$, which implies a static factor.

**Set F.2:** $\eta = 0.99$, which corresponds to a strong persistence of the factor.

The other parameters of the ARG process are fixed in order to get the unconditional factor mean equal to 1 and the unconditional variance equal to 0.5. By using the expression of the unconditional moments in Appendix 2 iii), this implies $\delta = 2$ and $\nu = 1/(2 + \eta)$.

The sensitivity parameters of the frailty are set to the following values:

**Set SF.1:** $b_1 = b_2 = 0$, which means no frailty effect.

**Set SF.2:** $b_1 = b_2 = 0.5$, to capture a common frailty.

The contagion parameters are set to:

**Set C.1:** $C^* = \begin{pmatrix} c_{11}^* & c_{12}^* \\ c_{21}^* & c_{22}^* \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, which corresponds to no contagion within types or between types.

**Set C.2:** $C^* = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}$, which captures a contagion within types and no contagion between types.

**Set C.3:** $C^* = \begin{pmatrix} 0 & 0 \\ 0.2 & 0 \end{pmatrix}$, which is a recursive model of contagion, able to describe the so-called double default effect encountered in credit risk (see e.g. Ebert, Luetkebohmert (2009), Gourieroux, Monfort (2010)).

The last parameters are fixed to $a_1 = a_2 = 1$.

### 3.2 Simulated paths

Simulated paths of the frailty and the two count series are displayed in Figures 1-5, and historical summary statistics: mean, variance, overdispersion, serial and cross-correlations, are computed.
on these series, are given in Table 3. These paths are simulated by using the nonlinear state space representation of the ARG(1) process [see Appendix 2 ii]).

[Insert Table 3: Summary Statistics]

We provide in Figure 1 the count trajectories corresponding to the sets of parameters F.1, SF.1 and C.1. The count processes are independent strong white noise processes with Poisson distribution and identical parameter $\lambda = 1$.

[Insert Figure 1: No Frailty and No Contagion Effect]

Figure 2 corresponds to the sets of parameters F.1, SF.2 and C.1. The shocks on the frailty process $F_t$ are now directly observed on the two mortality count series. This frailty effect generates instantaneous cross correlation between the two counting processes, and a phenomenon of overdispersion due to stochastic intensity (see Table 3, column 2).

[Insert Figure 2: Only Frailty with Uncorrelated Factor Effect]

We add in Figure 3 a serial dependence at the frailty level using the sets of parameters F.2, SF.2 and C.1. Since $\rho = \eta/(2 + \eta)$, the associated serial correlation is $\rho = 0.33$. The light persistence observed on the frailty level generates some persistence at the mortality counts level. We observe a clear clustering mortality effect, with several rather long periods associated with a high mortality rate. The serial correlation on the count variables is significant, but smaller than the serial correlation on the factor itself. Indeed, the counts are nonlinear noisy functions of the factors.

[Insert Figure 3: Only Frailty with Correlated Factor Effect]

Figure 4 corresponds to the sets of parameters F.2, SF.2 and C.2. The time persistence observed on the two counting processes has two distinct sources. The first one is the direct exposure to the frailty correlated factor. The second one is linked to the direct contagion effect, that is the mortality count of Type 1 at time $t - 1$ has an impact on the mortality count of Type 1 at time $t$. This increases the clustering effect observed on Figure 3.
We provide in Figure 5 the count trajectories corresponding to the sets of parameters F.2, SF.2 and C.3. The only difference with Figure 4 is the recursive behavior of the contagion effect, that is, the mortality count of Type 1 at time \( t - 1 \) has an impact on the mortality count of Type 2 at time \( t \). This recursive effect generates a time lead between the picks observed on the two mortality count series around \( t = 50 - 60 \).

3.3 Conditional overdispersion

The effects of frailty and contagion can be significantly different, but difficult to disentangle by considering standard summary statistics as historical moments, or autocorrelation function. This is easily understood from Proposition 1, since \( \sigma b \) can be identified from the first and second-order unconditional moments, but not \( \sigma \) and \( b \) separately. Gagliardini, Gourieroux (2011) propose to consider the conditional overdispersion:

\[
\tau(Y_{t-1}) = \frac{V(Y_t|Y_{t-1})}{E(Y_t|Y_{t-1})},
\]

(3.7)
in the case of a single type. Since an increase of conditional variance has an effect on the unconditional kurtosis, it is not surprising that the conditional overdispersion, which is more informative than the unconditional moment of order 4, allows for identifying \( \sigma \) and \( b \).

The conditional overdispersions have been estimated nonparametrically and are reported in Figure 6 for two simulation schemes, the first one with frailty only, the second-one with both frailty and contagion. We see that an increase of contagion changes the slope of this function. It has been noted in Gagliardini, Gourieroux (2011) that an increase of frailty implies a positive drift on the level of overdispersions.
4 Application

The dynamic Poisson model is applied to individual HF data reported in the June 2009 Lipper TASS database.

The database categorizes HF into "Live" and "Graveyard" funds. The "Live" funds are presented as still active. There are several reasons for a fund to be included in the Graveyard database. For instance, these funds i) no longer report their performance to TASS, ii) are liquidated, iii) are merged or restructured, iv) are closed to new investors. In our analysis, we consider only the HF, which are considered as "Live", or "Liquidated" (status code 1).

We follow another approach to separate dead and live funds and account for the time needed to pass from "Live" to "Graveyard" in TASS. We have transfered to the Graveyard database all funds of the "Live" database with at least missing data for April, May, June 2009 along the lines described in Appendix 1, Table 1.

4.1 The Data

The database consists of monthly returns, Asset Under Management (AUM) and other HF characteristics for individual funds from February 1977 to May 2009. We have only selected funds with Net Asset Value (NAV) in USD, with monthly reporting frequency. Nevertheless, we have also included the funds with quarterly reported frequency, when the intermediate monthly estimated returns were available (see Table 2).

The age of an individual HF is measured since the inception date reported in TASS. Thus, we do not take into account the incubation period, preliminary to inception, and the possible associated left truncation bias. Indeed, the data on incubation are either missing, or weakly reliable in the

9Graveyard status code: 1=fund liquidated; 2= fund no longer reporting to TASS; 3 = TASS unable to contact the manager for updated information; 4=fund closed to new investment; 5=fund has merged into another entity; 7=dormant fund, 9=unknown.
10Given the unregulated status of HF, there is in principle no way to find the information on either the date, or terms of the merger.
11More precisely, after such an event the fund is transferred into the Graveyard database. A hedge fund can only be listed in the Graveyard database after being listed in the Live database.
12Chan, Getmansky, Haas, Lo (2005) have regarded as liquidated all Graveyard funds in status code 1, 2 or 3.
13According to Fung, Hsieh (2000), the average incubation period is one year.
It is usual in the literature interested in performances and in the effect of performances on survival to consider HF with at least two years of track records [see e.g. Fung, Hsieh (2000)]. The aim is to avoid the over performances after inception date due to the incubation strategy. Since we do not introduce the effect of time dependent individual variables, such as returns, we do not follow this practice in our analysis.

In order to apply the Poisson approximation, within the management styles, we select the management styles with a sufficient size. They are Long/Short Equity (LSE) (39%), Event Driven (ED) (10%), Managed Futures (MF) (11%), Equity Market Neutral (EMN) (7%), Fixed Income Arbitrage (FI) (5%), Global Macro (GM) (7%), Emerging Markets (EM) (8%), Multi Strategy (MS) (10%), Convertible Arbitrage (CONV) (4%), with their frequency in the database given in parentheses [see Table 1 last column in Appendix 1]. Finally, to keep the interpretation in terms of individual funds, we eliminate the funds of funds. After this process of selection of funds, our sample include 6406 live and liquidated funds.

We provide in Figures 6 and 7, the evolutions of the subpopulation sizes and frequency counts of defunct HF for different management styles, without distinguishing the age of the HF. We observe from Figure 6 the market growth between 2000 and 2007 and the sharp decrease due to the 2008 financial crisis. However, the effect of the crisis is less pronounced for HF following a Global Macro strategy. Figure 7 shows mortality clustering both with respect to time and among categories. One mortality clustering corresponds to the Long Term Capital Management (LTCM) debacle in Summer 1998 and is especially visible for the Emerging Markets and Global Macro categories. Another mortality clustering arises in the 2008 crisis, but did not strongly reach the Global Macro strategy.

[Insert Figure 7: Subpopulation Sizes of HF]

[Insert Figure 8: Frequency Counts of Defunct HF]

Another left truncation bias which cannot be taken into account with the TASS database as well as other currently available HF databases, arises from the funds which have disappeared before the starting date of the database. Indeed, the database does not provide information for such funds, but does for funds of the same generation which are still alive at the starting date of the database.
Let us now focus on the age effect. We provide in Figure 9 the smoothed nonparametric estimates of the mortality intensity by categories, without taking time into account.

![Insert Figure 9: Smoothed Estimator of Mortality Intensity]

These estimators feature similar patterns, with a common bump around 4 years. However, by looking to the maximum and boundary values of the estimated intensities (see Table 4), we note that the intensity functions are not proportional, which leads to reject the hypothesis of proportional hazard model.

There can also exist cross-effects of time and age, which are difficult to observe with marginal analysis w.r.t. either time (see Figure 8), or age (see Figure 9). These cross-effects can be detected by means of the Lexis diagram. Each liquidated fund is reported on the diagram with the date of death on the x-axis and its age at death on the y-axis. By following the 45° line passing by the point characterizing this fund, we get all the liquidated funds of the same cohort. In particular, the intersection of this line with the x-axis provides the birth date of the fund (see Figure 10).

![Insert Figure 10: Lexis diagram]

The Lexis diagrams for the different categories are provided in Figures 11-19. In each case, we are looking for concentration of stars either in a band parallel to the x-axis (age effect), or parallel to the y-axis (time effect), or parallel to the 45° line (cohort effect). However, this analysis might be adjusted from the change of the cohort sizes over time. For the Emerging Markets strategy, we observe some age effect between 20 months, rather regular mortality for some cohorts starting on 1993 and time effect of the crisis. As expected, the Lexis diagram for Global Macro strategy reveals two time effects around 1998 (LTCM) and 2008-2009 (the recent crisis), whereas the two time effects are around Jan 2003 and the crisis for the Multi Strategy.

The strategy Managed Futures shows essentially an age effect, and so on. To summarize, we can expect the presence of the three types of effects, but they can differ significantly according to the strategy.

![Insert Figure 11: Lexis diagram]

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15based on the Kaplan-Meier estimators of the survival functions

22
4.2 Model with contagion only

Let us first focus on a Poisson regression model with lagged counts as regressors, where:

\[ Y_{k,h,t} \sim P \left( \gamma_{k,h,t}(a_k + \sum_{k'=1}^{K} c_{k,k'} \frac{Y_{k',t-1}^*}{\gamma_{k',t-1}^*}) \right), \]  

(4.1)

with two adjustments for cohort sizes. Without loss of information the data can be aggregated over the age to get:

\[ y_{k,t}^* \sim P \left[ \gamma_{k,t}^*(a_k + \sum_{k'=1}^{K} c_{k,k'} \frac{Y_{k',t-1}^*}{\gamma_{k',t-1}^*}) \right]. \]  

(4.2)

The model involves intercept parameters \( a_k, k = 1, ..., K \), and a matrix of contagion parameters \( c_{k,k'}, k, k' = 1, ..., K \). The estimated values of the intercepts are given in Table 5 with their t-statistics. The estimated contagion matrix is provided in Table 6, keeping the significant contagion coefficients only. The structure of the contagion matrix provides interesting information on the contagion routes. All strategies are interconnected either in one, or several steps. We observe the special role of the Fixed Income Arbitrage strategy which influences directly almost all other strategies.
We provide in Table 7 the eigenvalues and first eigenvectors of the contagion matrix.

### 4.3 Model with frailty and contagion

The model is now extended to include also the exogenous frailty:

\[
Y_{k,h,t} \sim \mathcal{P}[\gamma_{k,h,t}(a_k + b_k F_t + \sum_{k'=1}^{K} c_{kk'} \frac{Y_{k',t-1}^*}{\gamma_{k',t-1}^*})],
\]

(4.3)

with conditional independence. Without loss of information the data can be aggregated over the age to get:

\[
Y_{k,t}^* \sim \mathcal{P}[\gamma_{k,t}^*(a_k + b_k F_t + \sum_{k'=1}^{K} c_{kk'} Y_{k',t-1}^*)].
\]

(4.4)

From Proposition 1, we deduce the expectation, the variance and the serial covariance at order 1 of the observations \(Y_t^*\), by applying the appropriate size adjustments.

Let us assume that we are interested on the presence of frailty and/or contagion, and their respective effects on serial correlation. We can estimate parameters \(\sigma b, C, \eta\) of the model by a method based on these first and second-order moments. More precisely, parameters \(d = \sigma b, C, \eta\) can be estimated by minimizing the criterion:

\[
(\hat{d}, \hat{C}, \hat{\eta}) = \arg \min_{d, C, \eta} \left\{ ||\hat{\Gamma}(0) - C \hat{\Gamma}(0) C' - \text{diag} \hat{\mu} || + ||\hat{\Gamma}(1) - \text{diag} \hat{\mu} || \right\}
\]

\[
- \eta dd' - \eta d' - \eta \text{diag}(Id - \eta C)^{-1} dd' - \eta dd'(Id - \eta C)^{-1} C' || + ||\hat{\Gamma}(1) - \text{diag} \hat{\mu} ||,
\]

where \(\hat{\mu}, \hat{\Gamma}(0), \hat{\Gamma}(1)\) are the size adjusted sample counterparts of \(E(Y_t^*), V(Y_t^*), Cov(Y_t^*, Y_{t-1}^*)\), respectively.

The starting value for the algorithm used to minimize the calibration criterion are the estimates obtained in the model without frailty for \(a\) and \(C\) [see Section 4.2], \(b_0 = 0, \eta_0 = 0\).
We give in Table 6, the estimated coefficients \( \hat{d} = (\sigma \hat{b}), \hat{C}, \hat{\eta} \), with their standard errors. By considering the significance of coefficients \( d_k \), we are able to distinguish the management styles, which are not sensitive directly to an exogenous frailty.

However, they can be affected indirectly by means of endogenous contagion. We provide below the decomposition of the first-order autoregressive coefficient of the count variables to see the magnitude of both effects.

[to be completed]

5 Concluding remarks
References


Appendix 1

Characteristics of funds in the database

The Live database

The TASS base Farc1 includes a total of 6097 funds. It is filtered by currency code USD, in particular to avoid double count, since a same fund can have shares written in USD and EUR for example. It remains 3343 funds.

By eliminating the funds of funds and the small management styles, we get 2382 funds in the Live database. Their distribution by style is provided in the two first columns in Table 1. Among the funds in the “Live” database, 23 are considered as dead with our filter.

The Graveyard database

It contains 6767 funds, and only 5033 after applying the currency code filtering. Among them and excluding the funds of funds and the small management styles, 4024 are considered as dead. Their distribution by style is given in columns 3-4 of Table 1.
<table>
<thead>
<tr>
<th>Category</th>
<th>Farc1 (live)</th>
<th>(%)</th>
<th>Farc3 (dead)</th>
<th>(%)</th>
<th>Total</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONV Convertible Arbitrage</td>
<td>47</td>
<td>2,00%</td>
<td>170</td>
<td>4,20%</td>
<td>217</td>
<td>3,40%</td>
</tr>
<tr>
<td>EM Emerging Markets</td>
<td>235</td>
<td>9,90%</td>
<td>266</td>
<td>6,60%</td>
<td>501</td>
<td>7,80%</td>
</tr>
<tr>
<td>EMN Equity Market Neutral</td>
<td>132</td>
<td>5,50%</td>
<td>299</td>
<td>7,40%</td>
<td>431</td>
<td>6,70%</td>
</tr>
<tr>
<td>ED Event Driven</td>
<td>224</td>
<td>9,40%</td>
<td>434</td>
<td>10,80%</td>
<td>658</td>
<td>10,30%</td>
</tr>
<tr>
<td>FI Fixed Income Arbitrage</td>
<td>102</td>
<td>4,30%</td>
<td>239</td>
<td>5,90%</td>
<td>341</td>
<td>5,30%</td>
</tr>
<tr>
<td>GM Global Macro</td>
<td>181</td>
<td>7,60%</td>
<td>277</td>
<td>6,90%</td>
<td>458</td>
<td>7,10%</td>
</tr>
<tr>
<td>LSE Long/Short Equity Hedge</td>
<td>922</td>
<td>38,70%</td>
<td>1543</td>
<td>38,30%</td>
<td>2465</td>
<td>38,90%</td>
</tr>
<tr>
<td>MF Managed Futures</td>
<td>231</td>
<td>9,70%</td>
<td>454</td>
<td>11,30%</td>
<td>685</td>
<td>10,70%</td>
</tr>
<tr>
<td>MS Multi-Strategy</td>
<td>308</td>
<td>12,90%</td>
<td>342</td>
<td>8,50%</td>
<td>650</td>
<td>10,10%</td>
</tr>
</tbody>
</table>

**Total** 2382 100,00% 4024 100,00% 6406 100,00%

Table 1: The final database
Appendix 2

The autoregressive gamma process

Let us review the main properties of the ARG(1) process used in the paper [see Gourieroux, Jasiak (2006)].

i) The conditional distribution

An ARG(1) process \((F_t)\) is a Markov process with conditional distribution the noncentered gamma distribution \(\gamma(\delta, \eta F_{t-1}, \nu)\), where \(\delta, \delta > 0\), is the degree of freedom, \(\eta F_{t-1}, \eta > 0\), the noncentrality parameter and \(\gamma, \nu > 0\), a scale parameter.

Its first- and second-order conditional moments are:

\[
E_{t-1}(F_t) = \delta \nu + \eta \nu F_{t-1}, V_{t-1}(F_t) = \nu^2 \delta + 2 \eta \nu^2 F_{t-1}.
\]

ii) The state space representation

An ARG(1) process admits a state space representation, which is especially convenient for simulating the trajectories of the process. To get a simulated value of \(F_t^*\) given \(F_{t-1}\) and the parameters:

(*) we draw an intermediate value \(Z_t^*\) in a Poisson distribution \(\mathcal{P}(\eta F_{t-1})\);

(*) Then, \(F_t^*\) is drawn in the gamma distribution \(\gamma(\delta + Z_t^*, \nu)\).

iii) Stationarity condition and stationary distribution

The ARG(1) process is stationary if \(\rho = \nu \eta < 1\). Then the stationary distribution is a centered gamma distribution \(\gamma(\delta, 0, \frac{\nu}{1 - \nu \eta})\).

In particular, we get:

\[
E(F_t) = \frac{\nu \delta}{1 - \nu \eta}, V(F_t) = \delta \left(\frac{\nu}{1 - \nu \eta}\right)^2.
\]
Appendix 3

First and Second-Order Moments

Let us consider the Poisson model (3.4)-(3.5) with mortality intensity independent of age and an ARG frailty process.

i) Moments of order 1

We have:

\[ E_{t-1}(Y^*_t) = E_{t-1}E_{t-1}(Y^*_t \mid F_t) \]

\[ = E_{t-1}(a + bF_t + CY^*_t) \]

\[ = a + bE_{t-1}F_t + CY^*_t. \]

By taking expectation of both sides of the equation, and under the stationarity of \((Y^*_t)\), we get:

\[ E(Y^*_t) = a + b + CE(Y^*_t) \]

\[ \Leftrightarrow E(Y^*_t) = (Id - C)^{-1}(a + b). \quad (a.1) \]

ii) Moments of order 2

We have:

\[ E_{t-1}(F_tY^*_t) \]

\[ = E_{t-1}[E_{t-1}(F_tY^*_t \mid F_t)] \]

\[ = E_{t-1}[F_t(a + bF_t + CY^*_t)] \]

\[ = aE_{t-1}(F_t) + bE_{t-1}(F_t^2) + CE_{t-1}(F_t)Y^*_t \]

\[ = aE_{t-1}(F_t) + bE_{t-1}(F_t^2) + C(1 - \rho + \rho F_{t-1})Y^*_t. \]

34
By taking the expectation of both sides of the equation, we get:

\[ E(F_i Y_t^*) = a + b(1 + \sigma^2) + (1 - \rho)C(\text{Id} - C)^{-1}(a + b) + \rho CE(F_i Y_t^*). \]

We deduce that:

\[ E(F_i Y_t^*) = (\text{Id} - \rho C)^{-1}\{ba^2 + [\text{Id} + (1 - \rho)C(\text{Id} - C)^{-1}](a + b)\}, \]

\[ E(F_i Y_t^*) = (\text{Id} - \rho C)^{-1}b\sigma^2 + (\text{Id} - C)^{-1}(a + b). \]

In particular,

\[ \text{Cov}(Y_t^*, F_i) = \sigma^2(\text{Id} - \rho C)^{-1}b. \] (a.2)

Let us now consider the variance-covariance matrix of \( Y_t^* \). We have:

\[ E_{t-1}(Y_t^* Y_t^{*'}) \]

\[ = E_{t-1}(E_{t-1}[Y_t^* Y_t^{*'}|F_t]) \]

\[ = E_{t-1}[V_{t-1}(Y_t^*|F_t) + E_{t-1}(Y_t^*|F_t)E_{t-1}(Y_t^*|F_t)'] \]

\[ = E_{t-1}\{\text{diag}(a + bF_t + CY_t^{* -1})\} + E_{t-1}\{(a + bF_t + CY_t^{* -1})(a + bF_t + CY_t^{* -1})'\} \]

\[ = \text{diag}[a + bE_{t-1}(F_t) + CY_t^{* -1}] + bb'V_{t-1}(F_t) \]

\[ + E_{t-1}[a + bF_t + CY_{t-1}^*][E_{t-1}(a + bF_t + CY_{t-1}^*)]' \]

\[ = \text{diag}[a + bE_{t-1}(F_t) + CY_t^{* -1}] + bb'V_{t-1}(F_t) \]

\[ + [a + b((1 - \rho + \rho F_{t-1}) + CY_{t-1}^*][a + b(1 - \rho + \rho F_{t-1}) + CY_{t-1}^*]' \].

By taking the expectation of both sides, we deduce:
\[
E(Y^*_t Y^*_t) = \text{diag}[a + b + C(Id - C)^{-1}(a + b)] + bb'E[V_{t-1}(F_t)]
\]

\[
+ V[b\eta F_{t-1} + CY^*_{t-1}] + E[a + b(1 - \rho + \rho F_{t-1}) + CE(Y^*_{t-1})]
\]

\[
E[a + b[1 - \rho + \rho F_{t-1}] + CE(Y^*_{t-1})]^t
\]

\[
= \text{diag}[(Id - C)^{-1}(a + b)] + \sigma^2bb' + V[b\rho F_{t-1} + CY^*_{t-1}] + E(Y^*_t Y^*_t).
\]

Therefore, the variance-covariance matrix of \(Y^*_t\) satisfies the recursive equation:

\[
V(Y^*_t) = CV(Y^*_t)C^t + \text{diag}[(Id - C)^{-1}(a + b)] + \sigma^2bb' + \rho b\text{Cov}(F_t, Y^*_t)C^t + \rho CCov(Y^*_t, F_t)b^t.
\] (a.3)

The result is obtained by substituting the expression (a.2) of \(\text{Cov}(Y^*_t, F_t)\).

**iii) Autocovariance at order 1**

We have:

\[
\text{Cov}(Y^*_t, Y^*_t)
\]

\[
= \text{Cov}[E_{t-1}(Y^*_t), Y^*_t]
\]

\[
= \text{Cov}[E_{t-1}(a + b F_t + CY^*_{t-1}), Y^*_t]
\]

\[
= \text{Cov}(a + b[1 - \rho + \rho F_{t-1}] + CY^*_{t-1}, Y^*_t)
\]

Therefore:

\[
\text{Cov}(Y^*_t, Y^*_t) = \text{Cov}(b\rho F_{t-1} + CY^*_{t-1}, Y^*_t)
\]

\[
= b\rho \text{Cov}(F_{t-1}, Y^*_{t-1}) + CV(Y^*_t)
\]

\[
= CV(Y^*_t) + \sigma^2\rho bb'(Id - \rho C')^{-1}.
\] (a.4)
Figure 1: No Frailty and No Contagion Effect

Frailty with $\eta = 0$, $\delta = 2$, $\nu = 0.5$

Mortality Count Type 1 with $a_1 = 1$, $b_1 = 0$, $c_{1,1} = 0$, $c_{1,2} = 0$

Mortality Count Type 2 with $a_2 = 1$, $b_2 = 0$, $c_{2,1} = 0$, $c_{2,2} = 0$
Figure 2: Only Frailty with Uncorrelated Factor Effect

Frailty with $\eta = 0$, $\delta = 2$, $\nu = 0.5$

Mortality Count Type 1 with $a_1 = 1$, $b_1 = 1$, $c_{1,1} = 0$, $c_{1,2} = 0$

Mortality count Type 2 with $a_2 = 1$, $b_2 = 1$, $c_{2,1} = 0$, $c_{2,2} = 0$
Figure 3: Only Frailty with Correlated Factor Effect

Frailty with $\eta = 1$, $\delta = 2$, $\nu = 0.3$

Mortality Count Type 1 with $a_1 = 1$, $b_1 = 1$, $c_{1,1} = 0$, $c_{1,2} = 0$

Mortality Count Type 2 with $a_2 = 1$, $b_2 = 1$, $c_{2,1} = 0$, $c_{2,2} = 0$
Figure 4: Frailty with Correlated Factor and Direct Contagion Effects

Frailty with $\eta = 1$, $\delta = 2$, $\nu = 0.3$

Mortality Count Type 1 with $a_1 = 1$, $b_1 = 1$, $c_{1,1} = 0.5$, $c_{1,2} = 0$

Mortality count Type 2 with $a_2 = 1$, $b_2 = 1$, $c_{2,1} = 0$, $c_{2,2} = 0.5$
Figure 5: Frailty with Correlated Factor and Recursive Contagion Effects

Frailty with $\eta = 1$, $\delta = 2$, $\nu = 0.3$

Mortality Count Type 1 with $a_1 = 1$, $b_1 = 1$, $c_{1,1} = 0$, $c_{1,2} = 0$

Mortality count Type 2 with $a_2 = 1$, $b_2 = 1$, $c_{2,1} = 0.2$, $c_{2,2} = 0$
Figure 6: Conditional Overdispersion

Conditional dispersion with Frailty and Contagion
Figure 7: Subpopulation Sizes of HF
Figure 8: Frequency Counts of Defunct HF
Figure 9: Smoothed Estimator of Mortality Intensity

- **Convertible Arbitrage**
- **Emerging Markets**
- **Equity Market Neutral**
- **Event Driven**
- **Fixed Income Arbitrage**
- **Global Macro**
- **Long/Short Equity Hedge**
- **Managed Futures**
- **Multi-Strategy**
Figure 10: Lexis Diagram
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Table 3: Product description suite
Figure 19: The Contagion scheme
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Table 4: Statistics
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Table 5: Maximum and Boundary Values
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</tr>
<tr>
<td>Global Macro</td>
<td>(2.58)</td>
<td>(2.50)</td>
<td>(4.37)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long/Short Equity Hedge</td>
<td>0.61</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.29</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Managed Futures</td>
<td>(2.79)</td>
<td>(2.55)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multi-Strategy</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.21</td>
<td>0.00</td>
<td>0.00</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 7: $c_{k,k'}$ contagion parameters

\[
\begin{align*}
0.77 \\
-0.29 \\
0.11 + i 0.24 \\
0.11 - i 0.24 \\
0.3 + i 0.10 \\
0.3 - i 0.10 \\
-0.06 + i 0.10 \\
-0.06 - i 0.10 \\
0.12
\end{align*}
\]

Table 8: Eigenvalues and Eigenvectors of the Contagion Matrix